APPENDIX I
CORRESPONDENCES

Centre of Advanced Study in Education
Department of Education, Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat - 390 002.

To,
_(The Expert)_____
_____

Date: / /2018

Sub.: Need Assessment Questionnaire for Professional Enrichment of Mathematics Teachers (NAQPEMT) for Expert Validity.

Dear Sir / Madam,

With pleasure, I inform you that I have taken up a research study titled "**Development of Professional Enrichment Programme for the Secondary School Mathematics Teachers**" as a student of the Ph.D. Program at the Centre of Advanced Studies in Education (CASE), Department of Education, Faculty of Education and Psychology, The M. S. University of Baroda, Vadodara.

This **Need Assessment Questionnaire for Professional Enrichment of Mathematics Teachers (NAQPEMT)** has been designed to identify the areas of professional enrichment needs of Secondary Schools' Mathematics Teachers, based on which Professional Enrichment Programme modules will be developed.

You are requested to go through each item and statement of the questionnaire and suggest the relevance and clarity of the questions within the context of the research. You are requested to suggest the mining question, if any in the questionnaire.

Thanking you,
Yours sincerely,

Mrs. A. V. Beena
Research Scholar

Prof. K. Pushpanadham
Guide
Dept. of Educational Administration

Centre of Advanced Study in Education
Department of Education, Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat - 390 002.

To,
The Principal,
_____

Date: 10/04/2018

Sub.: Request for permission to administer a Need Assessment Questionnaire for Professional Enrichment of Mathematics Teachers (NAQPEMT)

Respected Sir / Madam,

I, Mrs. A. V. Beena, am a student of Doctoral Studies (Ph.D. Programme) at the Centre of Advanced Study in Education (CASE), Department of Education, Faculty of Education and Psychology, The Maharaja Sayajirao University of Baroda, Vadodara, in the area of development of a Professional Enrichment Programme for the Secondary School Mathematics Teachers.

I would like to inform you that Department of Education, Faculty of Education and Psychology, The M. S. University, Baroda, Vadodara, has been working in the area of development of teachers and providing academic support to several organizations.

Your esteemed institute has been chosen by me in carrying forward a Need Assessment study to identify the training needs (Professional Enrichment Programme needs) of Secondary School Mathematics Teachers. I would like to request you to grand me the permission and provide the administrative support for conducting this research.

Thanking you in anticipation,
Yours sincerely,

Mrs. A. V. Beena
Research Scholar

Forwarded through:
Prof. K. Pushpanadham
Guide
Dept. of Educational Administration

Centre of Advanced Study in Education
Department of Education, Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat - 390 002.

To,

Date: 10/04/2018

_____(Mathematics Teacher)
Secondary Section, _____School

Sub.: Professional Enrichment Need Assessment of Secondary School Mathematics Teachers

Dear Sir / Madam,

With pleasure, I inform you that I have taken up a research study titled '**Development of Professional Enrichment Programme for the Secondary School Mathematics Teachers**' as a student of the Ph.D. Program at the Centre of Advanced Studies in Education (CASE), Department of Education, Faculty of Education and Psychology, The M.S. University of Baroda, Vadodara.

This **Need Assessment Questionnaire for Professional Enrichment of Mathematics Teachers (NAQPEMT)** is to identify the professional enrichment needs of Secondary Schools Mathematics Teachers, based on which a Professional Enrichment Programme module will be developed and implemented in secondary schools.

While responding to the Questionnaire, feel free and be assured that your responses would be kept confidential and will be used only for research purpose. Appreciate your co-operation, in this regard.

Thanking you,
Yours sincerely,

Mrs. A. V. Beena
Research Scholar

Prof. K. Pushpanadham
Guide
Dept. of Educational Administration

Centre of Advanced Study in Education
Department of Education, Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat - 390 002.

To,
Dr. V. D. Pathak
Rtd. Professor, Dept. of Mathematics,
The M. S. University of Baroda, Vadodara.

Date: 28/12/2022

Sub.: Request for Validation of Modules

Respected Sir,

I am pleased to inform you that, my Research Study titled '**Development of Professional Enrichment Programme for the Secondary Schools Mathematics Teachers'** under the guidance of Prof. K. Pushpanadham at the Centre of Advanced Studies in Education (CASE), Faculty of Education and Psychology, The Maharaja Sayajirao University of Baroda, Vadodara, is at its final stage and as a part of it, I have developed 4 modules on 4 identified topics for the Professional Enrichment of Mathematics Teachers of Secondary Section.

I hereby request you to kindly find some time to go through the modules and give your valuable inputs on it. The developed modules are submitted to your good self for your expert validation on: **the clarity and quality of the content, language flow and its organization.**

Your inputs would be highly appreciated in order to improve the quality of these modules.

Yours sincerely,

A. V. Beena
Research Scholar

Prof. K. Pushpanadham
Guide
Dept. of Educational Administration

APPENDIX II

NAQPEMT

Centre of Advanced Study in Education
Department of Education, Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat - 390 002.

Need Assessment Questionnaire for Professional Enrichment of Mathematics Teachers (NAQPEMT)

(To be filed in by Mathematics Teachers of Secondary Section of the School)

Introduction:

This Need Assessment Questionnaire for Professional Enrichment of Mathematics Teachers (NAQPEMT) comprises the questions pertaining to the information regarding your academic and professional background, instructional practices and attitudes towards teaching Mathematics. Also it addresses the Professional Enrichment Needs of Secondary School Mathematics Teachers. Your ratings will help to determine the most important training needs of Secondary School Mathematics Teachers based on which suitable training modules for a Professional Enrichment Programme will be developed.

General Instructions & Guidelines:

- Fill in this Questionnaire carefully.
- Your answers are highly significant as they will determine the nature and content of the Professional Enrichment Program for Secondary School Mathematics Teachers.
- Your response to the questions would be kept confidential and will be used only for the research purpose.

Expecting and appreciating your co-operation in completing this questionnaire.

Mrs. A. V. Beena
Research Scholar

Prof. K. Pushpanadham
Guide
Dept. of Educational Administration

Centre of Advanced Study in Education
Department of Education, Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat - 390 002.

Background Information:

- Your Educational Qualification/s : ☐ B.Sc. ☐ M.Sc. ☐ B.Ed. ☐ M.Ed. ☐ Ph.D. ☐ Other _____
- Subjects studied : At Graduation : _____

  At Post Graduation : _____
- Gender : ☐ Male ☐ Female
- Medium of instruction during your schooling (till class 12) : ☐ English ☐ Hindi ☐ Gujarati ☐ Any other _____
- Type of the School where you are teaching at present? : ☐ Govt. ☐ Govt. Aided ☐ Private ☐ Other
- School is affiliated to : ☐ State Board ☐ CBSE ☐ ICSE ☐ Other
- Number of years of experience as Secondary School Mathematics Teacher in the present School : _____ Total Experience: _____
- Classes in which you are teaching : Class : IX / X / Both
- Number of students in each class : _____
- Have you undergone / completed any Professional Development Program in the past : ☐ Yes / ☐ No

  (If "Yes", please indicate the name of such program)

  Program : _____

  Conducted by : _____
- Do you have a teacher training certificate? (Apart from B.Ed./M.Ed.) ☐ Yes / ☐ No

  If "Yes", mention the Name : _____
- Are you a techno savvy person? ☐ Yes / ☐ No

  If "Yes", (1) Mention your proficiency : (Word / Excel / Power Point)
  (2) Are you able to integrate technology in teaching – learning ☐ Yes / ☐ No
- Do you believe in the networking of teachers to share the expertise and experience? ☐ Yes / ☐ No

- Whether your school Mathematics department has a head? ☐ Yes / ☐ No

1) If Yes, what is the role of the Head of the Department (HOD)?

_____

_____

_____

2) How often you have the departmental meeting? (Weekly / Monthly / Quarterly / Half Yearly)

- Whether your school promotes the peer observation of classes ☐ Yes / ☐ No
- Are you wanting to be a part of the Professional Enrichment Program which will be offered to you? ☐ Yes / ☐ No

I. Mathematics as a Subject:

To what extent do you agree or disagree with the following statements?

(Check one box in each row)

		Strongly Disagree	*Disagree*	*Neither Agree nor Disagree*	*Agree*	*Strongly Agree*
1)	Mathematics is primarily an abstract subject	☐	☐	☐	☐	☐
2)	Learning Mathematics helps students in facing the real-life situations and solving problems.	☐	☐	☐	☐	☐
3)	Some students have a natural talent for Mathematics.	☐	☐	☐	☐	☐
4)	Teachers should adopt the same approach to address the different learning difficulties of different students in Mathematics subject.	☐	☐	☐	☐	☐
5)	Teachers are required to use appropriate teaching aids for teaching different Mathematics topics.	☐	☐	☐	☐	☐
6)	Mathematics teachers need to have skills in					
	• Drawing	☐	☐	☐	☐	☐
	• Logical thinking	☐	☐	☐	☐	☐
	• Comprehension	☐	☐	☐	☐	☐
	• Basic computational skills	☐	☐	☐	☐	☐
	• Articulation	☐	☐	☐	☐	☐
	• Analytical approach to problem solving	☐	☐	☐	☐	☐
7)	A liking for and understanding of students are essential for teaching Mathematics.	☐	☐	☐	☐	☐
8)	It is not necessary for students to communicate their Mathematics ideas.	☐	☐	☐	☐	☐
9)	Technology distracts students from learning basic skills.	☐	☐	☐	☐	☐
10)	Understanding when, how and which a Mathematical technique to be used is important rather than just memorizing formulae and technique from memory.	☐	☐	☐	☐	☐
11)	Teacher should often remind students that a law of Mathematics may not be fun or interesting but it's important to learn it anyway.	☐	☐	☐	☐	☐

12)	While communicating with parents and students about performance, I tend to focus on student weaknesses instead of strengths.	☐	☐	☐	☐	☐
13)	The existing curriculum focuses more on learning procedures than increasing the students' understanding and reasoning capacity.	☐	☐	☐	☐	☐
14)	Mathematics is all about solving problems given in exercises of the text books.	☐	☐	☐	☐	☐

II. Pedagogy and Methodology Related:
In my Mathematics class, my students should: (Q. 1 – 14)

(Check one box in each row)

		Strongly Disagree	*Disagree*	*Neither Agree nor Disagree*	*Agree*	*Strongly Agree*
1)	Explain the reasoning behind an idea	☐	☐	☐	☐	☐
2)	Represent and analyze relationships using tables, charts or graphs	☐	☐	☐	☐	☐
3)	Work on problems for which there is no immediate and obvious solution	☐	☐	☐	☐	☐
4)	Use computers to solve exercises or problems	☐	☐	☐	☐	☐
5)	Write equations to represent relationships	☐	☐	☐	☐	☐
6)	Practice computational skills	☐	☐	☐	☐	☐
7)	Use calculators to solve exercises or problems	☐	☐	☐	☐	☐
8)	Link formal Mathematics with experimental learning.	☐	☐	☐	☐	☐
9)	Work individually without assistance from the teacher	☐	☐	☐	☐	☐
10)	Work individually with assistance from the teacher	☐	☐	☐	☐	☐
11)	Work together as a class with the teacher teaching the whole class	☐	☐	☐	☐	☐
12)	Work in pairs or small groups with assistance from the teacher.	☐	☐	☐	☐	☐
13)	In my school we have the practices of making					
	• Daily lesson plan	☐	☐	☐	☐	☐
	• Weekly lesson plan	☐	☐	☐	☐	☐
	• Monthly lesson plan	☐	☐	☐	☐	☐
	• Yearly lesson plan	☐	☐	☐	☐	☐
14)	My daily lesson plan never helps in achieving my targets in terms of syllabus completion.	☐	☐	☐	☐	☐
15)	I have the practice of using my own prepared teaching aids / models in classrooms.	☐	☐	☐	☐	☐
16)	I am confident of my content. So, I do not prepare for my class.	☐	☐	☐	☐	☐

17) I do not link the new topic to be introduced in the class with the previous knowledge, the students have.	☐	☐	☐	☐	☐
18) I introduce the topics by connecting the same with real life situation.	☐	☐	☐	☐	☐
19) I spend some time every day in my class to increase the basic computational skills of my learners.	☐	☐	☐	☐	☐
20) I am not able to reach every single learner of my class while delivering the concepts in terms of their understanding.	☐	☐	☐	☐	☐
21) Intelligent students of the class disturb me.	☐	☐	☐	☐	☐
22) I emphasis on 'Visualization and Representation' of the facts and topics	☐	☐	☐	☐	☐
23) I do not get adequate time to implement innovative teaching strategies.	☐	☐	☐	☐	☐
24) My main concern in my class is to complete the syllabus.	☐	☐	☐	☐	☐
25) Towards the end of every class, I do recapitalization of the important concepts.	☐	☐	☐	☐	☐
26) One should not assign Mathematics problems that can be solved in different ways, as that may confuse the students.	☐	☐	☐	☐	☐
27) It's not advisable for students to work together during Mathematics problem solving.	☐	☐	☐	☐	☐
28) When students solve the same problem using different strategies, teacher should ask them to share the same in class.	☐	☐	☐	☐	☐
29) I often learn from my students during Mathematics class because they come up with indigenous ways of solving problems that I have never thought of.	☐	☐	☐	☐	☐
30) I do not assign real-life Mathematics problems that are of interest to students.	☐	☐	☐	☐	☐
31) Teacher should make students feel that everybody can do Mathematics.	☐	☐	☐	☐	☐
32) I do not believe in communicating with my students' parents about student strength / weakness in learning Mathematics.	☐	☐	☐	☐	☐

33) I always focus on 'Mathematization' of students thought process in my Mathematics class.	☐	☐	☐	☐	☐
34) I practice 'activity – oriented' teaching in my class.	☐	☐	☐	☐	☐
35) When students are working on problems, I put more emphasis on getting the correct answer rather than on the process followed.	☐	☐	☐	☐	☐
36) I ask my students to master basic operations before they enter into complex problems.	☐	☐	☐	☐	☐
37) I refer various books (for content)	☐	☐	☐	☐	☐
38) I make students to practice from other reference books apart from Text book exercises.	☐	☐	☐	☐	☐
39) My classroom management skills are					
• Excellent	☐	☐	☐	☐	☐
• Good	☐	☐	☐	☐	☐
• Average	☐	☐	☐	☐	☐
• Need Improvement	☐	☐	☐	☐	☐
40) I blame the students for not having adequate basic knowledge for understanding a new concept.	☐	☐	☐	☐	☐
41) After teaching a concept,					
• I solve every problem given in exercise for student.	☐	☐	☐	☐	☐
• I ask students to solve the problems in the class	☐	☐	☐	☐	☐
• I solve few questions and rest I assign students to do at home.	☐	☐	☐	☐	☐

III. Assessment Practices:

(Check one box in each row)

	Strongly Disagree	*Disagree*	*Neither Agree nor Disagree*	*Agree*	*Strongly Agree*
1) I follow the Assessment patterns strictly as mentioned by CBSE.	☐	☐	☐	☐	☐
2) Apart from the Periodic Tests mentioned by CBSE, I take class tests after each lesson.	☐	☐	☐	☐	☐
3) I conduct diagnostic tests.	☐	☐	☐	☐	☐
4) I conduct remedial classes for the needy learners	☐	☐	☐	☐	☐
5) I find improvement in learners after providing the remedial classes.	☐	☐	☐	☐	☐
6) I change my teaching style after seeing the students' performance score to make it more impactful.	☐	☐	☐	☐	☐
7) I think the multiple modes of assessment, rather than the unique test pattern need to be encouraged	☐	☐	☐	☐	☐
8) Through my assessment pattern, I focus more on					
• Evaluating conceptual understanding	☐	☐	☐	☐	☐
• Evaluating the fast computational ability	☐	☐	☐	☐	☐
9) I follow the blue print, when I prepare Question Paper.	☐	☐	☐	☐	☐
10) I select items in Question Papers only from Text Books.	☐	☐	☐	☐	☐
11) The existing assessment practices really measure students' learning.	☐	☐	☐	☐	☐

12) Explain if any other practice of Assessment in your school:

_____

_____

_____

_____

IV. Technology Integration Practices:

(Check one box in each row)

	Strongly Disagree	*Disagree*	*Neither Agree nor Disagree*	*Agree*	*Strongly Agree*
1) I use the following technology integrated teaching – learning facilities in class rooms.					
• Computers with internet connection	☐	☐	☐	☐	☐
• Digital classrooms	☐	☐	☐	☐	☐
• E-learning facility	☐	☐	☐	☐	☐
2) I use the technology for Mathematics teaching – learning.					
• Every period	☐	☐	☐	☐	☐
• Once or Twice in a week	☐	☐	☐	☐	☐
• Once in a month	☐	☐	☐	☐	☐
• As per the demand of lessons / topics.	☐	☐	☐	☐	☐
3) My school has technology aided Mathematics laboratory.	☐	☐	☐	☐	☐
4) I use technology integrated Mathematics lab:					
• Every period	☐	☐	☐	☐	☐
• Once or Twice in a week	☐	☐	☐	☐	☐
• Once in a month	☐	☐	☐	☐	☐
• As per the demand of lessons / topics.	☐	☐	☐	☐	☐
5) I prepare my lecture notes using power point presentation and supplement it with detailed explanation on the board.	☐	☐	☐	☐	☐
6) I use Mathematical software like Mathematica, MATLAB, etc., specially developed for better understanding / visualization of the Mathematical concepts.	☐	☐	☐	☐	☐
7) I make students aware about the available teaching aids on the web and use such aids some time in my class.	☐	☐	☐	☐	☐
8) I find technology integrated learning more effective than normal teaching – learning practice	☐	☐	☐	☐	☐

V. Apart from content and pedagogy related areas which other area you wish the Professional Enrichment Program should focus on?

(Check one box in each row)

		Strongly Disagree	*Disagree*	*Neither Agree nor Disagree*	*Agree*	*Strongly Agree*
1)	Lesson planning	☐	☐	☐	☐	☐
2)	General teaching methods (e.g., cooperative learning or classroom management techniques)	☐	☐	☐	☐	☐
3)	Approaches to students' assessment and alternative assessment pattern.	☐	☐	☐	☐	☐
4)	Use of technology in instruction (e.g., computers, calculators)	☐	☐	☐	☐	☐
5)	Innovative technique / methods of teaching learning	☐	☐	☐	☐	☐
6)	Information on how students learn Mathematics effectively	☐	☐	☐	☐	☐
7)	Deepening your knowledge of Mathematics	☐	☐	☐	☐	☐
8)	Leadership development	☐	☐	☐	☐	☐
9)	Information on variety of Mathematical resources.	☐	☐	☐	☐	☐

10) Other: (Please be specify)

_____

_____

_____

_____

_____

_____

VI. Indicate your familiarity with each of the following:

(Check one box in each row)

		No such Document	*Not Familiar at all*	*Not much Familiar*	*Somewhat Familiar*	*Very Familiar*
1)	State Education Department Curriculum Guide / Manual.	☐	☐	☐	☐	☐
2)	State Education Department Assessment Specifications.	☐	☐	☐	☐	☐
3)	Central Board of Secondary Education (CBSE) Curriculum Guide / Manual.	☐	☐	☐	☐	☐
4)	CBSE Assessment pattern.	☐	☐	☐	☐	☐
5)	CBSE Manual on Expected Learning outcomes of teaching Mathematics at Secondary level.	☐	☐	☐	☐	☐
6)	'Position Paper' on Teaching of Mathematics by National Focus Group, NCERT.	☐	☐	☐	☐	☐
7)	NCF (National Curriculum Framework), 2005- Recommendations on Teaching Mathematics.	☐	☐	☐	☐	☐
8)	Different modules / guidelines on effective teaching of Mathematics.	☐	☐	☐	☐	☐
9)	National Council of Teachers of Mathematics (NCTM) Professional Standards for Teaching Mathematics.	☐	☐	☐	☐	☐
10)	The National Assessment for Educational Progress (NAEP) Assessment Frameworks / Specifications.	☐	☐	☐	☐	☐

VII. Content Related :

1. How much proficiency you have to teach the below topics for any class up to X:

(Check one box in each row)

		I do not teach these topics	*Needs Preparation*	*Not well confident*	*Somewhat confident*	*Very confident*
1)	Fractions, Decimals and Percentages	☐	☐	☐	☐	☐
2)	Ratios and Proportions	☐	☐	☐	☐	☐
3)	Number Systems	☐	☐	☐	☐	☐
4)	Measurement – Units, Instruments and Accuracy	☐	☐	☐	☐	☐
5)	Perimeter, Area and Volume	☐	☐	☐	☐	☐
6)	Geometric Figures – Definitions and Properties	☐	☐	☐	☐	☐
7)	Geometric Figures – Symmetry, Motions and Transformations, Congruence and Similarity	☐	☐	☐	☐	☐
8)	Coordinate Geometry	☐	☐	☐	☐	☐
9)	Algebraic Representation	☐	☐	☐	☐	☐
10)	Evaluate and Perform Operations on Algebraic Expressions	☐	☐	☐	☐	☐
11)	Solving Linear Equations, Inequalities and Quadratic Equations	☐	☐	☐	☐	☐
12)	Representation and Interpretation of Data in Graphs, Charts and Tables	☐	☐	☐	☐	☐
13)	Simple Probabilities – Understanding and Calculations	☐	☐	☐	☐	☐
14)	Introduction to Trigonometry	☐	☐	☐	☐	☐
15)	Theorems and Proofs	☐	☐	☐	☐	☐

2. Grade the following topics according to the difficulty level you face while teaching those topics: (Wherever Applicable)

The scale of rating is listed below:

VD (5)	= Very difficult (PEP Needed)
D (4)	= Difficult (PEP Desired)
ND/NE (3)	= Not so difficult and not so easy (Not sure about the need of PEP)
E (2)	= Easy (PEP not required)
VE (1)	= Very easy (PEP definitely not required)

Sr. No.	Area of Teaching / Topics		VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
	CLASS: IX						
01.	**NUMBER SYSTEMS**						
	1.	Irrational Numbers	☐	☐	☐	☐	☐
	2.	Real Numbers and their Decimal Expansions	☐	☐	☐	☐	☐
	3.	Representing Real Numbers on the Number Line	☐	☐	☐	☐	☐
	4.	Operations on Real Numbers	☐	☐	☐	☐	☐
	5.	Laws of Exponents for Real Numbers	☐	☐	☐	☐	☐
02.	**POLYNOMIALS**						
	1.	Polynomials in One Variable	☐	☐	☐	☐	☐
	2.	Zeroes of a Polynomial	☐	☐	☐	☐	☐
	3.	Remainder Theorem	☐	☐	☐	☐	☐
	4.	Factorization of Polynomials	☐	☐	☐	☐	☐
	5.	Algebraic Identities	☐	☐	☐	☐	☐
03.	**COORDINATE GEOMETRY**						
	1.	Cartesian System	☐	☐	☐	☐	☐
	2.	Plotting a Point in the Plane if its Coordinates are given.	☐	☐	☐	☐	☐
04.	**LINEAR EQUATIONS IN TWO VARIABLES**						
	1.	Linear Equations	☐	☐	☐	☐	☐
	2.	Solution of a Linear Equation	☐	☐	☐	☐	☐
	3.	Graph of a Linear Equation in Two Variables	☐	☐	☐	☐	☐
	4.	Equations of Lines Parallel to x-axis and y-axis	☐	☐	☐	☐	☐

Sr. No.	Area of Teaching / Topics	VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
05.	**INTRODUCTION TO EUCLID'S GEOMETRY**					
	1. Euclid's Definitions, Axioms and Postulates	☐	☐	☐	☐	☐
	2. Equivalent Versions of Euclid's Fifth Postulate	☐	☐	☐	☐	☐
06.	**LINES AND ANGLES**					
	1. Basic Terms and Definitions	☐	☐	☐	☐	☐
	2. Intersecting Lines and Non-intersecting Lines	☐	☐	☐	☐	☐
	3. Pairs of Angles	☐	☐	☐	☐	☐
	4. Parallel Lines and a Transversal	☐	☐	☐	☐	☐
	5. Lines Parallel to the same Line	☐	☐	☐	☐	☐
	6. Angle Sum Property of a Triangle	☐	☐	☐	☐	☐
07.	**TRIANGLES**					
	1. Congruence of Triangles	☐	☐	☐	☐	☐
	2. Criteria for Congruence of Triangles	☐	☐	☐	☐	☐
	3. Some Properties of a Triangle	☐	☐	☐	☐	☐
	4. Some More Criteria for Congruence of Triangles	☐	☐	☐	☐	☐
	5. Inequalities in a Triangle	☐	☐	☐	☐	☐
08.	**QUADRILATERALS**					
	1. Angle Sum Property of a Quadrilateral	☐	☐	☐	☐	☐
	2. Types of Quadrilaterals	☐	☐	☐	☐	☐
	3. Properties of a Parallelogram	☐	☐	☐	☐	☐
	4. Another Condition for a Quadrilateral to be a Parallelogram	☐	☐	☐	☐	☐
	5. The Mid-Point Theorem	☐	☐	☐	☐	☐
09.	**AREAS OF PARALLELOGRAMS AND TRIANGLES**					
	1. Figures on the same Base and between the same Parallels	☐	☐	☐	☐	☐
	2. Parallelograms on the same Base and between the same Parallels.	☐	☐	☐	☐	☐
	3. Triangles on the same Base and between the same Parallels.	☐	☐	☐	☐	☐

Sr. No.	Area of Teaching / Topics		VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
10.	**CIRCLES**						
	1.	Circles and its Related Terms : A Review	☐	☐	☐	☐	☐
	2.	Angle Subtended by a Chord at a Point	☐	☐	☐	☐	☐
	3.	Perpendicular from the Centre to a Chord	☐	☐	☐	☐	☐
	4.	Circle through Three Points	☐	☐	☐	☐	☐
	5.	Equal Chords and their Distances from the Centre	☐	☐	☐	☐	☐
	6.	Angle Subtended by an Arc of a Circle	☐	☐	☐	☐	☐
	7.	Cyclic Quadrilaterals	☐	☐	☐	☐	☐
11.	**CONSTRUCTIONS**						
	1.	Basic Constructions	☐	☐	☐	☐	☐
	2.	Some Constructions of Triangles	☐	☐	☐	☐	☐
12.	**HERON'S FORMULA**						
	1.	Area of a Triangle – by Heron's Formula	☐	☐	☐	☐	☐
	2.	Application of Heron's Formula in finding Areas of Quadrilaterals	☐	☐	☐	☐	☐
13.	**SURFACE AREAS AND VOLUMES**						
	1.	Surface Area of a Cuboid and a Cude	☐	☐	☐	☐	☐
	2.	Surface Area of a Right Circular Cylinder	☐	☐	☐	☐	☐
	3.	Surface Area of a Right Circular Cone	☐	☐	☐	☐	☐
	4.	Surface Area of a Sphere	☐	☐	☐	☐	☐
	5.	Volume of a Cuboid	☐	☐	☐	☐	☐
	6.	Volume of a Cylinder	☐	☐	☐	☐	☐
	7.	Volume of a Right Circular Cone	☐	☐	☐	☐	☐
	8.	Volume of a Sphere	☐	☐	☐	☐	☐
14.	**STATISTICS**						
	1.	Collection of Data	☐	☐	☐	☐	☐
	2.	Presentation of Data	☐	☐	☐	☐	☐
	3.	Geographical Representation of Data	☐	☐	☐	☐	☐

Sr. No.	Area of Teaching / Topics		VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
	4.	Measures of Central Tendency	☐	☐	☐	☐	☐
15.	**PROBABILITY**						
	1.	Probability – an Experimental Approach	☐	☐	☐	☐	☐

Sr. No.	Area of Teaching / Topics	VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
	CLASS: X					
01.	**REAL NUMBERS**					
	1. Euclid's Division Lemma	☐	☐	☐	☐	☐
	2. The Fundamental Theorem of Arithmetic	☐	☐	☐	☐	☐
	3. Revisiting Irrational Numbers	☐	☐	☐	☐	☐
	4. Revisiting Rational Numbers and their Decimal Expansions	☐	☐	☐	☐	☐
02.	**POLYNOMIALS**					
	1. Geometrical Meaning of the Zeroes of a Polynomial	☐	☐	☐	☐	☐
	2. Relationship between Zeroes and Coefficients of a Polynomial	☐	☐	☐	☐	☐
	3. Division Algorithm for Polynomials	☐	☐	☐	☐	☐
03.	**PAIR OF LINEAR EQUATIONS IN TWO VARIABLES**					
	1. Pair of Linear Equations in Two Variables	☐	☐	☐	☐	☐
	2. Graphical Method of Solution of a Pair of Linear Equations	☐	☐	☐	☐	☐
	3. Algebraic Methods of Solving a Pair of Linear Equations	☐	☐	☐	☐	☐
	Substitution Method	☐	☐	☐	☐	☐
	Elimination Method	☐	☐	☐	☐	☐
	Cross-Multiplication Method	☐	☐	☐	☐	☐
	4. Equations Reducible to a Pair of Linear Equations in Two Variables	☐	☐	☐	☐	☐
04.	**QUADRATIC EQUATIONS**					
	1. Quadratic Equations	☐	☐	☐	☐	☐
	2. Solution of a Quadratic Equation by Factorization	☐	☐	☐	☐	☐
	3. Solution of a Quadratic Equation by Completing the Square	☐	☐	☐	☐	☐
	4. Nature of Roots	☐	☐	☐	☐	☐
05.	**ARITHMETIC PROGRESSIONS**					
	1. Arithmetic Progressions	☐	☐	☐	☐	☐

Sr. No.	Area of Teaching / Topics		VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
	2.	nth Term of an AP	☐	☐	☐	☐	☐
	3.	Sum of First n Terms of an AP	☐	☐	☐	☐	☐
06.	**TRIANGLES**						
	1.	Similar Figures	☐	☐	☐	☐	☐
	2.	Similarity of Triangles	☐	☐	☐	☐	☐
	3.	Criteria for Similarity of Triangles	☐	☐	☐	☐	☐
	4.	Areas of Similar Triangles	☐	☐	☐	☐	☐
	5.	Pythagoras Theorem	☐	☐	☐	☐	☐
07.	**COORDINATE GEOMETRY**						
	1.	Distance Formula	☐	☐	☐	☐	☐
	2.	Section Formula	☐	☐	☐	☐	☐
	3.	Area of a Triangle	☐	☐	☐	☐	☐
	4.	Some More Criteria for Congruence of Triangles	☐	☐	☐	☐	☐
08.	**INTRODUCTION TO TRIGONOMETRY**						
	1.	Trigonometric Ratios	☐	☐	☐	☐	☐
	2.	Trigonometric Ratios of Some Specific Angles	☐	☐	☐	☐	☐
	3.	Trigonometric Ratios of Complementary Angles	☐	☐	☐	☐	☐
	4.	Trigonometric Identities	☐	☐	☐	☐	☐
09.	**SOME APPLICATIONS OF TRIGONOMETRY**						
	1.	Heights and Distances	☐	☐	☐	☐	☐
10.	**CIRCLES**						
	1.	Tangent to a Circle	☐	☐	☐	☐	☐
	2.	Number of Tangents from a Point on a Circle	☐	☐	☐	☐	☐
11.	**CONSTRUCTIONS**						
	1.	Division of a Line Segment	☐	☐	☐	☐	☐
	2.	Construction of Tangents to a Circle	☐	☐	☐	☐	☐
12.	**AREAS RELATED TO CIRCLES**						
	1.	Areas of Sector and Segment of a Circle	☐	☐	☐	☐	☐

Sr. No.	Area of Teaching / Topics		VD (5)	D (4)	ND/NE (3)	E (2)	VE (1)
	2.	Areas of Combinations of Plane Figures	☐	☐	☐	☐	☐
13.	**SURFACE AREAS AND VOLUMES**						
	1.	Surface Area of a Combination of Solids	☐	☐	☐	☐	☐
	2.	Volume of a Combination of Solids	☐	☐	☐	☐	☐
	3.	Conversion of Solid from One Shape to Another	☐	☐	☐	☐	☐
	4.	Frustum of a Cone	☐	☐	☐	☐	☐
14.	**STATISTICS**						
	1.	Mean of Grouped Data	☐	☐	☐	☐	☐
	2.	Mode of Grouped Data	☐	☐	☐	☐	☐
	3.	Median of Grouped Data	☐	☐	☐	☐	☐
	4.	Graphical Representation of Cumulative Frequency Distribution	☐	☐	☐	☐	☐
15.	**PROBABILITY**						
	1.	Probability – A Theoretical Approach	☐	☐	☐	☐	☐

Thank you for submitting your response.

APPENDIX III
FEEDBACK FORMS

Feedback Form - 1

Implementation of the Professional Enrichment Programme developed for the Secondary School Mathematics Teachers.
(Feedback on the offline training session conducted on 8th July 2023)

Dear Educator,

I thank you for attending the Professional Enrichment Programme (workshop) of Secondary School Mathematics Teachers, organized on 8th July 2023 at IIRA International School, Bhayli, Vadodara.

At the outset, I request you to fill in the attached Feedback Form about the session which will be used to study the effectiveness of the modules prepared by me and explained to you.

Your responses would be used only for research purposes and would be kept highly confidential.

General Information

1. Name of the Teacher: _____

2. Name of School: _____

3.Experience as a teacher in secondary section:

Years. _____

Month. _____

4. Educational Qualification:

UG: ☐

P G: ☐

B Ed: ☐

Other if any: _____

5.Whether explored any other module as part of your Professional Enrichment Programme:

Yes. ☐

No. ☐

6. Are you a computer savvy person?

Yes. ☐

No. ☐

[Rate out of 5: 5- Excellent, 4- Very Good, 3- Good, 2- Average, 1- Needs Modification]

1. Do you find the training session workshop meaningful?

2. Was it relevant in your area of work, teaching, Learning.

3. Rate on the content deliver y and presentation.

4. Where the teaching modules innovative?

5. Rate on the adaptability of the teaching modules presented to you.

6. Rate on the usability and quality of the following presented to you.
   a. Art integration ideas.

   b. Experiential learning aspects.

   c. Interdisciplinary aspects.

   d. Competency based teaching learning ideas.

e. Assessment methods discussed.

f. Video links shown.

7. If you have any specific feedback on any other aspects of the professional enrichment. Programme or on modules, please specify:

_____

_____

_____

_____

_____

_____

Feedback Form - 2

[About the Teaching Modules for the Professional Enrichment of Secondary Schools' Mathematics Teachers]

(Feedbck from the teachers who explored the modules and implemented the programme)

Dear Educator,

I thank you for exploring the modules prepared for the Professional Enrichment of Secondary School Mathematics Teachers.

At the outset, I request you to fill out the attached Feedback Form on the modules which will be used to study the effectiveness of the modules.

Your responses would be used only for research purposes and would be kept highly confidential.

General Information

1. Name of the Teacher: _____

2. Name of School: _____

3. Experience as a teacher in Secondary Section:

Years: _____

Months: _____

4. Educational Qualification:

UG: ☐

PG: ☐

B.Ed.: ☐

Other if any: _____

5. Whether explored any other module as part of your Professional Enrichment Program:

Yes: ☐

No: ☐

6. Are you a Computer Savvy person?

Yes: ☐

No: ☐

Part – I

1. Have you explored all 4 modules?

Yes: ☐

No: ☐

If No, Please state reason:

_____

_____

2. Date on which you started exploring the modules: _____

Date of completion: _____

3. How often did you access the modules?

Every day: ☐

Alternative Days: ☐

Rarely: ☐

4. Overall, how much time did you spend on the modules?

_____

Part – II

[Rate Out of 5: 5-Excellent, 4-Very Good, 3-Good, 2-Average, 1-Needs Modification]

1. Concept Map

1	2	3	4	5

2. Clarity of content

1	2	3	4	5

3. Organization of the content

1	2	3	4	5

4. Your View on

a. Warm-up Activities

1	2	3	4	5

b. Teaching Points

1	2	3	4	5

c. Teaching Process

1	2	3	4	5

d. Language Flow

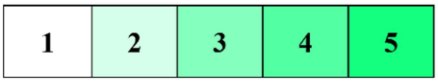
1	2	3	4	5

5. Rate the quality and usefulness of the video links shared.

1	2	3	4	5

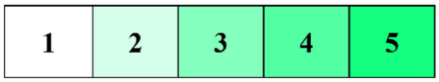
6. Rate the adaptability and usability of

a. Incorporated Art Integration ideas

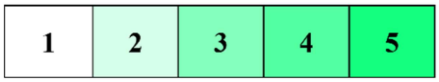


b. Experiential learning aspects

c. Interdisciplinary inputs



d. Assessment Method



7. About the above-mentioned aspects, if you have any specific feedback, please mention.

_____

_____

_____

_____

Part – III

1. Which feature/s did you like the most in the modules?

_____

_____

2. Would you like to suggest anything else to add to these modules:

If YES, please mention:

_____

_____

3. Would you like to prefer a few more training sessions on these modules?

Yes: ☐

No: ☐

If YES, which mode of training you would prefer?

Online: ☐

Offline: ☐

Part IV

General Feedback on Modules.

Rate your experience about using teaching modules in terms of the following:

1. User Friendliness (Was easy to access or not):

   Yes / No

2. Whether the content of the Modules is logically and sequentially arranged?

   Yes / No

3. Whether the learning experiences are relevant and meaningful to the learners.

   Yes / No

4. Do you feel it is adaptable?

   Yes / No

5. Do you feel the content of each module can be used as self-learning materials for your professional enrichment?

   Yes / No

6. Was it innovative?

   Yes / No

7. Whether modules are useful for the students to get clarification on different concepts?

   Yes / No

8. Whether the modules help in developing positive attitude towards learning Mathematics?

   Yes / No

9. Whether scholastic achievements of the students can be enhanced through the Implementation of these modules in the classroom.

   Yes / No

Thanks for your responses.

Ms. Beena Prashant
Researcher
Centre of Advanced Study in Education (CASE)
Department of Education
Faculty of Education and Psychology.
The M. S. U. of Baroda, Vadodara

FEEDBACK FORM-3

(To be filled by students)

[Feedback about the teaching – learning activities which are mentioned in modules and implemented by teachers for learners]

Date: 1st November'2023

Dear Student,

I thank you for taking out your precious time to fill this feedback form.

This is to be filled on the basis of your experience while exploring the activities of the teaching modules implemented in your classroom by your Mathematics teacher.

Your feedback will be used to study the effectiveness of the program developed by me for Mathematics teachers and your responses will be used only for the research purpose and will be kept highly confidential.

Thanks,

A.V.Beena
Research Scholar
Centre of Advanced Study in Education [CASE]
Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda, Vadodara

GENERAL INFORMATION

1. Name of the student: _____
CLASS: _____ACADEMIC YEAR: : _____

2. Name of School: _____

FEEDBACK

Details about the explored modules/ lessons

CLASS: IX

Sr. No.	MODULES	COULD EXPLORE? YES/NO	IF YES, NO. OF PERIODS UTILIZED
1	MODULE-I NUMBER SYSTEMS		
2	MODULE-II TRIANGLES		

CLASS: X

Sr. No.	MODULES	COULD EXPLORE? YES/NO	IF YES, NO. OF PERIODS UTILIZED
1	MODULE-III REAL NUMBERS		
2	MODULE-IV TRIANGLES		

[Rate Out of 5: 5-Excellent, 4-Very Good, 3-Good, 2-Average, 1-Needs Modification]

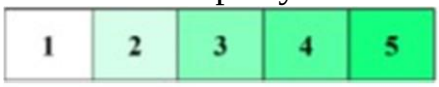
1. Whether the warm up activities helped you to recollect the previously learned concepts?

2. Rate on the clarity of the content implemented in class (whether logically and sequentially arranged and understood)

3. Whether the learning experiences were meaningful?

4. Whether it was useful to get clarity on different concepts?

5. Whether it helped you to enhance your positive attitude towards the subject



6. Whether the following aspects enhanced your overall scholastic achievements?

    a. Art Integration ideas:

    b. Experiential learning activities:

    c. Interdisciplinary aspects:

d. Videos incorporated:

7. If you have any specific feedback on any other aspects, please specify

_____

_____

_____

_____

_____

_____

APPENDIX IV
PAPER PUBLICATIONS

Cover Page

DOI: http://ijmer.in.doi./2021/10.07.238

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
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Peer Reviewed and Refereed Journal: VOLUME:10, ISSUE:7(12), July: 2021
Online Copy of Article Publication Available: www.ijmer.in
Digital certificate of publication: http://ijmer.in/pdf/e-Certificate%20of%20Publication-IJMER.pdf
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: 4th July
Publication Date:30th July 2021

INNOVATIVE PRACTICES IN TEACHING MATHEMATICS

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Atladara, Vadodara, Gujarat, India

ABSTRACT

Mathematics education is evolving constantly. Though the basic concepts in Mathematics remain the same, new trends and methods of teaching evolve as per the requirement of the changing the socio-cultural and psychological status of the learners. In order to meet the demands of the curriculum and the present century mathematics learners, in this rapidly changing world, it becomes necessary for mathematics teachers to have innovation in classrooms and also to have innovative teaching learning resources with them. Though most of the mathematics teachers are equipped with good content knowledge, innovation is needed in their pedagogical planning and in methodology of teaching. Resources in terms of the best curriculum text books, reference books, etc., are available for teachers, but there is a need for the development of innovative teaching – learning practices and strategies to enhance and enrich the teaching – learning happening in mathematics classrooms. This research paper highlights the need for the development of innovative teaching – learning resources for the present century Mathematics teachers and employing the innovative practices in Mathematics classrooms.

Keywords: Innovative Practices, Mathematics Education, Teaching – Learning Resources, Development of Innovative Strategies.

Introduction

"Destroying any nation does not require the use of atomic bombs or long missiles. It only requires lowering the quality of education... the collapse of education is the collapse of nation" (Sharma, M., 2017, Speaking Tree, TOI). The importance of a good education system for a country is well summarized in the above few lines.

Education system has a tremendous responsibility to transform a child into a fully developed individual. Over the ages, academicians and educationists of the country were relentlessly working to develop a system of education which can express and promote its social and cultural identity, a system which can full fill the requirement of the time. Continuously research studies are happening in education sector for the improvement of the existing system and to establish a system where in learners can be equipped with skills necessary to prepare them to face the technological world and also to prepare them for the current century learning and life. The new National Education Policy (NEP) – 2020 has emerged as a result for the need for such a reform.

Significance of the Study

Mathematics education being an unavoidable or integral part of the curriculum, if structured in the right manner, it can contribute to the progress of our Nation. All available means should be practiced to improve Mathematics education. However, the fact that the majority of students consider Mathematics as a difficult subject to learn, makes it urgent for schools to exploit all resources and strategies to help students understand it (Algani, 2019). Rapid technological advancement also indicates the importance of employing innovative strategies and teaching practices in Mathematics education by integrating technology in the classrooms.

Resources in terms of text books, reference books, good infrastructure, well set Mathematics laboratory, etc., are available for Mathematics teachers in majority of schools and most of the mathematics teachers are well equipped with their content knowledge too. But the variation in the learning outcomes in different students indicates that the quality of methods used in content explanation and the pedagogical planning need change and improvement. Many studies on the related topics are made mostly in Arabian countries, less studies are found in India on developing various strategies for improvement of Mathematics education. The present study is made with a purpose to understand the need of development of innovative teaching – learning resources for quality Mathematics education.

Objectives of the Study

The objectives of the study, based on the above purpose, are as follows

1) To study the different strategies and teaching practices in Mathematics education.
2) To study the need of development of innovative Mathematics teaching – learning resources.

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Methods and Procedure

The study was to investigate the present trends and practices in Mathematics education and to suggest few innovative strategies for Mathematics teachers. Qualitative approach-based on unstructured interviews, discussions and interaction with teachers of 3 different schools and also the observations of their classes-was adopted in the study. Recommendations and findings of previously conducted studies also gave strong foundation for this study and helped in getting an accurate qualitative result for the study. Answers and opinions collected from teachers through the unstructured interviews were analyzed qualitatively to interpret and to formulate the findings.

Reformations in Mathematics Education over the years

Mathematics as a compulsory subject is taught in School Education. Mathematical skills are crucial for a wide array of analytical, technological, scientific security and economic applications (Norris, 2012). Training students to become adept users of Mathematics and to appreciate its usefulness is of paramount importance for the future. Mathematics is not only needed for the understanding of the other Sciences, understanding of a basic level of numeracy is required for all in order to function in an increasingly complex world (Burghes, 2011).

Several efforts have been made through commissions and committees to improve the quality of education in general and Mathematics education in specific. Since last few decades, Mathematics education in school, both at elementary and secondary stage has made remarkable reformation by reforming curriculum, renewing textbooks and changing teaching – learning process. The major reforms in curriculum for all stages of school education came after National Policy on Education (NPE), 1968.The next major change occurred in terms of curriculum and teaching strategies with New Education Policy, 1986 which was subsequently amended in 1992.

National Policy on Education (1986) recommended a common core component in the school curriculum throughout the country. The policy also entrusted National Council of Educational Research and Training (NCERT) with the responsibility of developing the National Curriculum Framework and reviewing the framework at frequent intervals. The National Curriculum Framework (NCF) – 2005, is one of the four National Curriculum Frameworks published in 1975, 1988, 2000 and 2005 by NCERT. With the purpose to make education a joyful experience for children, the recommendations are being made in NCF-2005. It largely focused to provide a curriculum which is not only limited to text books, but to go beyond books and classrooms. More emphasis was made on activity-based learning and for the holistic development of children. ICT based learning was also emphasized in NCF-2005. Mathematics Laboratory activities were made compulsory in the recommendations made by NCF-2005.But in spite of wider spread acceptance of NCF-2005, the situation in majority of class rooms of Mathematics not changed because many teachers failed to translate the ideas mentioned in NCF into class room practices. Position Paper published by NCERT, 2006 by National Focus Group on Teaching of Mathematics also made good recommendations for improving the quality Mathematics education.

Though earlier policies were robust in conception and orientation, they have not delivered the desired results in terms of acceptable outcomes in the education sector. So MHRD felt the need for National Education Policy (NEP) – 2020. NEP-2020 is a much-needed reform specially in this digital era where there is an exploitation of technology, in every field. The emphasis made by NEP-2020 on learning systems like online learning, coding and for the digital courses like Artificial Intelligence, etc., indicates that this new policy has taken care of every step to achieve the goal of holistic education to produce the skillful youth for the country.

Objectives of Teaching Mathematics

The **Central Board of Secondary Education (CBSE)**is a Board of Education for public and private schools, under the Union Government of India, Ministry of Human Resource Development (MHRD) as its Parent Organization. CBSE, in all these years has set a good standard of education in India. With its influential educational policies, now it is one of the most preferred boards in the country. Its effectiveness is reflected on the number of schools affiliating with it each year. It has revised the curriculum and syllabus of Mathematics by considering the objectives laid by NCF-2005. Objectives of teaching Mathematics stated by CBSE are to help learners to:

- Consolidate the Mathematical knowledge and skills acquired;
- Acquire knowledge and understanding, particularly by the way of motivation and visualization of basic concepts, terms, principles and symbols and underlying processes and skills;
- Develop mastery of basic algebraic skills;
- Develop drawing skills;
- Feel the flow of reason while proving a result or solving a problem;
- Apply the Mathematical knowledge and skills to solve real life problems by developing ability to think, analyze and articulate logically;

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- Develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases;
- Develop necessary skills to work with modern technological devices such as calculators, computers, etc.;
- Develop interest in Mathematics as a problem-solving tool in various fields for its beautiful structures and patterns, etc.;
- Develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics.

But whether these objectives are achieved in their true sense through present Mathematics education is a million-dollar question. Achieving all the above objectives from a heterogeneous group of students with diversified capabilities is a big task on part of teachers. But to achieve these objectives, acquiring the required skills and practicing appropriate strategies and innovative methods of teaching are very important.

India has made great emphasis on educating all its children since its independence. India, with its strong Mathematical traditions, may be expected by the world to produce excellence in Mathematics (Ramanujam, Subramanian, 2012). But in reality, Mathematics is often referred as the difficult subject and in India, a large number of children fail or drop out before completing the schooling, because they are not able to cope with the demands of the curriculum (Rampal, A. & Subramanian, J., 2012).

Status of Mathematics Teachers

Teacher education institutes are making all the efforts in order to make the student teachers to acquire the skills during their course of training. But after the pre-service program when they enter in the profession, they find the methods of teaching, curricula and various other requirements in schools different from those advocated and implemented in teacher education institutes (NCTE, 2006). There is a need of a systematic mechanism for the academic support for teachers in terms of good resources and also for their professional development.

In many cases, Mathematics is taught by teachers who are not very confident of their Mathematics. Even in cases where qualified teachers teach the subject, their conceptual understanding was seen inadequate (Dewan, H., Batra, N., & Singh, C. I. 2012). Their understanding of the nature of Mathematics and attitude to it and its learning are very different from what is underlined in the NCF-2005 and now in NEP-2020. The lack of ability of teachers in Mathematics is probably the result of their preparation at the School and College level. It may also be because of the inadequate efforts at teachers' preparation and lack of innovative pedagogical and methodological planning. Lack of motivation of teachers also results into poor teaching – learning happening in Mathematics classrooms.

Need of Innovative practices in Mathematics Education

Mathematics is both a practical and theoretical subject. Its aim of teaching at school level being the development of many skills like: Critical Thinking, Problem Solving, Computation, Analytical Thinking, Logical Thinking, Reasoning (inductive and deductive), etc., and also the development of many Life skills too in learners. So, emphasis should be on both rote learning (memorizing the formula, theorem, etc.) and also in applying their meta cognition ability (ability to apply a formula or result in specific problem area).

The curriculum in India at school level, designed by the authorized body like NCERT (for CBSE & other Board) and other state board curriculum, etc., are considered to be the best in comparison with other countries' curriculum. But still students' learning outcomes in mathematics are not up to the expectation. This indicates the lack of strategic planning, innovative practices, innovative pedagogical and methodological planning suitable to the heterogeneous students, on part of teachers. The teachers must develop strategies. Ma'abrah, T. (2018) recommends that teachers should consider presenting the study material in sophisticated and innovative ways. According to Coe, Kristi (2018) there are various ideas that can be exploited and teachers can use innovative and creative ideas to encourage students to study mathematics and most importantly to understand it. It requires reconsidering the nature of the curriculum for mathematics itself, the quality of the methods used in its explanation and finally the extent to which the students accept their content and achieve good results at the end of each semester (Algani, 2019).

Employing different strategies and innovation is very vital in teaching Mathematics. Though teachers talk about their limitations or barriers in employing such strategies in class rooms as: Time barrier (due to the task of completion of entire syllabus during the stipulated time), Lack of motivation, Lack of interest in learners, Lack of professional development programs on innovation or Lack of innovative teaching – learning resources, etc., they must pay attention not only on the syllabus completion but on innovative strategies and creative planning which are proved to be effective for teaching – learning Mathematics.

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The following are few strategies which can be employed in class rooms by teachers for effective Mathematics teaching – learning.

(i) Blended Learning

Learning mathematics can be made easy and attractive if teachers integrate technology in classrooms for today's technology loving learners. Blended learning is a method of teaching that integrates the digital media with traditional teacher led classrooms. This method helps the learners to stay motivated and engaged. Blending Learning is now widely adopted in education with some scholars referring to it as the 'new normal' in course delivery (Dziuban, C., Grahan, C. R., 2018).

(ii) Experiential Learning

Experiential learning is a teaching – learning strategy where in learners learn by doing practically by appreciating the real-world relevance of the subject which helps the students to retain the concepts for a longer period. Such learning by nature, enables the development of a variety of capabilities, such as planning, team work, coping with stressful situations, responsibility and leadership (Davidovitch, N., Yavich, R., Keller, N., 2014).

(iii) Art Integration

Art-Integration is a cross-curricular pedagogical approach that utilizes various aspects and forms of art and culture as the basis for experiencing the learning of concepts across subjects. Art-integrated education, if embedded in classroom transactions, can not only create joyful classrooms, but also can help in imbibing the Indian ethos through integration of Indian art and culture in the teaching - learning process at every level.

Art Integration with mathematics concepts can encourage in demonstration and application of learning in real-life situation by the learners. CBSE has introduced Art – Integrated learning in all its affiliated schools vide circular dated 8th March, 2019.

(iv) Interdisciplinary Approach

It is an approach of curriculum integration which generates an understanding of concepts through different subjects and their connection with the real world. It means combining two or more subjects for enhancing the clarity of a single concept, through activities.

Results and Discussions

Curriculum planning, methods of teaching and pedagogical planning are few important aspects of school education and the quality of school education depends on the quality of these aspects.

NCERT being the curriculum advisor to the Ministry of Human Resource Development (HRD) and considering the important role of NCERT in country's educational activities like development of curriculum, methods of teaching, techniques of evaluation, etc., the researcher has made some suggestions to NCERT and other Policy Makers, based on the findings of the study.

- Along with curriculum, it is necessary to provide a well-structured pedagogical plan and period-wise lesson plans of each chapter for the teachers.The plan must be made mandatory for the teachers to follow.
- At the beginning of each chapter, a concept map to summarize the whole chapter for its overview can be given.
- The period-wise plan must give importance to the following methodological aspects on part of teachers:

1. Every period must start with some warm up activity / activities involving the previous knowledge checking.
2. Swift corrections of the previous day's assignment are very essential in the period itself, with proper positive comments. This every day's practice inculcates a habit in learners to complete their tasks and assignments on time.
3. Specific technology integration, as per the requirement of the chapter / topic to be done. This can be in terms of explaining an abstract concept with the help of a video, etc. Web link / video link and other links for references of related topics need to be provided in order to enhance the concept clarity and to develop the interest of learners in the subject.
4. At the end of every period, quick recapitalization of the learned concepts must be made as a habit.
5. The period must end by assigning some learning activities / practice works on the basis of the learned concepts of the day.

- After completion of the chapter / topic (by using the required number of periods):

1. A comprehensive series of self-assessment questions to be provided to do in class rooms or at home, at the convenience of the learners. (Self-assessment is being emphasized in NEP 2020 also)

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2. Projects based on "Art Integration" and "Experiential Learning" to be assigned by keeping in mind the mandated learning outcomes, from the topics / concepts taught.
3. One period must be utilized for the conduct of a Formative Assessment of the completed chapter.
- On the basis of the results of the Formative Assessment, a diagnostic test with required remedies to be provided to the learners who are in need of the same.

If a Mathematics teacher follows the exact style and the process mentioned above for the teaching of a Mathematics topic, the researcher guarantees the improvement in academic performance of the learners.

Policy makers need to think about lightening the burden of teachers with the heavy task of completing syllabus by removing the repetition of topics in different classes. This will give space for teachers to adopt creativity and innovation in classrooms. This will also make the teaching process interesting and attractive for learners because interest is a powerful motivational process that energizes learning (Harachiewicz, Smith, & Priniski, 2016).

Conclusion

The world is changing. Education in general and Mathematics Education in particular will continue to face challenges due to their evolving natures.

At present in India, for improvement in Mathematics teaching – learning, a system improvement (rather than the improvement of individual teachers) is needed. A system to be framed to engage more mathematicians, mathematics education researchers, etc., to study more about the development strategies which can enhance the quality of works by mathematics teachers. More contributions in this area in terms of developing strategies, practices, resources and professional enrichment program, etc., may solve the existing problems and may help in achieving the quality Mathematics Education.

National Education Policy (NEP) – 2020 has emerged as the result of search for an urgent requirement of reformation in educational practices. It gives a new ray of hope as it talks a lot about teaching career, professionalism and enriching and empowering teachers. It discusses about ensuring Performance Standards of teachers, and also of Teachers Audit or Performance Appraisal as a system. New Policy has also mentioned the need of innovative practices in teaching – learning by integrating technology and other various strategies.

India's education sector is eagerly waiting to see the immediate and urgent measures of implementing the same to achieve the years' long expectations and requirement of the desired improvement in Education System and Quality of teaching – learning, especially in Mathematics.

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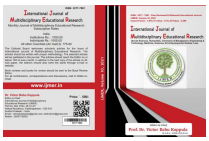
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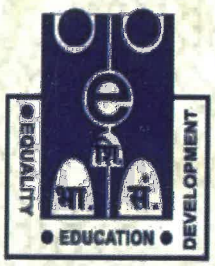

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Professional Development of Mathematics Teachers: Perspectives and Practices

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Abstract:

In order to have a fresh insight of the subject, Professional Development of teachers of the subject is very important. Like other subjects, Mathematics curriculum also is constantly evolving. So, to meet the demands of the present century Mathematics learners and curriculum, it is necessary for Mathematics teachers to have the practices of self-appraisal, reflective strategies and professional development. In India and other countries, Mathematics teacher education is a continuous process and it goes beyond the initial pre- service education-the Degree or Diploma. Hence Professional Development programs have become integral and significant parts of Mathematics education. Professional Development for teachers is offered in many different formats and it varies in its practices as per the demand and requirement of the curriculum and of teachers. This paper covers various aspects on the importance and needs of having Professional Development practices for Mathematics teachers. It explores the National and International practices in Professional Development of Mathematics teachers. Also, this paper gives deep and clear perspectives on the various trends and issues in Professional Development Programs for Mathematics Teachers.

Keywords: Professional Development, Mathematics teachers, Perspectives, Practices

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Introduction:

Mathematics is considered as the 'frightening' subject by majority of learners in India and abroad but a few students in each classroom consider Mathematics as a very scoring and interesting subject. The difference in the opinions is due to the difference in understanding the subject Mathematics. Mathematics teachers have very significant roles in making Mathematics, either as the frightening subject or as scoring or interesting subject. Students who learn under the teachers who do not have deep understanding of the subject may turn to be the ones who hate the subject and the learners who learn under a passionate subject expert may develop an interest and liking of the subject Mathematics in learners. Professional Development of teachers at regular interval is the key to raise the level of teaching – learning of Mathematics happening in the classrooms. Also, in order for Mathematics education to be dynamic and responsive, robust research in Mathematics education is urgently needed to look at global trends as well as local innovations and their systemic and systematic application and implementation (Aziz, D., 2015).

Importance of Learning Mathematics:

Mathematics is the subject, learning which helps the multiple abilities of learners to develop and nurture. It is very important to nurture the logical – mathematical abilities in every child as it encourages and develops important ways of thinking which are necessary for their lives. Ensuring children to have a good grounding in Mathematics will equip children for their future lives by developing the skills valued in industry and university (Ofsted, 2011). Mathematical skills are crucial for a wide array of analytical, technological, scientific security and economic applications (Norris, 2012). Mathematics is not only needed for the understanding of other sciences, understanding of a basic level of numeracy is required for all in order to function in an increasingly complex world (Burghes, 2011).

Getting the learners of Mathematics into the abstract thinking, logical thinking, Mathematization of things, developing their problem-solving skills, etc., are responsibilities of an efficient Mathematics teacher. It becomes necessary for the teachers to update and rebuild themselves fit to meet the demands of the current century Mathematics classrooms.

Status of Mathematics Teachers:

Mathematics is a subject which to be taught with the objective of developing Mathematization skills and capacity in learners, not just the skill of memorization of facts and concepts. A subject should be taught with an aim to develop skills of problem solving, reasoning, communicating mathematically and connecting the learned concepts of mathematics to the real life.

Mathematics teachers are expected to possess the skills required to prepare the learners for the 21st century's demands of life and work. NCF (2005) and now the New Education Policy (NEP- 2020) talks about the need of vocational skills development and connecting knowledge of Mathematics to the life outside the school. But actual fact is, in spite of having wider spread acceptance of the policies, the situation in majority of Mathematics classrooms are still conventional because majority of the teachers are failed to translate the ideas mentioned in such policies into real classroom practices. This shows the status of teachers – their need for a systematic mechanism for the academic support and for their skills and professional development.

At present in India, various mechanisms of building the capabilities and interests of Mathematics teachers are being evolved and many states started revising their D.Ed., B.Ed., curricula in accordance with the new framework (Rampal, A. & Subramanian, J., 2012). Attempts are also being made to strengthen the in-service training programs, to reach ideas to the teachers through ICT etc. But still there is a strong need for more such processes that would enable teachers to become more confident and to continue to engage those learning new things.

Need for Professional Development of Mathematics Teachers:

Professional Development is the enrichment training provided to teachers over a period of time to promote their development in all aspects of content and pedagogy.

Teacher preparation doesn't begin or end with Teacher Education Institutes. It is a continuous process – starts from pre-service to in-service till the end of the career (Manichander, 2016). Keeping updated and knowing the latest trends in Mathematics Education is very important for Mathematics Teachers of any level. Many cases are noted where Mathematics is taught by teachers who are not very confident of their Mathematics. Even in cases where qualified teachers teach the subject, their conceptual understanding was seen inadequate (Dewan, H., Batra, N. & Singh, C. I., 2012). Teachers' understanding of the nature of Mathematics and attitude to it and its learning are very different from what is underlined in the National Curriculum Frameworks (NCF) 2005 and now in National Education Policy (NEP) 2020. India has made great emphasis on educating all its children since its independence. India, with its strong Mathematical traditions, may be expected by the world to produce excellence in Mathematics (Ramanujam & Subramanian, 2012). But in reality, Mathematics is often referred as the difficult subject and in India, a large number of children fail or drop out before completing the schooling, because they are not able to cope with the demands of the curriculum (Rampal, A. & Subramanian, J., 2012). The lack of ability of teachers in Mathematics is probably the result of their preparation at the School and College level. It may also be because of the inadequate efforts of teachers' preparation as well as less opportunities in professional development programs. Lack of motivation of teachers also results into poor teaching – learning happening in Mathematics classrooms. The below mentioned are the few reasons that explain why a Mathematics teacher needs to look for Professional Development opportunities.

For certification purpose:

Usually, the district, state and central authorities and schools make it mandatory for Mathematics teachers to attend some certified Programs to enhance their professional competencies which ultimately helps in improving the learning outcomes of students. NEP (2020) also recommends the teachers to undergo 50 hours of Professional Development trainings in an academic year.

To understand new trends:

Though the basic concepts in Mathematics remain the same, like other subjects, Maths is constantly evolving. Very often, new trends and innovate methods of teaching come up. To become a competent teacher, it becomes important to know and get trained in these trends. Attending Professional development programs helps the mathematics teachers to have a fresh insight of the subject.

To stay connected with teachers' community:

In schools, there are not many teachers teaching the same grade. It is necessary for the teachers to connect other Mathematics teachers of the same grade of the other institutes to get new ideas and to check out innovative and creative methods used around. Professional development programs help teachers' community to connect with one another. NEP (2020) also has highly emphasized on collaborative learning.

Apart from the above needs of Mathematics teachers, there are other factors

which indicate or point out towards the necessity of Professional Development or Enrichment program for Mathematics Teachers. The most important reason is to ensure the required learning outcomes from all students of Mathematics. When Mathematics teachers are empowered with the facts and other skills of teaching Mathematics then students of the classrooms will definitely get empowered with the understanding and application of the subject. In order to make the teachers empowered with the various aspects of teaching Mathematics as per the change in curriculum and the new trends, Professional Development Program and practices for them are important. It is also necessary to have the practice of self-appraisal and reflective strategies for Mathematics teachers in order to meet the demands of the present century Mathematics learners.

The teachers must develop strategies. Ma'abrah, T. (2018) recommends that teachers should consider presenting the study material in sophisticated and innovative ways. According to Coe, Kristi (2018) there are various ideas that can be exploited and teachers can use innovative and creative ideas to encourage students to study Mathematics and most importantly to understand it. It requires reconsidering the nature of the curriculum for Mathematics itself, the quality of the methods used in its explanation and finally the extent to which the students accept the content and achieve good results at the end of each semester (Algani, 2019).

Rashtriya Madhyamik Shiksha Abhiyan (RMSA, 2009) Framework states that in-service teachers and Heads of schools to be trained for five days every year (Manichander, 2016).

So there is a strong need for more professional development programs which would enable teachers to become more competent, especially in this digital era where technology is taking its central position.

National and International Practices in Professional Development of Mathematics Teachers:

Professional Development Practices of Mathematics Teachers in India:

Mathematics teachers of India are now more open to the fact that their teaching approach needs to undergo fundamental change. It is widely acknowledged that in order to support change in classroom teaching, in terms of implementation of NCF (2005), there has to be system improvement involving teachers. Also, the course that pre-service teachers undergo has not involved much about the research and discussions carried out by the community of researchers in India. This situation creates a potential for change by way of designing or development of teacher's professional enrichment program to comprehend the vision of teaching and learning as articulated in the NCF (2005) (Kumar, Dewan & Subramaniam, 2012).

It has seen in India, despite of repeated reiteration of the need to professionalize the school teacher in policy documents and commission reports over the last 30 years, most teacher education programs continue to 'train' teachers to adjust to the need of an education system in which education is seen as the transmission of information, and learning reproduced from the textbooks (Naik, S., 2008).

The training needs of teachers in India's government schools are really daunting. The pre-service training curriculum followed in the country needs rejuvenation. It does not fully develop teachers to handle the challenges of their role innovatively. This scenario tells us that the in-service teacher education and professional development is one of the greatest challenges in ensuring the quality Mathematics education to learners.

But in self-financed (private) schools, the scenario is little different. At present in India, a large number of students are studying Central Board of Secondary Education (CBSE) curriculum. CBSE has made larger influence on many States Boards curriculum also, in terms of academic content development. CBSE Bye laws State that every teacher working with the CBSE affiliated schools should undergo a minimum of 25 hours of training (now as per NEP-2020, 50 hours of training) or Professional Development programs in an academic year. In this regard CBSE has also empowered various agencies like 'Centre of Excellence (COE)' and Schools' association like 'Sahodaya' to execute this task of training teachers or developing teachers professionally, in various subjects' content and pedagogical areas. So it has become a practice for the schools of the country affiliated to CBSE to train the teachers under the subject experts as Resource Persons (RP) appointed by the agency (Centre of Excellence - COE) or Sahodaya and maximum teachers are getting benefited out of this for their Professional Development. Mathematics subject experts of the country are chosen as the RP for such workshops to ensure the quality of such in-service training programs.

Now- a- days there are Private Agencies, Publication Houses etc., working for the same with the aim of their business development along with teachers' Professional Development. But quality of the same depends on the Resource Persons chosen by them.

International Practices of Professional Development of Mathematics Teachers:

Professional Development practices in different countries differ depending on the curriculum needs of each country. Hence teacher education activity and their Professional Development practices focus on changing teaching practice in line with curriculum needs. But in every country Mathematics education and Professional Development of Mathematics teachers are given much importance.

The book International Approaches to Professional Development for Mathematics teachers (Bednar, N., Florentine, D. & Human, R., 2011) reveals different international perspectives on alternative approaches and experiences established to support the Professional Development of Mathematics teachers.

In USA whatever Mathematics the learners learn (the curriculum) is directly connected with their real life. There, the Professional Development practices are strongly connected with experienced teachers. Teachers' leadership refers to mentor teachers working with novice teachers (Rhodes & Wilson, 2011). Well experienced teachers work as mentors to help novice teachers by providing advices and sharing

experiences.

More or less, same practices are implemented in countries like China and Portugal. In China, the expert and experienced teachers give public lessons, supervise novice teachers and share their developed teaching materials to novice teachers to enhance their Professional Development. In Portugal, expert and experienced teachers conduct teachers' group meetings, prepare the novice teachers for conducting group meetings, supervise and evaluate their classroom practices. In Portugal Professional Development means a subject teacher working with other teachers in a school (Nunes, 2011).

Finland being a top performing country (in terms of providing education) in many international comparisons (OECD, 2003; 2010; 2014) and Finnish Mathematics education being proved excellent according to PISA testing, the mathematics teacher education program of Finland and their Professional Development is an area the researchers across the globe peeped into, with much interest and curiosity. A special feature that differentiates the Finnish teacher education from that of many other countries is research studies in teacher education (Jyrhama & Maaranen, 2012; Niemi & Jakka – Sihvanen, 2006). The leading principle has been that teachers need a thorough knowledge of the most recent research advances in the subjects they teach. (Niemi, 2015). In Finland, Mathematics teachers have much freedom, but they are responsible for their students' learning too. Teachers have the freedom to choose the books, customize the curriculum as per the learning needs of individual learner. Their schooling itself has good impact on developing themselves as teachers. Finland system of pre-service teacher education also helps to generate Mathematics teachers with high quality.

The success in PISA testing reveals about their effective pre-service education good teaching profession culture, the impact of in-service teacher education and the efforts the country puts in to develop Mathematics education.

In Japan, the 'Lesson Study' is the main practices for the Professional Development of Mathematics teachers. The practices of lesson study involve a group of teachers carefully planning a lesson on a particular topic. These teachers conduct the lesson which is observed by the other teachers. In the process of planning the lesson, the teachers share various ways in which they can teach the lesson, which will help them to reach their goals (Lewis, C., 2000).

Japanese Lesson Study has come under increasing attention from educators in the West and throughout South – East Asia. This form of professional development is now no longer restricted to Japan. While Lesson Study takes place across all curriculum areas in Japan, it is perhaps most commonly practiced in Mathematics and this has tended to be case in other countries too (Doig, B & Groves, S. ,2011).

Significance and Objectives of the Study:

Mathematics education being an unavoidable and integral part of the curriculum, if structured in the right manner, can contribute to the progress of the

Nation. All available means should be practiced to improve Mathematics education. However, the fact that the majority of students consider Mathematics as a difficult subject to learn, makes it urgent for schools to exploit all resources and strategies to help students understand it (Algani, 2019). Rapid technological advancement also indicates the importance of employing innovative strategies and teaching practices in Mathematics education by integrating technology in the classrooms.

Resources in terms of text books, reference books, good infrastructure, well set Mathematics laboratory, etc., are available for Mathematics teachers in majority of schools and most of the mathematics teachers are well equipped with their content knowledge too. But the variation in the learning outcomes in different students indicates that the quality of methods used in content explanation and the pedagogical planning need change and improvement. Many studies on the related topics are made mostly in Arabian and other foreign countries, but fewer studies are found in India on developing various strategies for improvement in Mathematics education. The present study is made with a purpose to understand the need of development of innovative teaching – learning resources and professional development programs to improve quality of Mathematics education.

Objectives of the Study: The objectives of the study, based on the above purpose, are as follows:

1) To investigate various professional development practices in India and in other countries.
2) To study the need of professional development programs for Mathematics teachers in India.
3) To study different perspectives of Mathematics teachers on professional development.

Methods and Procedure:

The study is aimed to investigate the present trends and practices in Professional development of Mathematics teachers and also to collect various perspectives on professional development practices. A qualitative approach was adopted for the study, based on discussions, interaction unstructured interviews etc., with Mathematics teacher of different schools in Gujarat and few schools in Kerala Opinions of School Principals also were collected through unstructured interviews. Recommendations and findings of previously conducted studies also gave strong foundation for this study and helped in getting an accurate qualitative result for the study. Answers and opinions collected from teachers through the unstructured interviews were analysed qualitatively to interpret and formulate the findings of the present study.

Results and Discussion:

The analysis of the practices of Mathematics teachers' professional development in India and other countries leads to the perspective that Mathematics teacher education and professional development have been given much importance in most of the

countries. By understanding the importance of the subject and also due to the constantly evolving nature of Mathematics curriculum, Mathematics teacher education and development has been an extensive research area across the globe. But out of the many good research findings, very few are only incorporated and implemented in the system by the policy makers. Hence, the researcher strongly makes some suggestions to NCERT and other Policy makers of Indian education system, based on the findings of the present study.

Apart from putting efforts on creating various Mechanisms and effective modules the following easily implementable practices can be involved while developing the system to enhance the professional development of Mathematics teachers.

- A system where teachers learn from one another to enhance their practices and their professional development.
- Experienced / leader teacher supervising, guiding and demonstrating for novice teachers.
- Peer monitoring and discussions
- Involving novice teachers for research and experiment new practices.
- System to have a training plan for the whole academic year focusing various areas of content, pedagogy and other skills required so that teachers can chose and select the program, based on their needs.
- Use of technology: experienced teacher demonstrating the difficult topics through virtual classroom teaching and novice teachers observing the classes.
- Creating a bank of good videos on teaching the basic and key topics and to make them available for all teachers.

Also, the core of Japanese Professional Development practice – Lesson Study can be involved in system and its practice can give a paradigm shift in Mathematics teachers' development.

Sharing the formal and Informal experiences of Mathematics teachers with one another can contribute in their professional development. The practical knowledge, knowledge through collaboration with other teachers also can enhance their professional development.

Conclusion:

Whatever practices for the Professional Development of Mathematics teachers are there in the system, the ultimate aim should be to create accountability and responsibility on part of Mathematics teachers for the quality of students learning Mathematics. To achieve this quality teaching – learning happening in Mathematics, the system needs to focus on the needs of every Mathematics teacher and accordingly teacher preparation and professional development to be planned and to be executed at regular intervals.

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APPENDIX V
Ph.D. Course Work Certificate

THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

CERTIFICATE

[As per O.Ph.D. 2 under UGC (Minimum Standards and Procedure for Awards of M.Phil./Ph.D. Degree) Regulation, 2009 for 15 Credits to be earned by Ph.D. Scholars]

This is to certify that **A. V. Beena,** Research Scholar, registered under UGC (*Minimum Standards and Procedure for Awards of M.Phil./Ph.D. Degree) Regulation, 2009,* vide Registration Certificate Number **226** dated **04/02/2017**, for pursuing Ph.D. on has undertaken and completed the course work with the Grade B.

STATEMENT OF CREDITS EARNED

Name of Research Scholar: **A. V. Beena**

Faculty/Institution: Faculty of Education and Psychology

Department: Department of Education

Paper Number	Course Title	Course Credits	Grade Earned
Core Courses – 09 Credits *[Offered at University Level]*			
I.	Introduction to Research and Research Writing	3	C
II.	Quantitative Research Techniques	3	C
III.	Introduction to Basic Computer Functions and Applications for Research	3	B
Departmental Courses – 06 Credits *[Offered at Departmental Level]*			
IV.	Review of Related Literature	3	A
V.	Conceptual Framework of Research Problem	3	A
	Overall Grade		B

FoEdu/226
DC : 130/28122017
UC : 85/5-17/250517 to 300617

Date of Issue: 20/01/2018
Place: Vadodara

Registrar (OSD)

Grade Conversion Table and Grade Calculation Formula

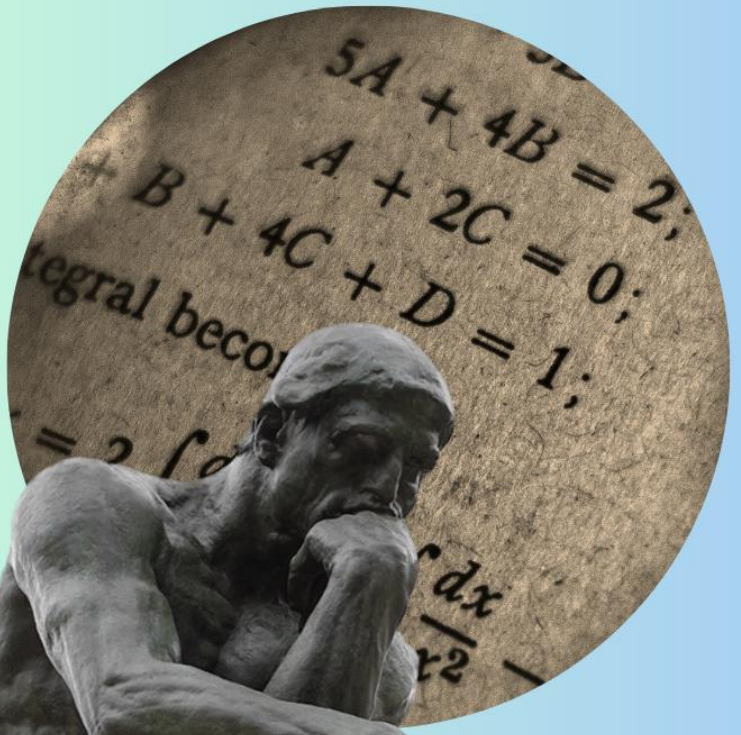
Grade	Grade Points	Range
O	10	Above 9.01
A	9	8.01 – 9.00
B	8	7.01 – 8.00
C	7	6.01 – 7.00
D	6	5.01 – 6.00
E	5	4.01 – 5.00
F	4	Below 4.00

$$Overall\ Grade = \frac{\sum (Grade\ Points \times Credits)}{\sum Credits}$$

Appendix - VI Modules

TEACHING MODULES
ON
NUMBER SYSTEMS & TRIANGLES.

(FOR CLASS IX & X MATHEMATICS TEACHERS)



Mrs. A. V. Beena.
Researcher

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Department of Education
Faculty of Education and Psychology
The Maharaja Sayajirao University of Baroda,
Vadodara, Gujarat - 390002

DECLARATION BY AUTHOR

These modules are the results of the research done by the author, in the area of Mathematics Education of Secondary Section, in India.

For developing these modules, a survey was conducted by taking Secondary Section Mathematics teachers as samples, to identify their needs. The topics of the modules have been identified after the statistical analysis of the data collected through the survey.

While developing these modules, every effort has been made to avoid errors in typing and printing. Despite this, if you notice any mistake or any other discrepancy which might have crept in, may be brought to the notice which shall be rectified further.

The content of these modules is presented in the blended learning form. Links/QR codes for the self-created videos are given, wherever it's applicable, for the teachers' reference.

I would be highly gratified if these modules can be of any help to teachers who are using them to improve student's performance in the mentioned areas.

Key features of each module:

- It has been presented as per the syllabus prescribed by **CBSE/NCERT, before Covid Pandemic.**
- **Objectives** of each module, in terms of achieving the expected learning outcomes are given at the beginning.
- **Keywords** of the chapter are given at the beginning of each module which helps in getting an understanding of the main concepts of the chapter.
- A **concept map** of the content given at the beginning of each module, summarizes the whole chapter, giving an overview of the content of the chapter.
- Every module is divided into a number of units on the basis of the main teaching points.
- **Index** given at the beginning helps to understand the whole module at a glance.
- Each unit of the module starts with an explanation to do some **warm-up activities** in the class which will help the learners to recollect the previously learned matters, required for the content grasping.

- Unit wise planning given for each module emphasizes on the following **methodological aspects:**
  1. Every unit compels the teachers to discuss or check the **previous knowledge** of learners, related to the main teaching points of the unit, in the form of warm-up activity.
  2. At the end of each unit, the appropriate **teaching methodologies** are suggested which can be adopted by the teachers, as per the demand of the content.
- **Blended Learning / Technology Integration aspects in each module:**
  Technology integration using the required digital media, in a specific area is given as per the requirement of the topic. Video links, Activity links, etc. provided are aimed to give some additional information about the content, which may help the teachers to deliver the content in an interesting manner.
- **Suggestions for multiple resources** for references are given along with these modules, referring to which may help the teachers to acquire some added information and understanding.
- Each module has a reference to the Self-Assessment series of questions related to the topics. Self-Assessment is strongly emphasized in the **National Education Policy (NEP) -2020,** with an aim to help teachers to make the students get away from rote learning.
- Each module contains suggestions for **Art Integrated** projects for students. Similar kinds of projects can be assigned to learners.

[Art Integration is made mandatory in all CBSE affiliated Schools vide Circular No. Acad-12/2019 dated 8th March 2019, by CBSE, to ensure the mandated Learning Outcomes (LO) Based Teaching – Learning process].

- Suggestion for assignments, focusing on **Experiential Learning** is given in each module.
- At the end of each module (after completion of the whole chapter), Assessment methods are suggested. Based on the results of Assessments, teachers are expected to conduct **Diagnostic Test/s** followed by providing required remedies to the learners who need the same.
- An **Inter-disciplinary** approach of teaching Mathematics by correlating another subject/s is also emphasized in each module.

If the teachers follow the exact style and the process mentioned in the module, the improvement in the academic performance of students in the mentioned area/topics can be rest assured or guaranteed.

MODULE – I

TOPIC: NUMBER SYSTEMS

Subject: Mathematics **Class: IX**

"It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions..."

- Pierre Simon Laplace

Introduction:

This module aims to help the teachers to make the learners very clear with the whole concept of Number systems. Teachers are expected to deliver each unit mentioned in the module, in the way each is explained so that each learner of the class will be benefitted from the clarity of the concepts like various sets of numbers, problems based on rational and irrational numbers, their representations on the number line, etc. This module also explains Real numbers, their decimal expansions and representation on the number line, operations of Real numbers, and the Laws of Exponents for Real Numbers which give a clear understanding of the complete Number Systems and enhance the learners' knowledge and skills to solve the related problems in their real-life situations. If implemented effectively by teachers, it can tremendously improve the learner's performance.

Objectives / Expected Learning Outcomes:

After doing with all units of this Module, teachers will be able to enable the learners to:

- Know with clarity, which are the different sets of numbers and what are the major differences between sets of numbers.
- Understand the difference between Rational and Irrational numbers and their positions on Number lines.

- Understand, there is a one-to-one correspondence between the Points on the Number Line and Real numbers.
- Express Real numbers as decimals (Know the decimal expansions of real numbers.)
- Represent Real numbers on the Number line.
- Carry out various operations on Real Numbers.
- Apply Laws of Exponents for solving problems of Real Numbers.
- Know how to rationalize the denominators of an expression involving Real Numbers.
- Solve various problems on Real numbers by applying the learned results and knowledge.

It will also enable the teacher to:

- Understand the various strategies to introduce the mentioned topics.
- How to blend the other teaching techniques with the traditional methods of teaching for the mentioned topics.

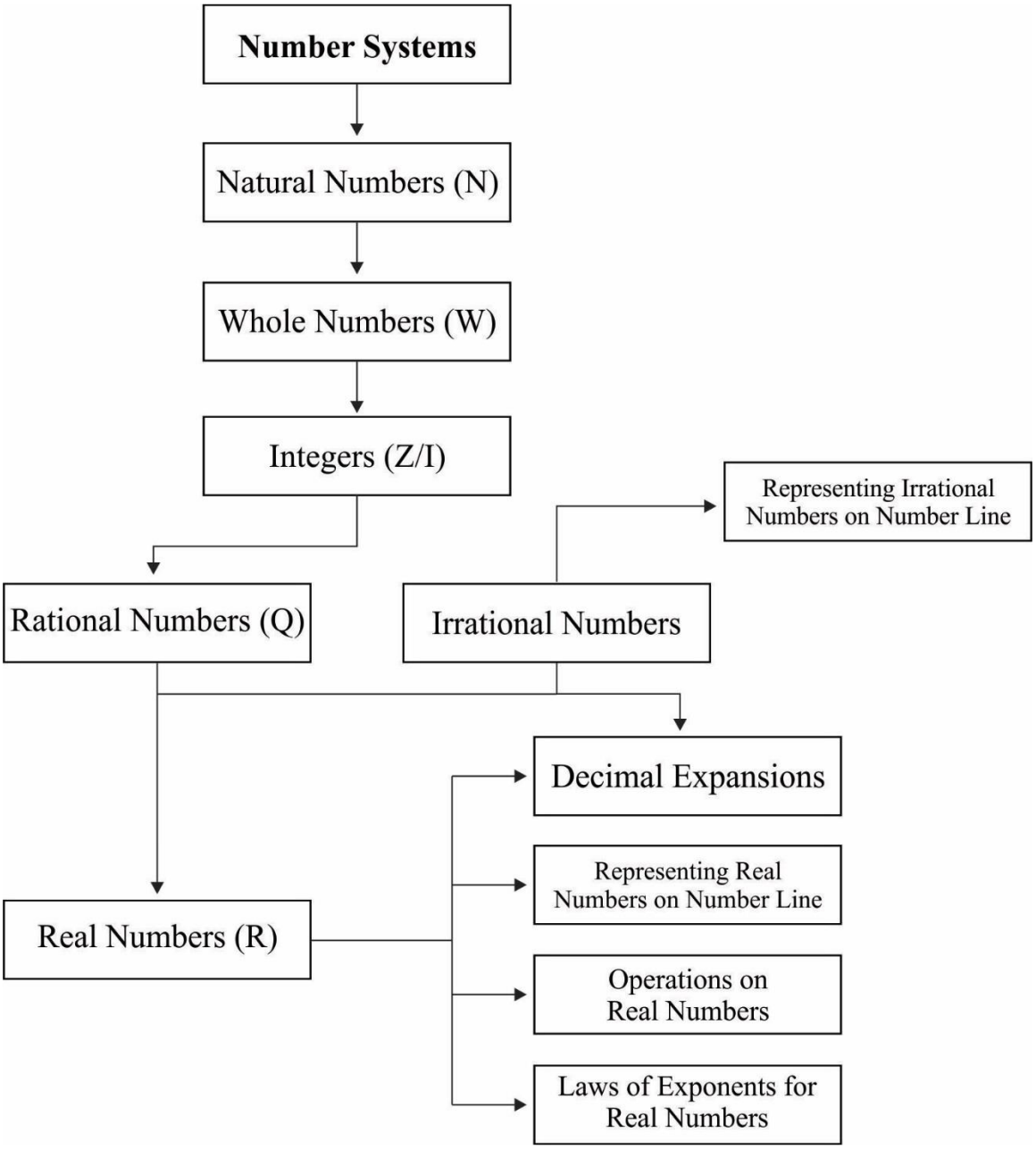
Pre-requisite knowledge required for learners.

To know about all sets of numbers learned in earlier classes, operations on various sets of numbers, and properties of operations on numbers, representing various types of numbers on the number line.

Key Concepts / Important Terms:

1. Real numbers
2. Rational and Irrational numbers
3. Decimal expansions
4. Division Lemma
5. Division Algorithm

CONCEPT MAP


Number Systems
Natural Numbers (N)
Whole Numbers (W)
Integers (Z/I)
Rational Numbers (Q)
Irrational Numbers
Representing Irrational Numbers on Number Line
Real Numbers (R)
Decimal Expansions
Representing Real Numbers on Number Line
Operations on Real Numbers
Laws of Exponents for Real Numbers

INDEX

UNIT – 1

REVISIT ON NUMBER SYSTEMS

Teaching Points: Revision of already learned information about various kinds of Numbers and Number Systems.

Warm-up activities for learners:

Discuss the following with learners:

How ancient people used to count?

Who introduced Numbers?

Discuss the ancient symbols of numbers and India's contribution to numbers. Make group discussions on the Number line and the position of numbers on the number line.

Teaching Process:

Ask the learners to recall the smallest set of Number Systems and how they position on the Number Line.

The set of Numbers starting from 1 is the smallest set of numbers, which is called the set of Natural Numbers denoted by N.

$N = \{1, 2, 3, 4 ...\}$

Now along with this set of Natural numbers, if we take 0, then we get the set of whole numbers, denoted by W.

$W = O \cup^* N$

$= \{0, 1, 2, 3....\}$

So, $N \subset^* W$

$\cup^*$ = Union $\subset^*$ = Subset, part of

Now what is Integer?

A set of Whole numbers (W) and the numbers falling on the opposite side of 0 (opposite of all natural numbers) make the set of Integers, denoted by I or Z.

Discuss:

Why is a set of Integers denoted by Z?

Z is from the German word 'Zahlen' means 'to count'.

I OR Z = { -1, -2, -3 .....} $\cup$ W

= {..... -3, -2, -1, 0, 1, 2, 3.....}

So, N $\subset$ W $\subset$ Z

Let the learner answer:

Is there any number remaining that is known to us in any different forms, other than Integers?

What about the numbers in the form of $\frac{3}{4}$, $\frac{5}{7}$, $\frac{100}{101}$, $\frac{-3}{7}$, etc.?

Since these numbers are in the form of ratios, this kind of collection of numbers is called Rational Numbers denoted by Q. (Q denoting quotient).

Definition of Rational Number:

A number *r* is called a rational number if it can be written in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

Discuss:

Why rational numbers are of the form $\frac{p}{q}$, $q \neq 0$?

What is a number divided by zero?

A few important facts about Rational Numbers can be brought to the learners through the following discussions:

Is every integer a rational number?

Yes, it is because it can be expressed in the form of $\frac{p}{q}$ Eg. 25 = $\frac{25}{1}$

i.e., the rational numbers include natural numbers, whole numbers, and integers.

i.e., N $\subset$ W $\subset$ Z $\subset$ Q and the rational numbers do not have a unique representation in the form $\frac{p}{q}$, p, q $\in$ Z, $q \neq 0$.

Ask learners to recall the equivalent fractions.

E.g. : $\frac{3}{4} = \frac{6}{8} = \frac{15}{20}$ ....., the lowest or the reduced form being $\frac{3}{4}$.

They are called **Equivalent Rational Numbers.**

When we represent a rational number in $\frac{p}{q}$ form, the HCF (p, q) should be 1.

i.e., p and q should not have common factors other than 1. (Means p and q should be co-prime numbers)

Discuss:

What are co-prime numbers? Is 0 a rational number?

How to find the rational numbers between two integers?

Illustrations to be given.

Find 5 rational numbers between 3 and 4.

Explain that to find a rational number between r and s, you can add r and s and divide the sum by 2, that is $\frac{r+s}{2}$ lies between r and s. So, $\frac{7}{2}$ is a number between 3 and 4. In this manner, we can find four more rational numbers between 3 and 4.

Discuss the following statements: –whether they are true or false.

Let the learners justify their answers.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

Justification:

(i) False, because zero is a whole number but not a natural number.

(ii) True, because every integer 'm' can be expressed in the form of $\frac{m}{1}$, and so it is a rational number.

(iii) False, because $\frac{3}{5}$ is not an integer but is a rational number.

Ask the learners to find 5 or 6 rational numbers:

1. In between any two consecutive negative integers.
2. In between any two fractions with the same denominator.

Discuss how to make a journal on:

1. Every natural number is a whole number.
2. Every integer is a whole number.
3. Every rational number is a whole number.

Recommended Teaching Methodologies:

- **Demonstration method**
- **Inductive method**
- **Lecture method**

Notes:

UNIT – 2

IRRATIONAL NUMBERS

Teaching Point: Irrational Numbers.

Warm-up activities for learners:

Let the learners recall the previously learned results on Numbers and Number systems. Ask the learners to define the rational numbers.

Teaching Process:

Let the learners think about numbers that cannot be expressed in the form of $\frac{p}{q}$.

Let them recall the value of π and $\sqrt{2}$.

The value of π they learned in lower grade as 3.14.... which is a non-terminating and non-recurring decimal.

Similarly, there are infinitely many decimals that are non-terminating and non-recurring, which cannot be converted in the form of $\frac{p}{q}$.

Such numbers are irrational.

If a decimal is recurring and non-terminating, then it can be converted to $\frac{p}{q}$ form.

Ask the learners to recall the method of conversions.

Definition of Irrational Numbers:

A number *s* is called irrational if it cannot be written in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

Now let the learners know that all irrational numbers also can be placed on the number line.

Explain that when we consider all rational and irrational numbers, then there is no number left out in the numbers system.

The collection of all rational numbers and irrational numbers together makes up the collection or set of Real numbers, denoted by R.

So, R is the superset of all other set of numbers. $N \subset W \subset Z \subset Q \subset R$.

Explain that Irrational Numbers $\subset$ R (Subset of Real Numbers).

Irrational numbers can be located on the number line.

In earlier classes, learners have learned that all Natural numbers, Whole numbers, Integers and Rational numbers have their places on the number line.

Ask learners to recall the Pythagoras theorem.

Illustrate the methods to locate the irrational numbers on the number line.

Example: Locate $\sqrt{2}$ and $\sqrt{3}$ on the number lines.

Let the learners understand that any irrational numbers such as $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$ etc., can be located on the number line with the help of Pythagoras theorem and using the geometrical construction.

Activity:

Ask the learners to collect information about Pythagoras and his contributions in Mathematics.

Discuss:

The square roots of all positive integers. Are they all rational or irrational?

Now let the learners know that the set or collection of all Rational numbers and Irrational numbers make the set or collection of Real Numbers, denoted by R.

So, a Real number is either rational or irrational.

Can we represent all Real numbers on the number line? Yes.

Is there any point on the number line other than the real number? No.

Every real number is represented by a unique point on the number line. And every point on the number line represents a unique real number.

There is a one–to–one correspondence between the set of real numbers and the number line.
Discuss more on this with illustrations.

Recommended Teaching Methodologies:

- **Blended learning method**
- **Demonstration method**
- **Illustration method**
- **Lecture method**

UNIT – 3

REAL NUMBERS AND DECIMAL EXPANSIONS

Teaching Points: Real Numbers and their Decimal Expressions.

Warm-up activities for learners:

Ask the learners to recall and state all known results related to Number Systems.

Let them recall Rational numbers, Irrational numbers, Real numbers, and decimal expressions of a number.

Teaching Process:

Discuss:

Looking at the decimal expansion of any number, is it possible to identify a rational and an irrational number?

Let the learners do an experiment by finding the decimal expansions of a few fractions.

Fractions are always rational numbers. So, after finding the decimal expressions of a few fractions, it is easy to make the learners understand that the decimal representations of rational numbers either terminate or recur.

This means, after a stage of actual division, there are two situations.

1. The reminder becomes zero if the decimal representation terminates.

OR

2. The reminder never becomes zero. i.e., the decimal expression is non-terminating and recurring.

E.g., : $\frac{1}{2}, \frac{3}{7}$, etc,

So, any decimal expansion which is terminating OR non-terminating, but recurring, then it is a rational number.

Now, let the learners think about the decimal expression of irrational numbers. It is easy to conclude that since there is the possibility of only one property remaining, the decimal expansions of irrationals can be non-terminating and non-recurring.

Using this property of decimal expansion of irrational numbers, we can find many irrational numbers in between two rational or irrational numbers.

Share the history of finding the digits in the decimal expansion of the irrational number π through the below video link.

[π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?]

Video Link: INTERESTING FACT ABOUT PIE

Discuss:

Is it possible to find the decimal expressions of the fractions with denominator 7, without doing the actual division?

Share the surprising factor while expressing the rational number 0.9999……. in the form of $\frac{p}{q}$.

Ask the learners to experience the uniqueness in the rational number $\frac{1}{17}$ by actually doing the division.

Recommended Teaching Methodologies:

- **Blended learning method**
- **Demonstration method**
- **Lecture method**
- **Induction and deduction method**

UNIT – 4

REPRESENTING REAL NUMBERS ON THE NUMBER LINE

Teaching Point: Representing Real Numbers on the Number line.

Warm-up activity for learners:

Ask them to arrange the sequence of number systems, starting from Natural Numbers to Real Numbers.

Teaching Process:

The set of Real numbers is the union of the set of rational and irrational numbers. Every real number has a decimal expansion.

It is necessary and important for the learners to understand that every real number, (whether rational or irrational) can be located or represented on the number line.

It is very important to make the learners understand the important aspects of number lines like :

1. On a number line, the numbers are marked at equal intervals.
2. It has an order relation on the number line which is as shown below.

   ….. -5 < -4 < -3 < -2 < -1 < 0 < 1 < 2 < 3…….

   (i.e., the number on the left side of another is always less than the other)
3. Points are marked on the number line to mark the corresponding numbers.
4. Infinite set of numbers can be marked by putting arrow marks at the end of the line,
5. Directed line segments are used to show the displacements of the number from 0 towards the left.

E.g.: -5 is 5 units away from 0 towards the left.

Ask the learners to recall the process of finding the decimal expansion of any rational numbers or irrational numbers.

There is a method to find the position of any decimal expression on the number line.

Let the learners know that any real number can be represented on the number line with the help of its decimal expansion.

Video Link: [METHOD OF SUCCESSIVE MAGNIFICATION]

This method of visualizing, enlarging, and representing the numbers on the number line is called the process of **Successive magnification.**

Through the successive magnification method, it is easy for the learners to understand that every real number, whether rational or irrational, can be located on the number line.

Recommended Teaching Methodologies:

- **Demonstration method**
- **Lecture method**
- **Blended learning method**

Notes:

UNIT – 5

OPERATIONS ON REAL NUMBERS.

Teaching Points: Operations on Real Numbers.

Warm-up activity for learners:

Let the learners recall and state all learned results. Ask them to recollect the various operations on numbers, what they mean by commutative, associative, and distributive laws for addition and multiplication, and what they mean by the closure property of addition, subtraction, multiplication, and division on Rational numbers.

Teaching Process:

With examples, let the learners understand the fact that irrational numbers also satisfy the commutative, associative, and distributive laws for addition and multiplication, but irrational numbers are not 'closed' with respect to the operations of addition, subtraction, multiplication, and division.

That means Sum, Difference, Products and Quotients of two irrational numbers are not always irrational.

Use various illustrations to explain the same.

E.g.: $\frac{\sqrt{3}}{\sqrt{3}} = 1$ is rational.

$\sqrt{7} + -\sqrt{7} = 0$, is rational etc.,

Let the learners be exposed to various problems involving the operations – addition, subtraction, multiplication, and division of irrational numbers to understand the fact that the answers are not always irrational.

By solving such problems, learners be made clear about the following facts:

(i) The sum or difference between a rational number and an irrational number is irrational.

(ii) The product or quotient of a **non-zero** rational number with an irrational number is irrational.

(iii) If we add, subtract, multiply, or divide two irrationals, the result may be rational or irrational.

In earlier classes, learners have learned that if *a* is a natural number, then.

$\sqrt{a} = b$ means $b^2 = a$, $b > 0$.

The same is applicable to any positive real numbers.

i.e**., if a > 0 is a real number, then $\sqrt{a}$ = b means b^2 = a, b > 0.**

Also, learners have learned how to represent $\sqrt{n}$ for any positive integer 'n' on the number line.

Now let them be exposed to find or locate $\sqrt{x}$ for any given positive real number x, geometrically.

Video Link: LOCATING REAL NUMBERS ON THE NUMBER LINE

In earlier classes, learners have learned square roots, cube roots, 4^{th} root…. nth root, etc., of positive integers.

These operations are all applicable for a positive real number also.

i.e., if a > 0 is a real number and n is a positive integer, then $\sqrt[n]{a}$ = b or b^n = a, b > 0

Thus, they learn, the symbols used in $\sqrt{a}$, $\sqrt[3]{a}$, $\sqrt[4]{a}$, $\sqrt[n]{a}$ etc., are called radical signs.

Here $\sqrt[n]{a}$ means, the positive nth root of the real number *a*.

Learners should be made acquainted with how to find and interpret the roots of the real numbers coming under the radical sign and the table of roots should be made familiar and squares, cubes, square roots, and cube roots of possible numbers can be asked to memorize which may help in their quick computations and calculations.

Using various examples and an inductive approach, learners are to be asked to derive the following identities related to square roots, which are useful in solving many problems on real numbers.

Let *a* and *b* be positive real numbers.

(i) $\sqrt{ab} = \sqrt{a}\text{-}\sqrt{b}$

(ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+ \sqrt{b})\ (\sqrt{a}\text{-}\sqrt{b}) = a - b$

(iv) $(a + \sqrt{b})\ (a - \sqrt{b}) = a^2 - b$

(v) $(\sqrt{a}+ \sqrt{b})\ (\sqrt{c} - \sqrt{d}) = \sqrt{ac} - \sqrt{ad} + \sqrt{bc} - \sqrt{bd}$

(vi) $(\sqrt{a}+ \sqrt{b})^2 = a + 2\sqrt{ab} + b$

Learners are to be exposed to various problem situations involving irrational numbers and their simplifications.

Learners are to be made clear about the rationalization process and why it is required.

Locating an irrational number like $\frac{1}{\sqrt{7}}$ on the number line would be easy if we convert the denominator to a rational number. So, rationalization is a process of converting irrational denominators to rationales.

To do the rationalization, knowing the identities involving square roots are very essential.

So, when the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called rationalizing the denominator.

Learners are to be asked to solve many problems involving the rationalization process.

Learners should be explained about very complex numerical computations can be made easy by applying appropriate rules and properties of numbers in various problem situations.

Video Link: IDENTITIES AND ITS APPLICATION IN RATIONALIZATION

Recommended Teaching Methodologies:

- **Inductive method**
- **Blended learning method**
- **Lecture method**
- **Demonstration method**

Notes:

UNIT – 6

LAWS OF EXPONENTS FOR REAL NUMBERS.

Teaching Points: Laws of Exponents for Real Numbers.

Warm-up activity for learners:
Let the learners recall all identities involving square roots of real numbers and ask them to state laws of exponents for natural numbers.

Teaching Process:

In earlier classes, the learner has learnt if *a*, *n* and *m* are natural numbers, *a* as the base, and *m* and *n* are exponents:

(i) $a^m . a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\frac{a^m}{a^n} = a^{m-n}$, m>n

(iv) $a^m b^m = (ab)^m$ (v) $a^o = 1$

Now using the results (iii) and (v), let the learners know that the above results can be extended to negative exponents also.

$a^o = 1$ and $\frac{a^m}{a^n} = a^{m-n}$

So $\frac{a^o}{a^n} = a^{o-n}$

i.e., $\frac{1}{a^n} = a^{-n}$

Now let them verify, if the base is a positive real number and exponents are rational numbers, whether laws are applicable or not.

Earlier it is taught that for a real number $a > 0$, and a positive integer n,

$\sqrt[n]{a}$ = b, if $b^n = a$ and $b > 0$.

Also, learners have learned in earlier classes**:**

Exponent means, how many times the base is to be multiplied by itself. In fact, multiplication leads to exponents.

2^n = 2 x 2 x 2 ……. x 2 (n times)

a^n = a x a x a ……. x a (n times)

a^n means, the number ‘a’ for which ‘n’ times it is taken as a factor.

Let the learners understand, the process of finding power and root are inverse processes.

i.e., if $3^2 = 9$ then $\sqrt[2]{9} = 3$

If $2^4 = 16$ then $\sqrt[4]{16} = 2$

Same way, if $2^n = b$ then $\sqrt[n]{b} = 2$.

In general, if $a^n = b$ then $\sqrt[n]{b} = a$

Now using exponents, we define, $\sqrt[n]{a} = a^{1/n}$

[Here if $b = a^{1/n}$, then $b^n = (a^{1/n})^n = a$]

e.g., $\sqrt[3]{5} = 5^{1/3}$

What is $4^{\frac{3}{2}}$?

Can be solved in two ways:

(i) $4^{\frac{3}{2}} = (4^{\frac{1}{2}})^3 = (\sqrt{4})^3 = 2^3 = 8$

(ii) $4^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = \sqrt{64} = 8$

From the above, let us derive the following :

If $a > 0$ is a real number and 'm', 'n' are integers such that 'm' and 'n' have no common factors other than 1, and $n > 0$, then.

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

If $a > 0$ is a real number and p and q are rational numbers, then.

(ii) $a^p \cdot a^q = a^{p+q}$ (iii) $(a^p)^q = a^{pq}$

(iv) $\frac{a^p}{a^q} = a^{p-q}$ (v) $a^p b^p = (ab)^p$, where $b > 0$ is also a real number.

Illustrations of different problems involving the laws of exponents to be done and various problems situations to be assigned to the learners to solve.

Video Link: **MORE ABOUT LAWS OF EXPONENTS**

Recommended Teaching Methodologies:

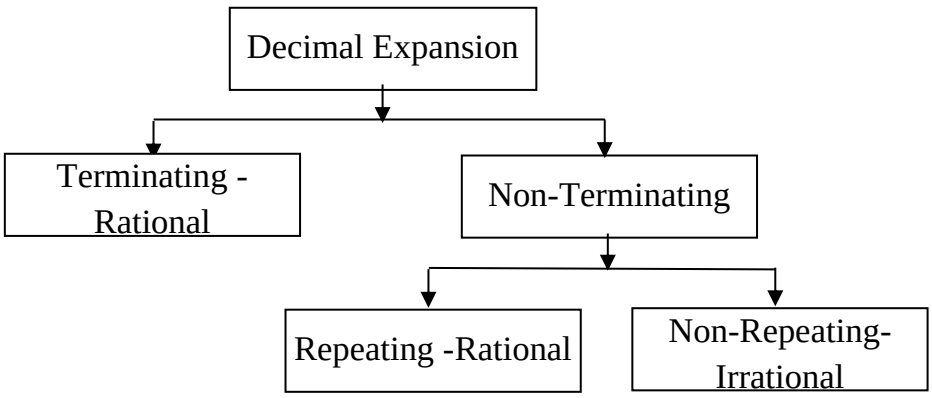
- **Lecture method**
- **Inductive method**
- **Demonstration method**

Notes:

UNIT – 7

SUM UP OF THE CHAPTER

1. Set of Natural Numbers is the smallest set of numbers.
2. $N \subset W \subset Z \subset Q$
3. A number *r* is called a rational number if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
4. A number *s is* called an irrational number, if it cannot be written in the form of

   $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
5. All rational and irrational numbers together make a set of Real numbers (R).
6. $N \subset W \subset I \subset Q \subset R$, also irrational number $\subset R$.
7. The decimal expansion of a rational number is either terminating or non-terminating recurring. Moreover, a number whose decimal expansion is terminating, or non-terminating recurring is rational.
8. The decimal expansion of an irrational number is non-terminating and non-recurring. Moreover, a number whose decimal expansion is non-terminating and non-recurring is irrational.
9. 

10. Every Real Number can be located on the Number Line.

11. Rational and irrational numbers with decimal expansions non-terminating recurring and non-terminating non-recurring respectively can be located on the number line using the process of successive magnification.

12. For positive real numbers a and b, the following identities hold:

    (i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$ (iv) $(a+\sqrt{b})((a-\sqrt{b}) = a^2 - b$

(v) $(\sqrt{a}+\sqrt{b})^2 = a + 2\sqrt{ab} + b$

13. To rationalize the denominator of $\frac{1}{\sqrt{a}+b}$ we multiply this by $\frac{\sqrt{a}-b}{\sqrt{a}-b}$, where a and b are integers.

14. Let a > 0 be a real number and p and q be rational numbers. Then

(i) $a^p . a^q = a^{p+q}$ (ii) $(a^p)^q = a^{pq}$

(iii) $\frac{a^p}{a^q} = a^{p-q}$ (iv) $a^p b^p = (ab)^p$

Notes:

UNIT – 8

NEP-2020 ASPECTS

NEP- 2020 Aspects:

Below given are a few activities and projects which can be explained to learners to make them understand about few aspects of NEP-2020 to develop creative mathematical thinking ability in Mathematics Education.

This will indirectly encourage the learners to find interest in the subject and will enhance their concept clarity.

Art Integrated Project:

Ask the learners to understand and use the concept of **Lippan Art** to represent the entire Number Systems to make a wall piece display.

Experiential learning activity:

Learners can be asked to visit a nearby jeweler/goldsmith to understand the successive magnification concept involved while making the very small, weighted gold ornaments.

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An Interdisciplinary aspect:

Subject: ENGLISH

A 'limerick' writing using the concept of a 'Number System.'

For a number to be rational,
It would have to be fractional.
An irrational will simply refuse,
To be a fraction, or to reduce.
But in any case, they both will still be real.

UNIT – 9

ASSESSMENT METHODS

Every topic/ content presented in the class by the teacher and learned by the learner is to be assessed on the very next period / next day, by using the warm -up activities mentioned in each unit.

After the completion of the chapter, a **Formative Assessment** to be conducted to know the learning status. Based on the performance of the learners, teachers need to make modifications in the teaching activities and in the methodology, if required.

A **diagnostic test** can be conducted to know the specific problem area of learning of different learners and accordingly required remedies to be provided.

Notes:

UNIT – 10

SUGGESTED READINGS & REFERENCES

Sharma, R.D. (2023, April 24). *R.D. Sharma Class 9 - Chapter 1 Number Systems updated for 2023-2024*

IGNOU (2017). *Teaching Learning of Mathematics*. New Delhi

.Egyankosh. https://egyankosh.ac.in/bitstream/123456789/46799/1/BES-143B2-E.pdf

NCERT (2012). *Pedagogy of Mathematics*, NCERT, New Delhi

James, A. (2005). *Teaching of Mathematics*. Neelkamal Publication, New Delhi.

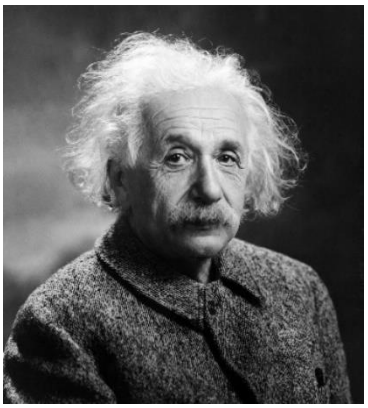
Self-assessment series of questions:

MTG Learning Media (P)Ltd. (2020). *100 PERCENT MATHEMATICS*, Class 9, Gurugram, New Delhi.

MODULE – II

TOPIC: REAL NUMBERS

Subject: Mathematics **Class: X**



"The formulation of a problem is often more essential than its solution which may be merely a matter of mathematical or experimental skills."

- Albert Einstein

Introduction:

This module is prepared with an aim to enhance the teachers to employ a different teaching strategy in clear perception about the topic, Real Numbers. The methodology of teaching mentioned in the module gives space to the learners to explore the world of Real Numbers, specifically revisiting irrational numbers. While using this module, teachers should focus on helping the learners to develop the ability to particularize Euclid's Division Lemma, the Fundamental Theorem of Arithmetic, and other important concepts. Revisiting irrational numbers and decimal expansions of rational numbers, explained in this module shall help the teachers to give a deeper knowledge of those concepts and can pass it on to the learners in implementing or applying the knowledge for solving related real-life problems.

Objectives / Expected Learning Outcomes:

After doing the units, teachers should be able to help the learners to :

- Gain deep and diversified knowledge of Real Numbers.
- Understand Euclid's Division Lemma.
- Know what Euclid's Division Algorithm is.
- Know Prime and Composite numbers and the Fundamental Theorem of Arithmetic.
- Apply the relation between HCF and LCM in solving related problems.
- Understand in detail about irrational numbers. Theorems on irrational numbers and how to prove a number, irrational.
- Understand in depth the decimal expansions of rational numbers and related theorems.

- Apply the Fundamental Theorem of Arithmetic and other results on Real Numbers in solving various problems.

<u>It will enable the teachers to:</u>

- Understand the various strategies to introduce the topics mentioned.
- Understand how other teaching methodologies can be adopted for the mentioned topics.

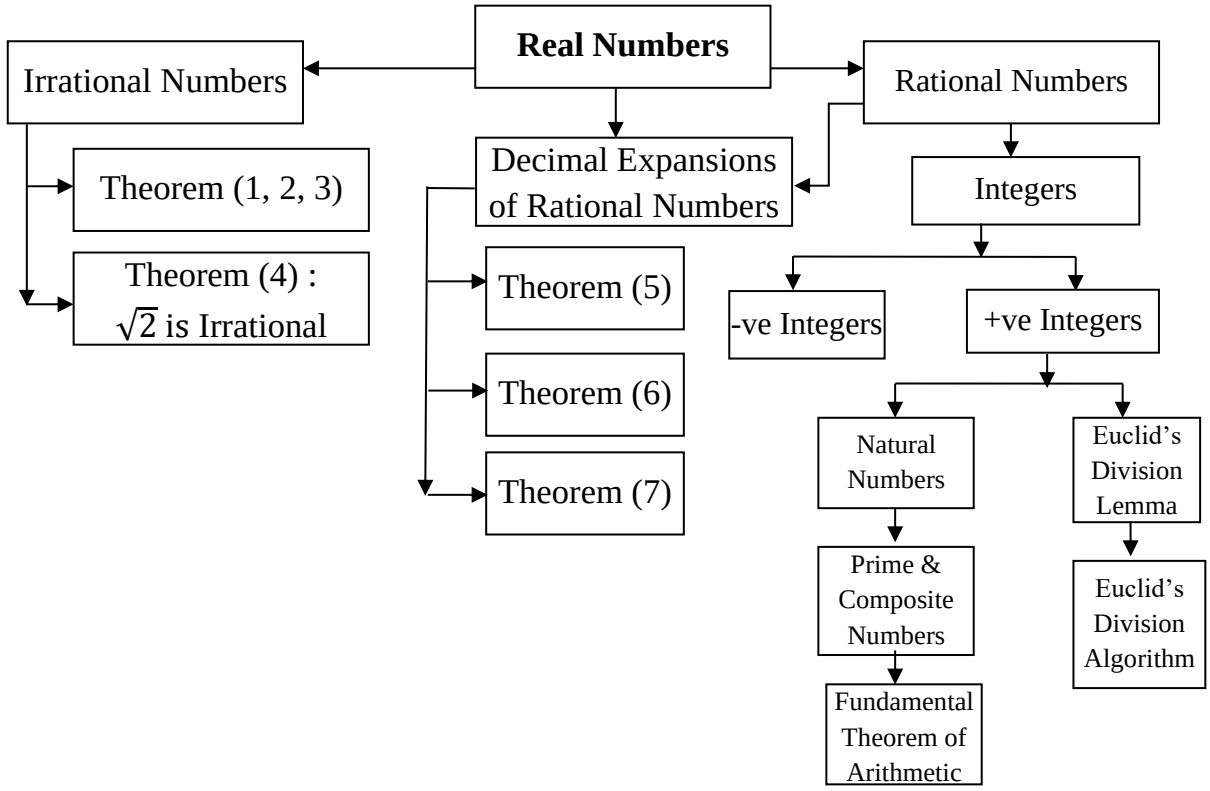
Pre-requisite knowledge is required for learners.

1. Basic ideas of the Number Systems.
2. Prime, Composite numbers, HCF, LCM, and Decimal Expansions.
3. Difference between Irrational and Rational numbers.

Key Concepts / Important Terms:

1. Real numbers
2. Rational and Irrational numbers
3. Decimal expansions
4. Division Lemma
5. Division Algorithm

CONCEPT MAP



Theorem (1): Euclid's Division Lemma

Theorem (2): Fundamental Theorem of Arithmetic

Theorem (3): If a prime number p divides a^2 then p divides a where a is a positive integer.

Theorem (5): Let 'x' be a rational number whose decimal expansion terminates, then 'x' can be expressed in the form $\frac{p}{q}$, where 'p' and 'q' are co-prime and the prime factorization of 'q' is of the form $2^n 5^m$, where 'n', 'm' are non-negative integers.

Theorem (6): Let 'x' $= \frac{p}{q}$ be a rational number, such that the prime factors of 'q' is of the form $2^n 5^m$ where 'n', and 'm' are positive (no negative) integers. Then 'x' has a decimal expansion which terminates.

Theorem (7): Let 'x' $= \frac{p}{q}$ be a rational number, such that the prime factorization of 'q' is not of the form $2^n 5^m$, where 'n' and 'm' are non-negative integers. Then 'x' has a decimal expansion which is non-terminating repeating (recurring).

INDEX

UNIT – 1

EUCLID'S DIVISION LEMMA

Teaching Point: Euclid's Division Lemma

Warm-up activity for learners:

Let the learners recall the learned results on Number Systems.

1. State the set of Positive Integers.
2. Give examples of Rational and Irrational Numbers
3. What are HCF and LCM?
4. Give examples for Prime and Composite Numbers.

Teaching Process:

Since Primary classes, learners have learned about division, how to derive division facts from multiplication facts, and visa-a-versa.

Let the learners recall the division through various examples.

There are 5 chocolates. They need to be equally given to 2 children. How many chocolates each child will get? How it can be mathematically presented?

$$5 = 2 \times 2 + 1$$

Learners understand, after equal distribution, 1 chocolate will remain.

Similarly, how we can pack 11 mangoes equally in 4 boxes?

$$11 = 4 \times 2 + 3$$

Let the learners understand: for any two positive integers 11 and 4, there exist two integers 2 and 3 such that.

$$11 = 4 \times 2 + 3$$

Similarly, for two positive integers 17 and 6, there exist two unique numbers, 2 and 5 such that $17 = 6 \times 2 + 5$

Let the learners know the fact that 2 and 5 are unique because they are the only integers satisfying the above.

To generalize the above fact, it can be stated that for any two integers a and b (a being the dividend b being the divisor) there are two unique integers q and r (q = quotient and r = remainder) such that

$$a = bq + r,\ o \leq r < b$$

This simple-long division process, which learners have been doing since small classes, is known as **Euclid's Division Lemma** which can be stated in the form of a theorem.

Theorem (1) Euclid's Division Lemma: Given positive integers '*a*' and '*b*', there exist, unique integers '*q*' and '*r*' satisfying $a = bq + r,\ o < r < b$

Thought to Ponder:

1. **What is a Lemma?**

A Lemma is a proven statement used for proving another statement.

2. **What is an algorithm? From where did the word come?**

An algorithm is a series of well-defined steps that give a procedure for solving a type of problem.

Now let the learners know about **Euclid's Division Algorithm:**

It is a technique to compute the Highest Common Factor (HCF) of two given positive integers, which is based on Euclid's Division Lemma.

Illustrations to be made on how Euclid's Division Algorithm can be used to find the HCF of two positive integers.

Let the learners reiterate the steps in Euclid's Division Algorithm:

To find the HCF of two positive integers say 'c' and 'd', where c > d, then

Step (1): Using Euclid's Division Lemma to 'c' and 'd' and find two unique whole numbers 'q' and 'r' such that $c = d \times q + r,\ 0 \leq r < d$

Step (2): If r = 0, d is the HCF of c and d.

If $r \neq 0$, then apply division Lemma to 'd' and 'r'.

Step (3): Continue till the remainder becomes zero.

Divisor at this stage will be the required HCF.

The learners are to be explained that the division Lemma / Algorithm has several applications related to the properties of numbers.

Various examples are to be illustrated.

It is also to be explained to the learners that the application of Euclid's division algorithm is not only for calculating the HCF, but it is one of the earliest examples of an algorithm that a computer had been programmed to carry out.

Also, the learners must understand the fact that though Euclid's Division Algorithm is stated for the positive integers, it is applicable for all integers except 0. (In the advanced study, this fact will be learned in detail, by the learners.)

Various applications of Euclid's division Lemma / Algorithm in finding the HCF of big numbers and in finding the properties of numbers – to be illustrated and to be asked to solve by learners.

1. Find the HCF of 37728 and 12156 by Euclid's division algorithm.

   Solution:

   Here 37728 > 12156

   Apply Euclid's division lemma to get 37728 = 12156 x 3 + 1260

   Reminder = 1260 ≠ 0

   Apply Euclid's division lemma to get 12156 = 1260 x 9 + 816.

   Proceed in the same manner, till the remainder becomes 0.

   1260 = 816 x 1 + 444

   816 = 444 x 1 + 372

   444 = 372 x 1 + 72

   372 = 72 x 5 + 12

   72 = **12** x 6 + 0

   So, by Euclid's division algorithm.

   HCF (37728, 12,156) = 12.

Let the learners try by themselves to solve the following kind of problems:

1. Show that the square of any odd integer is of the form 4q + 1, for some integer q.
2. Show that for a positive integer "n" one and only one out of n, n + 1, n + 2 is divisible by 3.

Such needful assignments are to be given from the textbook and other reference books, till the learners get clarity on the process.

Learners are to be asked to repeatedly recall Euclid's division lemma as well as Euclid's division algorithm.

Video Link: **EUCLID'S DIVISION LEMMA & ALGORITHM**

Recommended Teaching Methodologies:

- **Induction method**
- **Illustration method**
- **Lecture method**
- **Blended learning method**

Notes:

UNIT – 2

FUNDAMENTAL THEOREM OF ARITHMETIC.

Teaching Point: Fundamental Theorem of Arithmetic.

Warm-up activities for learners:

Ask the learners to:

1. State Euclid's Division Lemma and Algorithm.
2. Recall the prime factorization of a number.
3. Recall composite numbers.

Teaching Process:

Let the learners be aware of the following important facts about **prime** and **composite** numbers which they have learned in the lower grades.

1. Any integer greater than 1 is either a prime number or can be expressed or factorized as the product of prime numbers (Prime factors).
2. A number is called a composite number, if it has at least one factor, other than 1 and the number itself.
3. 1 is neither considered a prime nor a composite number.
4. 2 is the only even prime number and 2 is the smallest prime number.
5. If two numbers have no common factors other than 1, (i.e., HCF = 1) then they are called **co-prime** numbers.

In earlier classes, learners learned about the prime factorization of numbers. They need to be made aware of the fact that by multiplying some prime numbers we can produce a collection of positive integers. Since there are infinitely many prime numbers, we can get infinitely many positive integers by multiplying prime numbers.

e.g., : $2 \times 3 \times 5 = 30$,

$2^3 \times 3 \times 73 = 8232$,

$7 \times 11 \times 13 = 1001$, etc.

A composite number is a number having factors other than 1 and that number itself.

Now the question in the mind of learners is, whether every composite number can be expressed as the product of primes or not.

Let the learners try to express any taken composite number as the product of primes.

3825 = 5 x 765

= 5 x 5 x 153

= 5 x 5 x 3 x 51

16380 = 2 x 8190

= 2 x 2 x 4095

= 2 x 2 x 3 x 1365

= 2 x 2 x 3 x 3 x 455

= 2 x 2 x 3 x 3 x 5 = 91

= 2 x 2 x 3 x 3 x 5 x 7 x 13

Through such verification, let the learners understand that every composite number can be written as the product of primes. This property of composite numbers is stated as a theorem called **The Fundamental Theorem of Arithmetic** which is very important in the study of integers.

Theorem (2) Fundamental Theorem of Arithmetic or Unique Factorization Theorem:
Every composite number can be expressed or factorized as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.

Explain the learners that the Fundamental Theorem of Arithmetic has many applications, not only in Mathematics but in other fields as well.

Let the learners understand that from this theorem, many other results on numbers can be derived. This theorem states that there is a unique way of expressing the prime factors because we are particular about the order in which the primes occur.

This fact can be stated as:

The prime factorization of a natural number is unique, except for the order of its factors.

In general, if x is a composite number, $x = p_1p_2p_3....p_n$, where $p_1, p_2,....p_n$ are primes and $p_1 \leq p_2 \leq p_3 \leq ..... \leq p_n$.

Learners are to be asked to understand the various applications of the Fundamental Theorem of Arithmetic in different problem areas through illustrations and assignments.

For any natural number *n*, check whether 2^n can be a number with the end digit 0 or 5.

Let the learners justify the answer. Make the learners understand that they have already learned how to find the HCF and LCM of two positive integers, using the Fundamental Theorem of Arithmetic in earlier classes, without realizing it!

This method is also called the prime factorization method. Let them recall this method through examples.

Find the LCM and HCF of 12 and 28 by the prime factorization method.

Solution:

We have:

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$28 = 2 \times 2 \times 7 = 2^2 \times 7$

HCF (12, 28) = 2^2 and LCM (12, 28) = $2 \times 2 \times 3 \times 7 = 84$, as done in earlier classes.

HCF= Product of the smallest power of each common prime factor in the numbers.

LCM =Product of the greatest power of each prime factor, involved in the numbers.

Through such examples, learners to be asked to verify the fact that**:**

for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b

Explain that this fact can be used to find the LCM of two positive integers if we have already found the HCF of those two positive integers.

Let the learners also verify the fact that this is not true in the case of 3 numbers.

Means HCF (a, b, c) x LCM (a, b, c) ≠ a x b x c.

i.e., the product of three numbers is not equal to the product of their HCF and LCM.

Learners 'concept clarity is to be made clearer by illustrating and assigning different problems to solve, from textbooks as well as from reference books.

Ask the learners to try themselves to solve the following kind of problems, to verify their concept clarity and understanding.

1. A circular sports ground has a circumference of 540 m. The students, Ram and Shyam start cycling together at constant speeds of 3 m/s and 6 m/s respectively. Find out, after how many minutes, they will meet again at the starting point.

Solution:

Circumference: 540 m

Time taken by Ram to complete one round = $\frac{540}{3}$ = 180 sec.

Time taken by Shyam to complete one round: = $\frac{540}{6}$ = 90 sec.

The required number of minutes, when they meet again, is the LCM of 180 and 90.

$180 = 2^2 \times 3^2 \times 5$

$90 = 2 \times 3^2 \times 5$

LCM (180, 90) = $2^2 \times 3^2 \times 5$ = 180 seconds= 3 minutes.

For solving the above kind of problems, learners should have the ability or the skill to understand the specific operation which needs to be adopted to find the solution.

The ability to identify a procedure/formula/operation, which to be applied in a specific problem area to find its solution is termed as Metacognition ability. In Mathematics, developing the Metacognition ability/skill of learners is the role of the mathematics teacher.

The learners are to be asked to explore more such problems.

If x and y are two positive integers, expressed in terms of primes as $x = p^2q^3$ and $y = p^3q$, then verify whether their LCM is a multiple of HCF or not.

Video Link: TIPS TO IDENTIFY - HCF AND LCM

Ask the learners to collect information about the Mathematician Carl Friedrich Gauss and his contributions to arithmetic.

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Blended learning method**
- **Illustrative method**

Notes:

UNIT – 3

REVISITING IRRATIONAL NUMBERS.

Teaching Point: Revisiting Irrational Numbers.

Warm-up activity for learners:

Recall all the properties of Irrational Numbers.

Teaching Process:

In earlier classes, learners have learnt how to represent irrational numbers on the number line.

Explain to the learners that, using the Fundamental Theorem of Arithmetic, it can be proved that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$.... in general, $\sqrt{P}$ is an irrational number, where P is a prime number.

Before providing it as a theorem, they need to know about another theorem based on the Fundamental Theorem of Arithmetic.

Theorem (3):

Let *p* be a prime number. If *p* divides a^2, then *p* divides *a*, where *a* is a positive integer.

Proof: Let the prime factorization of *a* be as follows:

$a = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots, p_n$ are primes, not necessarily distinct. Therefore, $a^2 = (p_1 p_2 \ldots p_n)(p_1 p_2 \ldots p_n) = p_1^2 p_2^2 \ldots . p_n^2$.

Now, it is given that p divides a^2. Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2. However, using the uniqueness part of the Fundamental Theorem of Arithmetic, let the learners realize that the only prime factors of a^2 are $p_1, p_2, \ldots, p_n$. So p is one of $p_1, p_2, \ldots, p_n$. Now, since $a = p_1 p_2 \ldots . p_n$, p divides a.

Now the learners will be able to understand the proof - $\sqrt{2}$ is irrational. The proof is based on a technique called 'proof by contradiction'. (This technique is discussed in detail in Appendix 1 of the NCERT Textbook of Class X)

Theorem (4):

$\sqrt{2}$ is irrational.

Proof: Ask the learners to assume, to the contrary that $\sqrt{2}$ is rational.

So, the learners can find integers *r and s* ($\neq 0$) such that $\sqrt{2} = \frac{r}{s}$.

(Suppose r and s have a common factor other than 1. Then, we divide it by the common factor to get $\sqrt{2} = \frac{a}{b}$, where a and b are co-prime.)

So, b $\sqrt{2}$ = a.

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2.

Now, by Theorem 3, it follows that 2 divides a.

So, it to be written that a = 2c for some integer c.

Substituting for a, $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. This means that 2 divides b^2, and so 2 divides b (again using Theorem 3 with p = 2).

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of the incorrect assumption that $\sqrt{2}$ is rational. So, it can be concluded that $\sqrt{2}$ is irrational.

Let the learners understand that many such results can be proved by **proof by contradiction**.

In earlier classes, learners have learned:

- The sum or difference between a rational and an irrational number is irrational and
- The product and quotient of a non-zero rational and irrational number is irrational.

Using these results and through proof by contradiction, some other cases can be proved.

Video Link: Proof by Contradiction

Let the learners try by themselves the following kinds of problems.

Show that $7 - 2\sqrt{3}$ is irrational.

Solution:

Suppose that $7 - 2\sqrt{3}$ is rational.

That is, learners find co-prime a and b ($b \neq 0$) such that $7 - 2\sqrt{3} = \frac{a}{b}$.

$7 - \frac{a}{b} = 2\sqrt{3}$·

$\frac{7}{2} - \frac{a}{2b} = \sqrt{3}$

But $7 - \frac{a}{b}$ is rational.

So $\sqrt{3}$ is rational.

Which is a contradiction. So, the supposition is wrong.

i.e., $7 - 2\sqrt{3}$ is irrational.

Learners are to be asked to practice and prove all these kinds of facts and results.

1. If $\sqrt{pq}$ is an irrational number, then prove that $\sqrt{p} + \sqrt{q}$ is an irrational number.

Recommended Teaching Methodologies:

- **Inductive method**
- **Demonstration method**
- **Lecture method**
- **Blended learning method**

Notes:

UNIT – 4

RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

Teaching Points: Revisiting Rational Numbers and their Decimal Expansions.

Warm-up activities for learners:

Ask the learners to recall rational numbers and their decimal expansions.

Decimal expansions of rational numbers can be of how many types?

Teaching Process:

In earlier classes learners have studied that decimal expansions of rational numbers are either terminating or non-terminating recurring.

Now the learners are to be explained about the exact nature of rational numbers and their decimal expansions, i.e., when the decimal expansion of the rational number $\frac{p}{q}$ is going to be terminating and when it is non-terminating, recurring.

Learners to be asked to convert decimal expansions of various rational numbers, which are terminating to convert in the form of $\frac{p}{q}$ in its reduced form.

(i.e., numerator and denominator are co-prime)

1) $0.65 = \frac{65}{100} = \frac{65}{10^2} = \frac{13 \text{ x } 5}{(2 \text{ x } 5)^2} = \frac{13\ x\ 5}{2^2 \text{ x } 5^2} = \frac{13}{2^2 x\ 5}$

2) $3.225 = \frac{0.225}{1000} = \frac{(15)^2}{(10)^3} = \frac{(3 \text{ x } 5)^2}{10^3} = \frac{2^2 \text{x } 5^2}{(2 \text{ x } 5)^3}$

$= \frac{3^2\ x\ 5^2}{2^3\ x\ 5^3} = \frac{3^2}{2^3 \text{ x } 5} = \frac{7}{2^3 \text{ x } 5}$

Here, the learners to be asked to observe a pattern :

i.e., the terminating decimal expansions can be converted into a rational number of the form $\frac{p}{q}$, where p and q are co-prime and the prime factorization of the denominator, q has only power of 2 or power of 5 or both.

This property can be verified with any decimal expansion of rational number which is terminating. This helps the learners to generalize and conclude in the form of given theorem.

Theorem (5):

Let ‘x’ be a rational number whose decimal expansion terminates, then ‘x’ can be expressed in the form $\frac{p}{q}$, where ‘p’ and ‘q’ are co-prime and the prime factorization of ‘q’ is of the form $2^n 5^m$, where ‘n’, ‘m’ are non-negative integers.

Now let the learners think about the reverse of this theorem.

If a rational number $\frac{p}{q}$, whose denominator q can be converted as the power of 10, (means if q is of the form $2^n 5^m$), then what about the decimal expansion of that rational number?

Let the learners do various examples of rational numbers whose denominators can be converted in power of 10 (in the form $2^n 5^m$)

i) $\frac{3}{5} = \frac{3 \text{ x } 2}{5 \text{ x } 2} = \frac{6}{10} = 0.6$

ii) $\frac{8}{40} = \frac{2^3}{2^2 \text{ x } 10} = \frac{2}{10} = 0.2$

iii) $\frac{7}{8} = \frac{7}{2^3} = \frac{7 \text{ x } 5^3}{2^3 \text{x } 5^3} = \frac{7 \text{ x } 125}{(2 \text{ x } 5)^3} = \frac{875}{1000} = 0.875$

Such rational numbers can have decimal expansions, only terminating.

Let the learners convert this fact in the form of another theorem.

Theorem (6):

Let 'x' = $\frac{p}{q}$ be a rational number, such that the prime factors of 'q' is of the form $2^n 5^m$ where 'n', 'm' are positive (no negative) integers. Then 'x' has a decimal expansion which terminates.

Now ask the learners to consider few rational numbers whose decimal expansions are non-terminating, but recurring.

$\frac{3}{7}$ = 0.428571428571.....

```
   0.4285714
7) 30
   -28
   ------
    20
   -14
   ------
     60
    -56
    ------
      40
     -35
     ------
      50
     -49
     ------
       10
        -7
        ------
         30
         -28
         ====
```

Let the learners understand that here the denominator can never be converted in power of 10 or in the form $2^n 5^m$.

So, there are rational numbers where denominators cannot be converted as the power of 10 or in the form of $2^n 5^m$. Such all rational numbers have their decimal expansions non-terminating, recurring.

This fact can be expressed as another theorem, given below:

Theorem (7):

Let 'x' $= \frac{p}{q}$ be a rational number, such that the prime factorization of 'q' is not of the form $2^n 5^m$, where 'n' and 'm' are non-negative integers. Then 'x' has a decimal expansion which is non-terminating repeating (recurring).

Form theorems 5, 6, 7, let the learners conclude that the decimal expansion of every rational number is either terminating or non-terminating repeating (recurring).

Video Link: **FUNDAMENTAL THEOREM OF ARITHMETIC**

Ask the learners to go through various illustrations and assignments to be given to make their concept clearer with respect to theorem 5, 6 and 7.

Recommended Teaching Methodologies:

- **Induction method**
- **Demonstration method**
- **Blended learning method,**
- **Lecture method**

Notes:

UNIT – 5

SUM UP OF THE CHAPTER

1. Euclid's division lemma: Given positive integers a and b, there exist whole numbers q and r satisfying $a = bq + r$, $0 \leq r < b$.
2. Euclid's division algorithm: This is based on Euclid's division lemma. According to this, the HCF of any two positive integers a and b, with $a > b$, is obtained as follows:
   Step 1: Apply the division lemma to find q and r where $a = bq + r$, $0 \leq r < b$.
   Step 2: If $r = 0$, the HCF is b. If $r \neq 0$, apply Euclid's lemma to b and r.
   Step 3: Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b). Also, HCF (a, b) = HCF (b, r).
3. The Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.
4. If 'p' is a prime number and 'p' divides 'a^2' then p divides 'a', where 'a' is a positive integer.
5. $\sqrt{2}$, $\sqrt{3}$, etc. can be proved as irrational numbers.
6. Let x be a rational number whose decimal expansion terminates. Then we can express x in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers.
7. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^n5^m, where n and m are non-negative integers. Then x has a decimal expansion which terminates.
8. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^n5^m, where n and m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

UNIT – 6

NEP-2020 ASPECTS

NEP- 2020 Aspects:

Below given are a few activities and projects which can be explained to learners to make them understand about few aspects of NEP-2020 to develop creative mathematical thinking ability in Mathematics Education.

This will indirectly encourage the learners to find interest in the subject and will enhance their concept clarity.

Art Integrated Project:

Integrate the Art form **Applique work** of Saurashtra, Gujarat to make an Algorithm to find out the HCF of 2 numbers.

Experiential learning activity:

Apply the concept of Euclid's Division Lemma to distribute a packet of 100 chocolates equally to 40 students of grade 9, on the Birthday of a student of the class. Ask the learners to identify the 'q' and 'r' of the Euclid's Division Lemma in this case**.**

An Interdisciplinary Aspect:

Subject: Computer Science

Algorithm to find HCF.

Step 1: Initialize a variable 'hcf' to return the answer i.e., hcf=1.

Step 2: Find the minimum of two numbers, 'n1' and 'n2'

Step 3: Run a loop from 1 to 'min' value.

Step 4: For each value of 'i', check if 'i' completely divides 'n1' and 'n2', then set the value of 'hcf' to 'i'

Step 5: Return value of 'hcf' variable.

Algorithm of factorial of a number

Step 1: Start

Step 2: Read a number n.

Step 2: Initialize variables i = 1, fact = 1

Step 3: If i<= n go to step 4 otherwise go to Step 7: Step 4: Calculate fact = fact * i

Step 5: Increment the i by 1 (i= i+1) and go to step 3

Step 6: Print the fact.

Step 7: Stop

Notes:

UNIT – 7

ASSESSMENT METHODS

Every topic/content presented in the class by the teacher and learned by the learner is to be assessed on the very next period/next day, by using the warm -up activities mentioned in each unit.

After the completion of the chapter, a **Formative Assessment** to be conducted to know the learning status. Based on the performance of the learners, teachers need to make modifications in the teaching activities and in the methodology, if required.

A **diagnostic test** can be conducted to know the specific problem area of learning of different learners and accordingly required remedies to be provided.

Notes:

UNIT – 8

SUGGESTED READINGS & REFERENCES

References:

Sharma, R.D. (2023, April 24). *R.D. Sharma Class 10 - Chapter 1Real Numbers updated for 2023-2024*

IGNOU (2017). *Teaching Learning of Mathematics*. New Delhi .Egyankosh. https://egyankosh.ac.in/bitstream/123456789/46799/1/BES-143B2-E.pdf

NCERT (2012). *Pedagogy of Mathematics*, NCERT, New Delhi

James, A. (2005). *Teaching of Mathematics*. Neelkamal Publication, New Delhi.

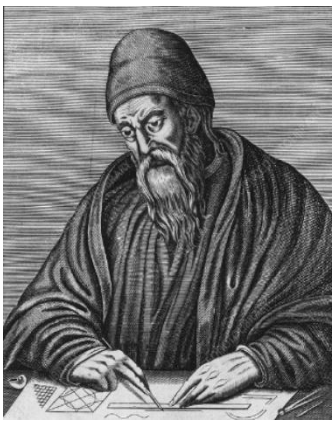
Self-assessment series of questions:

MTG Learning Media (P)Ltd. (2020). *100 PERCENT MATHEMATICS*, Class10, Gurugram, New Delhi

MODULE - III

TOPIC: TRIANGLES

Subject: Mathematics **Class: IX**



"The laws of nature are but the Mathematical thoughts of God."

- Euclid: Father of Geometry

Introduction:

This module on **triangles** aims to help the teachers to make the learners clear about the concept of triangles at large, the congruence of triangles, and some of their properties in specific. Reading the explanation given in the module, implementing the same in the class, and using the e-support given, can help the teachers in making the learners visualize, analyze and grasp the concept. It helps the teachers to achieve the expected learning outcomes from the learners and in helping the learners to solve similar real-life problems.

Objectives/Expected Learning Outcomes:

After doing all units, the teacher will be able to help the learners to:

- Recall the terms and basic properties already learned by them.
- Identify the congruency in triangles by verifying the congruency rules.
- Understand what the inequalities of triangles are.
- Investigate the congruency rules of Triangles in various life situations.
- Understand opposite side to angle and opposite angle to the side of a triangle.
- Understand and define the properties of various types of triangles based on the sides and angles.

- Do the exercise and solve similar problems in real life.

The Module also may help the teacher to:

- To understand the various strategies to introduce the topic Triangle.
- To blend the other teaching techniques with the traditional ones.

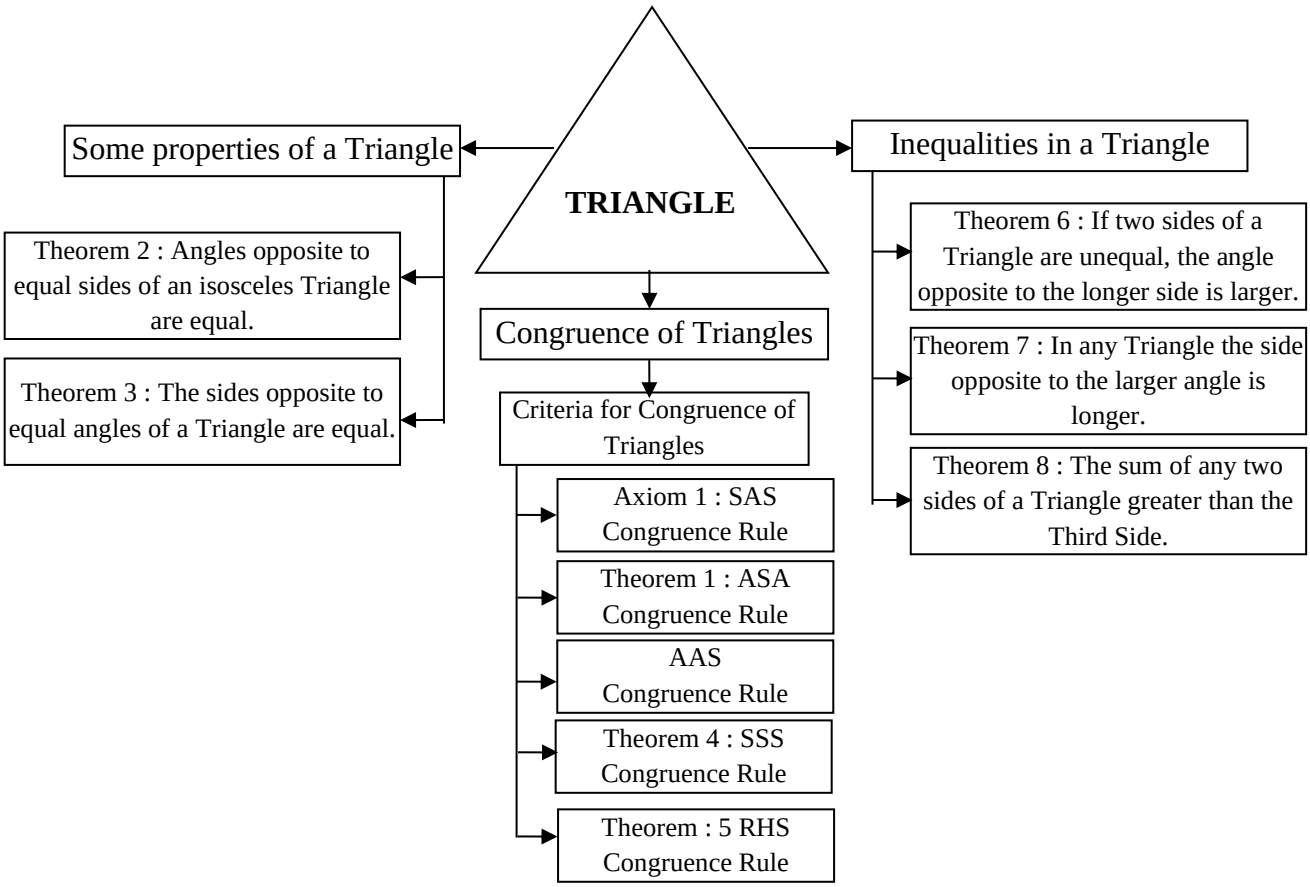
Pre-requisite knowledge, required for learners:

1. Meaning of the terms: Triangle, sides, angles, vertices, interior, exterior of the triangle, properties, congruence.
2. Properties of triangles learned in lower classes.

Key Concepts / Important Terms:

1. Congruence
2. Correspondence
3. CPCT, Axiom
4. Theorem
5. Congruence criteria.

CONCEPT MAP



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Unit – 1	Introduction : Congruence of Triangles.
Unit – 2	Criteria for Congruence of Triangles Axiom – 1 (SAS Congruence Rule) Theorem – 1 (ASA Congruence Rule) AAS Congruence Rule
Unit – 3	Some properties of a Triangle. (Theorem – 2 and 3)
Unit – 4	Some more properties of a Triangle (Theorem 4 and 5)
Unit – 5	Inequalities in a Triangle. (Theorem 6, 7, 8)
Unit – 6	Sum up of the Chapter (of all Units).
Unit – 7	NEP-2020 Aspects: • Art Integrated Project. • Experiential Learning Activity. • An Interdisciplinary aspect.
Unit – 8	Assessment Methods.
Unit – 9	Reference / Suggested Readings

UNIT – 1

TRIANGLES & CONGRUENCE OF TRIANGLES

Teaching Points: Triangles, Congruence of Triangles, Criteria of Congruence of Triangle,

Warm-up activities for learners:

Ask the learners to recall the previously learned matters about Triangle.

1. State the basic components of a triangle.
2. State the elements of a triangle.
3. Recollect some of the properties of triangles.
4. Recall: Scalene triangle, Equilateral triangle, Isosceles triangle, Acute, Obtuse and Right-angled triangle, and the Angle sum property of a triangle.

Teaching Process:

Discuss with learners about the similar shaped, similar sized figures present in the classroom.

Place one notebook on another. Are they covering each other completely?

Yes, such figures are called congruent figures.

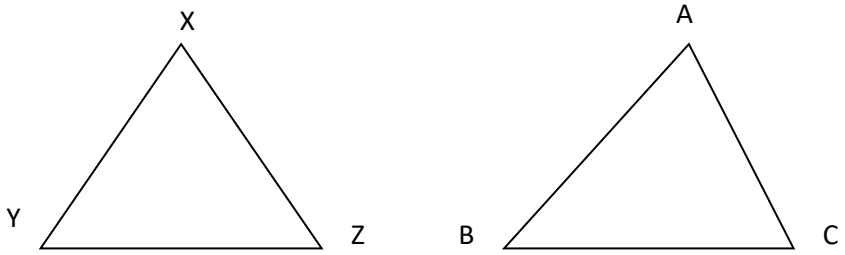
Ask the learners to think some more examples of congruent figures available in classrooms.

Ask the learners to draw different triangles and super impose them to identify congruent triangles.

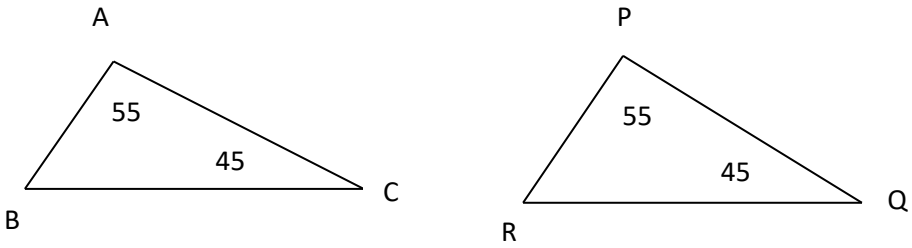
When two triangles can be congruent?

What is correspondence?

Discuss about all possible correspondences between the two triangles.



Under which correspondence, they can be congruent?



A $\Leftrightarrow$ P

B $\Leftrightarrow$ R

C $\Leftrightarrow$ Q

Let the learners understand:

Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.

i.e., ΔABC is congruent to ΔPRQ, which can be mathematically denoted as

$\Delta ABC \cong \Delta PRQ$ (Explain the learners, writing ΔPQR is not correct here)

Important:

Let the learners understand then fact that, corresponding parts of congruent triangle (CPCT) are always equal.

Also, two triangles are congruent only under one correspondence out of 6 correspondences between them.

Ask the learners to identify the congruence correspondence, between two congruent triangles, by mentioning all possible correspondences. (Through various examples)

Video Link: [The Correspondence between Two Triangles]

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Blended learning method**

Notes:

UNIT – 2

CRITERIA FOR CONGRUENCE OF TRIANGLES

Teaching Points: Criteria for Congruence of Triangles. (SAS, ASA/Theorem-I, and AAS Congruence Rule)

Warm-up activities for learners:

Ask the learners to recall the congruence conditions of two figures:

- How many correspondences can be formed between two triangles?
- Under how many correspondences, two triangles can be congruent?

Teaching Process:

Ask the learners: Is it necessary that we need to check all corresponding sides and angles (altogether 6 basic elements of components to predict about the congruence of two triangles?

Discuss:

Why are angles not considered as basic components of triangle?

Is there any possibility that with 2 equal sides and its included angle, any other triangle can be drawn so that it is not congruent with the given triangle? No.

Through specific examples and illustrations, learners to be explained that, under a given correspondence if two sides and included angle of one triangle are equal to the corresponding sides and angle of the other without measuring the other sides and angles, we can predict that triangles are congruent.

This is known as SAS congruence Rule (Axiom)

Axiom (1) (SAS Congruence rule)

Two triangles are congruent if two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle.

Ask the learners to recall, what is an axiom.

A statement accepted as true as that cannot be proved with the help of previously known results is called an Axiom.

Also, through various examples, let the learners understand that to predict the congruence of two triangles, equality of two angles and the included side with the corresponding angles and included side is sufficient.

This result is called Angle-Side-Angle (ASA) criterion and since it can be proved, it is stated as a theorem.

Theorem (1) (ASA – Congruence Rule):

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of another triangle.

While explaining the proof of the theorem, it to be emphasized to follow the necessary pattern or steps of proof.

So, two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal. We may call it as the AAS Congruence Rule.

Activity:

Ask the learners to draw triangles with angles 40o, 50o and 90o. How many such triangles can be drawn?

They understand that they can draw many triangles, with different lengths of sides.

Let the learners understand the fact that the triangles may or may not be congruent to each other.

So, equality of three angles is not sufficient for congruence of triangles. Therefore, for congruence of triangles, out of three equal parts, one part must be a side.

Discuss:

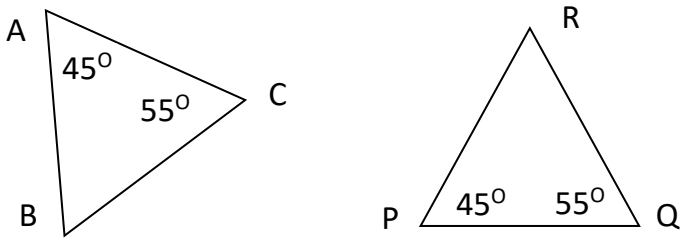
Is AAA congruence rule hold?

Is SSA congruence rule hold?

Ask the learners to verify the congruence of triangles through SAS congruence rule and ASA congruence rule by drawing a few pairs of triangles with corresponding side-included Angle-side and Angle Side-Angle equal.

Discuss by drawing specific pair of triangles in which 2 adjacent pair of angles are equal with the corresponding adjacent pair of angles under a correspondence between the two.

E.g., Under the correspondence ABC ~ PRQ.



Here learners understand that two pairs of angles are equal ($\angle A = \angle P$, $\angle C = \angle Q$) and one pair of corresponding sides are equal (AB = PR).

Let the learners think whether the triangles are congruent or not. Let them come out with the reasons for the congruence.

Reasons:

1. Sum of the 3 angles of a triangle is 180^o.
2. If two pairs of angles are equal, the third pair is also equal.
3. Then the corresponding sides given equal will become the included side of two angles.
4. So, by the SAS rule, triangles are congruent.

So, two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

This is AAS Congruence Rule.

Learners are to be given various illustrations and assignments to have conceptual clarity on this congruence rule.

Recommended Teaching Methodologies:

- **Demonstration method**
- **Illustration method**
- **Inductive method**
- **Blended learning method**

Notes:

UNIT – 3

SOME PROPERTIES OF TRIANGLES

Teaching Points: Some properties of Triangles.

Warm-up activity for learners:

Ask the learners to recall the learned congruence rules – SAS, ASA, AAS.

Teaching Process:

Discuss with learners:

By drawing a few pairs of isosceles triangles, ask them to measure the angles opposite to the equal sides.

Learners understand that the angles opposite to the equal sides are equal, which is a very important result, true for any isosceles triangles.

This result is stated as Theorem (2).

Theorem (2): (Isosceles Triangles Theorem)

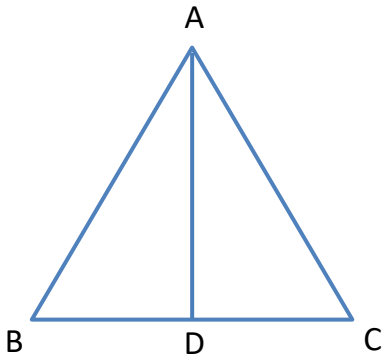
Angles opposite to equal sides of isosceles triangles are equal.

Proof of the theorem to be explained by emphasizing the necessary steps (Given, to prove, etc.) and by constructing the required:

Given:

An isosceles triangle ABC, in which

AB = AC



To prove: $\angle B = \angle C$

Proof:

Draw an angle bisector of ∠A which intersects the side BC at D.

Now in ΔABD and ΔACD.

AB = AC (given)

∠BAD = ∠CAD (by construction)

AD = AD (Common)

So, Δ ABD ≅ ΔACD (SAS Rule)

So, ∠ABD = ∠ACD (Corresponding sides of congruent triangles)

So, ∠B = ∠C is proved.

Activity:

Ask the learners to draw any two triangles say ΔABC and ΔPQR in such a way that the bases BC and QR are of any length and ∠B = ∠C = 50o and ∠Q = ∠R = 70o. Now ask them to measure the sides opposite to ∠B and ∠C. Also, to measure the sides opposite to ∠Q and ∠R.

The learners understand that the sides opposite to equal angles are equal.

This is again a theorem related to isosceles triangles, which is the opposite of the previous theorem.

Theorem (3):

The sides opposite to equal angles of a triangle are equal.

Activity:

Ask the learners to draw two equilateral triangles and let them find the measures of each angle.

They understand that each angle of an equilateral triangle is 60.

Various problems to be discussed and to be assigned to make the above learned concepts (on properties of triangles) clearer for learners.

Recommended Teaching Methodologies:

- **Lecture method**
- **Inductive method**
- **Demonstration method**

Notes:

UNIT – 4

MORE CRITERIA FOR CONGRUENCE OF TRIANGLE

Teaching Points: Some more Criteria for Congruence of Triangle (Theorem 4 and Theorem 5)

Warm-up activity for learners:

Ask the learners to recall and write so far learned Congruence Rules and Theorems.

1. **Axiom (1):** SAS Congruence Rule
2. **Theorem (1):** ASA Congruence Rule
3. **Theorem (2) and (3):** AAS Congruence Rule.

Teaching Process:

Activity: Ask the learners to draw 2 pairs of triangles with the same measures of sides.

Let them cut and place them on each other. What does it reveal?

Learners themselves come out with another rule for congruence. Which is stated in Theorem 4, given below:

Theorem (4): (SSS Congruence Rule):

If the 3 sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

Video Link: SSS Congruence Rule & Platonic Solids

Activity:

Now ask the learners to construct two right triangles with a hypotenuse equal to 5 cm and one side equal to 4 cm.

Ask the learners to cut them out and place one triangle over the other with equal sides placed on each other.

They understand that two right triangles are congruent if one pair of sides and the hypotenuse are equal.

Make the fact very clear to them that the right angle is not the included angle.

This is another congruence rule stated as theorem 5, given below:

Theorem (5): (RHS Congruence Rule) (RHS – Right angle Hypotenuse Side)

If in two right triangles, the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Let the learners do various problems involving the concepts of SSS and RHS congruence Rules.

Video Link: GENERAL BRIEFING ABOUT CONGRUENCE RULES

Recommended Teaching Methodologies:

- **Illustration method**
- **Blended learning method**
- **Lecture method**
- **Inductive method,**
- **Demonstration method**

Notes:

UNIT – 5

INEQUALITIES IN A TRIANGLE.

Teaching Points: Inequalities in a Triangle.

Warm-up activities for learners:

Ask the learners to recall and state all congruence rules. State Theorem 2 and 3.

Teaching Process:

In earlier classes, learners learned about inequalities and comparisons of two figures.

Make the learners understand that there are many situations in real life where we do need to compare the shapes and sizes of triangles to come to the solutions to various problem areas.

So, studying the comparison of angles and sides of triangles is important.

Activity:

Ask the learners to carry out the activity mentioned here to understand an important result of inequality in a triangle.

Fix the base of ΔABC as BC and vary the third vertex A of the triangle at various distances. Then the $\angle ABC$, which is opposite to the side AC varies as the length of the side AC varies.

Here the learners realize that the side and angle opposite to each other are connected.

Also, ask the learners to measure the various parts (Sides and Angles) of a few scalene Triangles.

It reveals a very important result of inequalities in a triangle, which can be stated as the theorem given below.

Theorem (6):

If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater)

Activity:

Ask the learners to draw a line-segment AB. Now with A as the center and any radius, draw an arc and mark different points say P, Q, R, S, or T on it.

Join each of these points with A as well as with B. Learners.

Let the learners think and observe that as we move from P to T, $\angle A$ is becoming larger and larger. What is happening to the length of the side opposite to it? They observe that the length of the side is also increasing that is $\angle TAB > \angle SAB > \angle RAB > \angle QAB < \angle PAB$ and $TB > SB > RB > QB > PB$.

i.e., the side opposite to the largest angle is the longest which is stated as another theorem.

Theorem (7):

In any triangle, the side opposite to the larger (greater) angle is longer.

Now ask the learners to draw few different triangles and measure each side.

Is there any triangle with sum of any two sides coming less than the third side? No, the learners understand that this is not possible. This property of triangle is stated as another theorem.

Theorem (8) :

The sum of any two sides of a triangle is greater than the third side.

Recommended Teaching Methodologies:

- **Lecture method**
- **Inductive method**
- **Illustration method**
- **Demonstration method**

UNIT – 6

SUM UP OF THE CHAPTER

1. Two figures are congruent if they are of the same shape and of the same size.
2. Two circles of the same radii are congruent.
3. Two squares of the same sides are congruent.
4. If two triangles ABC and PQR are congruent under the correspondence, $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$, then symbolically, it is expressed as $\Delta ABC \cong \Delta PQR$.
5. If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent (SAS Congruence Rule).
6. If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent (ASA Congruence Rule).
7. If two angles and are sides of one triangle are equal to two angles and the corresponding side of the other triangle, then the two triangles are congruent (AAS Congruence Rule).
8. Angles opposite to equal sides of a triangle are equal.
9. Sides opposite to equal angles of a triangle are equal.
10. Each angle of an equilateral triangle is of 60^o.
11. If three sides of one triangle are equal to three sides of the other triangle, then the two triangles are congruent (SSS Congruence Rule)
12. If in two right triangles, the hypotenuse and one side of a triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent (RHS Congruence Rule)
13. In an isosceles triangle, angles opposite to equal sides are equal.
14. A triangle having two equal altitudes, is an isosceles triangle.
15. In a triangle, the angle opposite to the longer side is larger (greater).
16. In a triangle, the side opposite to the larger (greater) angle is longer.
17. Sum of any two sides of a triangle is greater than the third side.

UNIT – 7

NEP- 2020 ASPECTS

NEP- 2020 Aspects:

Below given are a few activities and projects which can be explained to learners to make them understand about few aspects of NEP-2020 to develop creative mathematical thinking ability in Mathematics Education.

This will indirectly encourage the learners to find interest in the subject and will enhance their concept clarity.

Art Integrated Project:

Make a beautiful **Worli Painting** using only congruent triangles as borders.

Experiential learning activities:

- A wooden or aplastic ornament/ chain can be prepared by using congruent triangles showing all 5 congruence rules-SAS, ASA, AAS, SSS & RHS- appearing on both sides at equal distance.
- Study the platonic solid: tetrahedron thoroughly. Identify all the congruent Triangles in it. Make a net of the tetrahedron and come out with more properties of it. Also identify the other platonic solids with congruent triangles as faces.

An Interdisciplinary aspect:

Subject: Science

Congruence and Congruence transformations theory.

Congruence transformation is a phenomenon in which a moved figure retains its same size, shape, angles, side lengths. The figures are exactly in equal but formed after some movements of the actual figure or image.

The 3 types of the process which shows congruence transformations are:

1. Reflections
2. Rotations
3. Translations.

UNIT – 8

ASSESSMENT METHODS

Every topic/ content presented in the class by the teacher and learned by the learner is to be assessed on the very next period / next day, by using the warm -up activities mentioned in each unit.

After the completion of the chapter, **a Formative Assessment** to be conducted to know the learning status. On the basis of the performance of the learners, teachers need to make modifications in the teaching activities and in methodology, if required.

A **diagnostic test** can be conducted to know the specific problem area of learning of different learners and accordingly required remedies to be provided.

Notes:

UNIT – 9

SUGGESTED READINGS & REFERENCES

References:

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MODULE – IV

TOPIC: TRIANGLES

Subject: Mathematics **Class: X**

"There is Geometry in the Humming of the Strings, there is music in the Spacing of the Spheres."

- Pythagoras

Introduction:

This module on Triangles aims to enhance the teachers and hence the learners to give a clear idea about similar figures, particularly about the similarity of Triangles, and to apply this knowledge to prove Pythagoras Theorem and other relevant concepts. This module also aims to help the teachers to give a clear understanding to learners about the Basic Proportionality theorem, its converse, and different criteria on the similarity of Triangles. Also, it helps in classifying and solving problems related to similar triangles, and other real-life problems through the application of the knowledge gained through this module.

Objectives / Expected Learning Outcomes:

After doing all units of this module, the teachers should be able to help the learners to :

- Understand the fact that congruent figures are similar but similar figures need not be congruent.
- Know that circles with different radii are similar.
- Know that shadow of an object and the object are similar but may not be congruent.
- Identify similar triangles.
- Understand and prove basic proportionality theorem and its converse.
- Identify (i) AAA, (ii) SAS and (iii) SSS Similarity criteria.

- Prove that ratio of the areas of similar triangles is equal to the square of the ratio of their corresponding sides.
- Prove Pythagoras theorem and its converse.
- Solve, all problems involving the application of the above results, theorems, and knowledge.

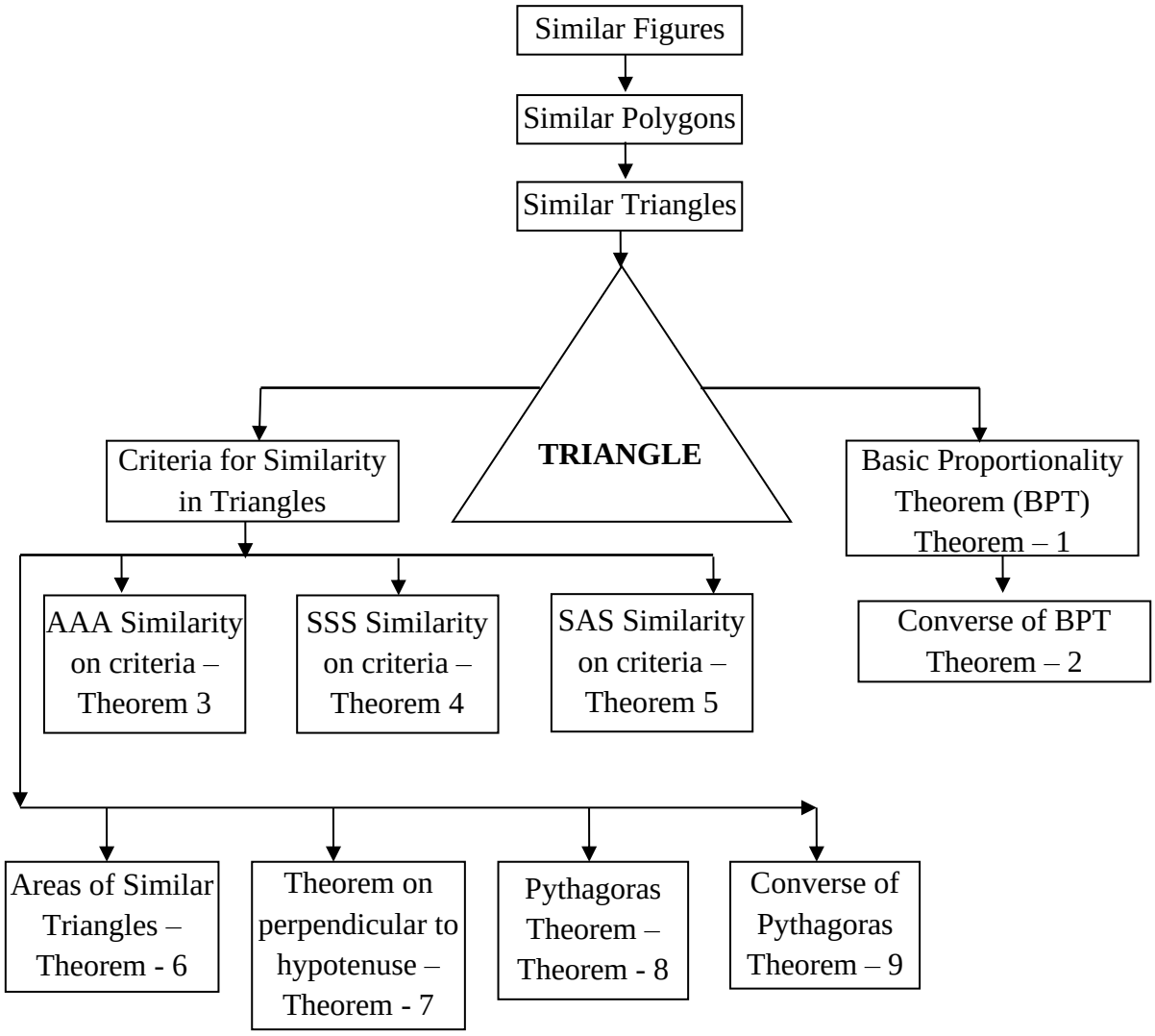
Pre-requisite knowledge, required for learners.

1. Congruency of Triangles
2. Inequalities in Triangles
3. Properties of triangles learnt from earlier classes.

Key Concepts / Important Terms:

1. Similar figures
2. Criterion for similarity
3. Basic proportionality theorem
4. Pythagoras
5. Areas of similar triangles.

CONCEPT MAP


Similar Figures
Similar Polygons
Similar Triangles
TRIANGLE
Criteria for Similarity in Triangles
Basic Proportionality Theorem (BPT) Theorem – 1
AAA Similarity on criteria – Theorem 3
SSS Similarity on criteria – Theorem 4
SAS Similarity on criteria – Theorem 5
Converse of BPT Theorem – 2
Areas of Similar Triangles – Theorem - 6
Theorem on perpendicular to hypotenuse – Theorem - 7
Pythagoras Theorem – Theorem - 8
Converse of Pythagoras Theorem – 9

INDEX

UNIT – 1

INTRODUCTION, SIMILAR FIGURES

Teaching Points: Introduction, Similar Figures

Warm-up activities for learners:

Ask the learners to recall the congruence rules learned in Class 9.

Recall the other rules and properties of congruent triangles.

Teaching Process: Ask the learners to explain their understanding of congruent figures and similar figures. Discuss by taking various examples.

Let the learners come out with the difference between similarity and congruence.

All congruent figures are similar, but the similar figures need not be congruent.

Discuss:

What is the need of learning about similar figures?

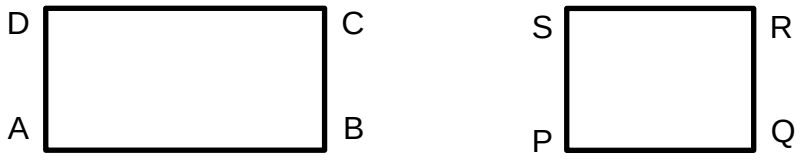
What is the height of Mt. Everest? Whether it is measured directly? No.

Heights and distances have been calculated using the idea of indirect measurements which is based on the principle of similarity of figures.

Can a circle and a square be similar? No.

Draw two rectangles and discuss:

Figure



Are they looking similar?

They appear to be similar, but we are not sure.

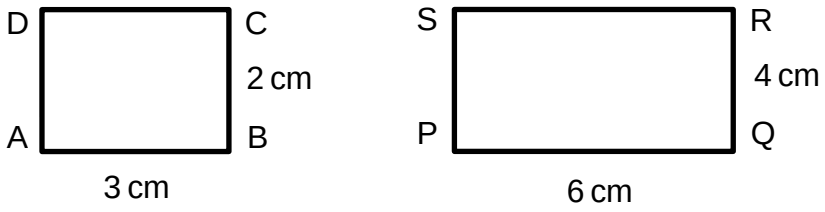
So, how similarity can be defined?

In the above rectangles, angles are always equal.

But the length of the sides is not.

Are the corresponding sides in proportion? No, not in this case.

Draw another pair of rectangles with the following measurement.



Are they looking similar? Yes, but we are not sure.

Here every side of the rectangle ABCD is enlarged (increased) in the ratio 1:2 and angles are equal.

This is the important aspect of the similarity in any polygons.

Two polygons of the same number of sides are similar if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio.

In rectangle ABCD and rectangle PQRS,

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, $\angle D = \angle S$.

and $\frac{AB}{PQ} = \frac{3}{6} = \frac{1}{2}$

$\frac{BC}{QR} = \frac{2}{4} = \frac{1}{2}$

$\frac{CD}{RS} = \frac{3}{6} = \frac{1}{2}$

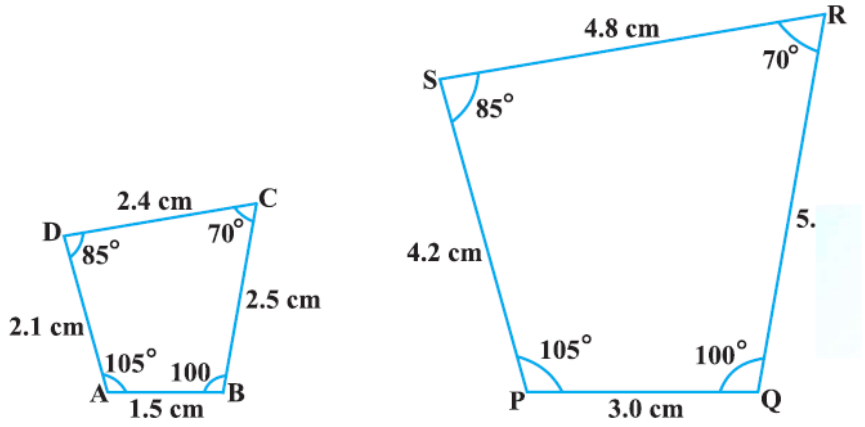
$\frac{AD}{PS} = \frac{2}{4} = \frac{1}{2}$

All corresponding sides are in the same ratio. So, the rectangles are similar.

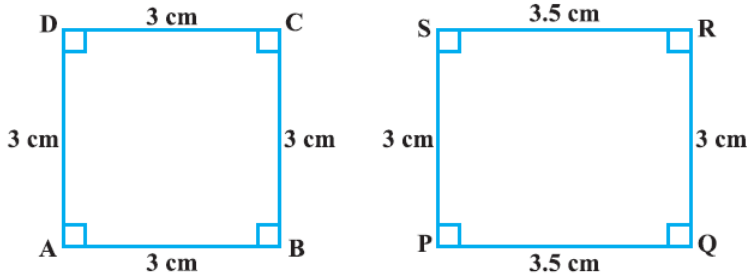
Activity:

Ask the learners to draw similar-looking pairs of polygons and measure them and verify the similarity between them.

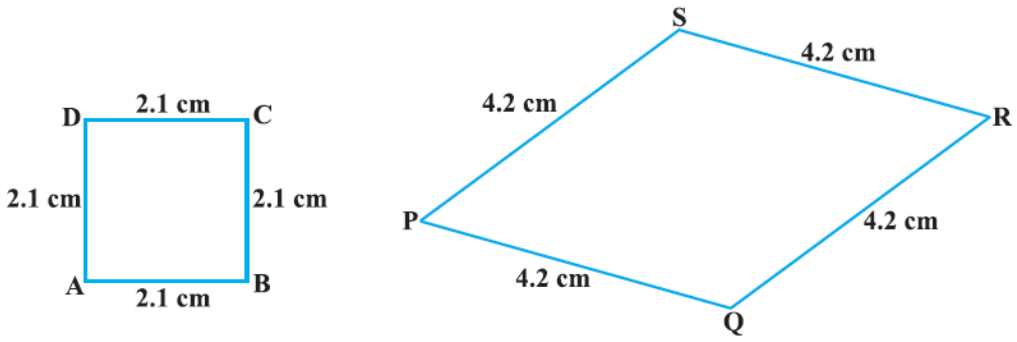
(a)



(b)



(c)



From the above, quadrilaterals ABCD and PQRS of Figure (a) are similar.

Let the learners understand the fact:

If one polygon is like another polygon and the second polygon is like a third polygon, then the first polygon is like the third polygon.

Also, note, in the two quadrilaterals (a square and a rectangle) of figure (b), corresponding angles are equal, but their corresponding sides are not in the same ratio. So, the two quadrilaterals are not similar.

Similarly, in the two quadrilaterals (a square and a rhombus) of figure (c), corresponding sides are in the same ratio, but their corresponding angles are not equal. Again, the two polygons (quadrilaterals) are not similar.

Thus, either of the two conditions (i) and (ii) given previously for the similarity of two polygons is not sufficient for them to be similar.

Various problem situations to be assigned to the learners to make them understand about the difference between similar and non-similar figure.

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Illustration method**
- **Inductive method**

Notes:

UNIT – 2

SIMILARITY OF TRIANGLE, BPT & ITS CONVERSE

Teaching Points: Similarity of a Triangle, Basic Proportionality Theorem, and Converse of BPT.

Warm-up activity for learners:

Ask the learners to differentiate between congruence and similarity.

Teaching Process: Learners have learned the similarity conditions of two polygons in the previous class.

Triangles also being the polygon, the same conditions can be stated for the similarity of two triangles.

Two triangles are similar if.

1. Their corresponding angles are equal and
2. Their corresponding sides are in the same ratio (or proportion)

Also, make the learners recall an equilateral Triangle and an equiangular triangle.

- A triangle with all sides equal is called an equilateral triangle.
- If the corresponding angles of two triangles are equal, they are equiangular.

By drawing two equiangular triangles, find the ratio of their corresponding sides.

It reveals that the ratio of any two corresponding sides in two equiangular triangles is always the same. This important result was given by the Greek Mathematician Thales.

A theorem called Basic Proportionality Theorem (BPT) also known as ‘Thales Theorem’ was used in stating the above result.

Theorem (1):

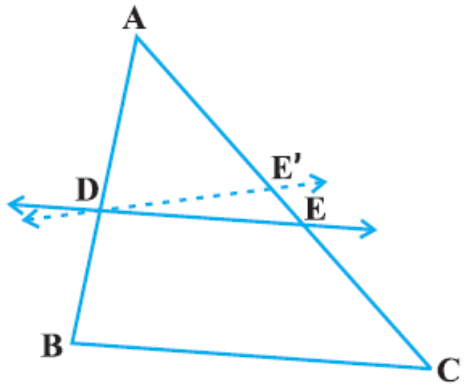
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Let the learners understand that the converse of the Basic Proportionality Theorem (BPT) is also correct.

Theorem (2):

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

This theorem can be proved by taking a line DE such that $\frac{AD}{DB} = \frac{EA}{CE}$ and assuming that DE is not parallel to BC (Figure).



If DE is not parallel to BC, draw a line DE' parallel to BC.

So, $\frac{AD}{DB} = \frac{AE'}{E'C}$ (by Theorem (1))

Therefore, $\frac{AE}{EC} = \frac{AE'}{E'C}$

Discuss:

Adding 1 to both sides of above, E and E' must coincide. Why?

Let the learners know that based on the Basic Proportionality Theorem (BPT), some important results as given below can also be derived.

- If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.
- The internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.
- Diagonals of a trapezium divide each other proportionally.

Video Link : BPT AND CONVERSE

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Illustration method**
- **Inductive method**

Notes:

UNIT – 3

CRITERIA FOR SIMILARITY OF TRIANGLES

Teaching Points: Criteria for similarity of triangles (AAA, SSS and SAS similarity criteria).

Warm-up activity for learners:

Ask the learners to recall:

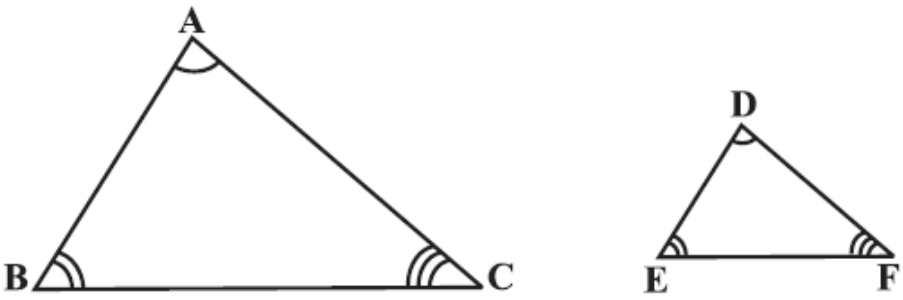
1. BPT
2. The converse of BPT
3. Equiangular Triangles

Teaching process:

Let the learners recall the symbol used to denote the congruence between two triangles say $\Delta ABC \cong \Delta DEF$.

The similarity of these two triangles can be symbolically written as $\Delta ABC \sim \Delta DEF$

Here it to be noted that the similarity of two triangles should be symbolically expressed by using the correct correspondence only of their vertices.



In ΔABC and ΔDEF

(i) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

So $\Delta ABC \sim \Delta DEF$

Let the learners understand the fact the

we cannot write, $\Delta ABC \sim EDF$ or $\Delta ABC \sim FED$

Discuss:

Can we write ΔABC ~ ΔEDF in the above-mentioned example?

Let the learners think now, for predicting the similarity of two triangles, is it necessary to check the equality of all corresponding angles and the equality of the ratios of their corresponding sides?

Here are the discussions to be made on various criteria for similarity.

This result is called Angle – Angle – Angle (AAA) Similarity criterion and is stated as a theorem with proof given below.

Theorem (3):

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion), and hence the two triangles are similar.

This theorem can be proved by taking two triangles ABC and DEF such that $\frac{AB}{DE} = \frac{CB}{FE} = \frac{AC}{FD}$.

Discuss:

If two angles of a triangle are equal to the corresponding angles of another triangle, what we can state about the similarity of the two triangles?

Learners understand that by the angle sum property of a triangle, their third angles also will be automatically equal.

Thus, the AAA similarity criterion can also be stated as the AA similarity criterion as follows.

AAA similarity criterion:

If two angles of one triangle are equal to respectively two angles of another triangle, then the two triangles are similar.

Video Link: [BOW-TIE TRIANGLES & AAA SIMILARITY]

Let the learners think about: Is there any other criterion involving only few elements of triangles by which the similarity between two triangles can be predicted?

Theorem (4):

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This theorem can be proved by taking two triangles ABC and DEF such that $\frac{AB}{DE} = \frac{CB}{FE} = \frac{AC}{FD}$ (< 1)

Theorem (3) and (4) reveal that it is not necessary to check both AAA and SSS criterion to predict the similarity of triangles as one condition implies the other.

Discuss:

If two triangles are similar, what about the ratio of their perimeters?

Let the learners understand that the ratio of their perimeters is equal to the ratio of their corresponding sides.

$\Delta ABC \sim \Delta DEF$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB + BC + AC}{DE + EF + DF}$$

Let the learners understand the fact that by considering one pair of corresponding angles of two triangles and sides including their angles, we can get another similarity criterion called Side – Angle – Side (SAS) similarity criterion.

Theorem (5):

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

As before, this theorem can be proved by taking two triangles ABC and DEF such that $\frac{AB}{DE} = \frac{AC}{DF} (< 1)$ and $\angle A = \angle D$.

Video Link: **CONGRUENCE AND SIMILARITY**

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Illustration method**
- **Inductive method**
- **Blended learning method**

Notes:

UNIT – 4

AREAS OF SIMILAR TRIANGLES

Teaching Points: Areas of similar triangles and related theorem.

Warm-up Activity for learners:

Ask the learners to:

Recall the formula to find area of a triangle and the unit to express area of a triangle.

Ask them to state all similarity criteria between two triangles.

Teaching Process:

Ask the learners:

Is there any relation between ratio of the areas of two similar triangles and the ratio of their corresponding sides?

Since area is depending on two measures (base and height) of the triangle, there is a possibility that the ratio of the areas may be equal to the square of the ratio of their corresponding sides.

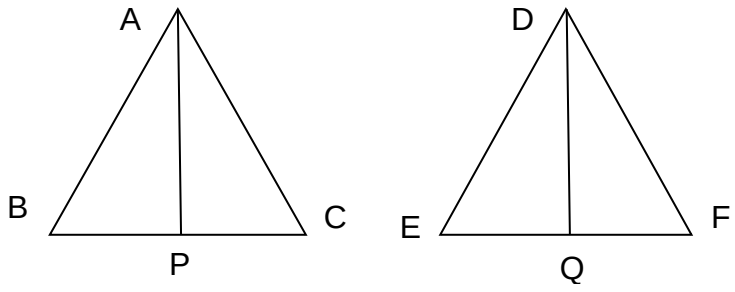
This is true and can be proved as a theorem.

Theorem (6):

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Let the learners understand that using the concept of the ratio of areas of two similar triangles and related theorem, the following important results also can be verified and proved.

*If two triangles are similar, then the ratio of their corresponding sides is the same as the ratio of the squares of their corresponding medians.

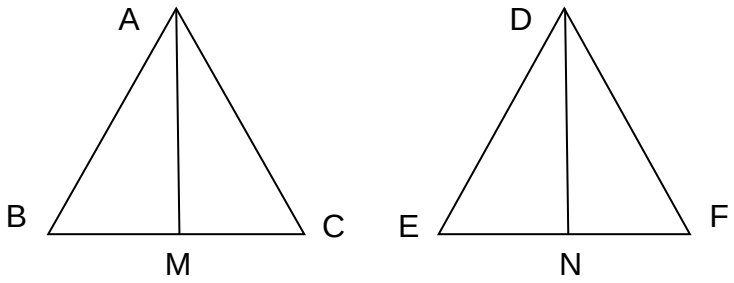


i.e., if ΔABC ~ ΔDEF,

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AP}{DQ}$

If two triangles are similar, then the ratio of their corresponding sides is the same as the ratio of the squares of their corresponding altitudes.

i.e., if ΔABC ~ ΔDEF, then $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AM}{DN}$



Various problems to be assigned to the learners to make the concept of the ratio of areas of similar triangles clearer and to make them equipped to solve the related real-life problems.

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Illustration method**
- **Inductive method**
- **Blended learning method**

Notes:

UNIT – 5

THEOREMS

Teaching Points: Theorem on Perpendicular to Hypotenuse, Pythagoras Theorem, and its Converse.

Warm-up activities for learners:

Ask the learners to:

1. State BPT and its converse.
2. State all 3 criterions of similarity of triangles.
3. State Pythagoras theorem.

Teaching Process:

Let the learners recollect the Pythagoras theorem which has been already learnt in class IX with its proof.

Now by using the concept of similarity, the below theorem can be proved correct.

This can be summarized as a theorem given below.

Theorem (7):

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are like the whole triangle and to each other.

Let the learners understand that the theorem can be used to prove the Pythagoras Theorem.

Theorem (8):

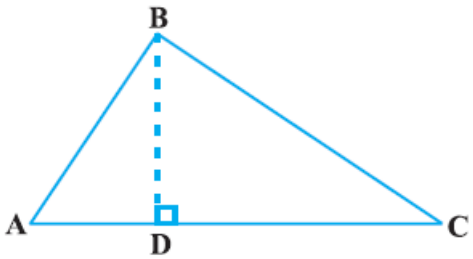
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: A right triangle ABC right angled at B.

To prove: $AC^2 = AB^2 + BC^2$

Proof:

Let us draw BD ⊥ AC (Figure).



Now, ΔADB ~ ΔABC (Theorem 7)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

or $AD \cdot AC = AB^2$ (1)

Also, ΔBDC ~ ΔABC (Theorem 7)

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or $CD \cdot AC = BC^2$ (2)

Adding (1) and (2),

$$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$$

or $$AC\,(AD + CD) = AB^2 + BC^2$$

or $$AC \cdot AC = AB^2 + BC^2$$

or $$AC^2 = AB^2 + BC^2$$

Before Pythagoras, this theorem was given by an ancient Indian mathematician Baudhayana (about 800 B.C.E.) in the following form and is referred as the Baudhayana theorem.

The diagonal of a rectangle produces by itself the same area as produced by its both sides. (Length and breadth)

Also in the earlier classes, learners have verified the converse of Pythagoras theorem and found it's true. Now this can be proved in the form of another theorem.

Theorem (9):

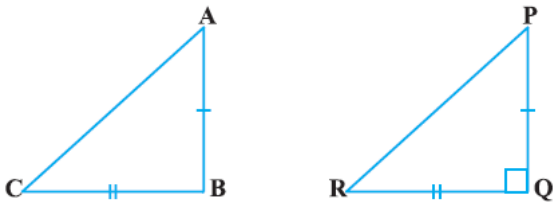
In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$.

To prove: $\angle B = 90°$.

Proof:

We construct a Δ PQR right angled at Q such that PQ = AB and QR = BC (Figure).



Now, from Δ PQR, we have :

$PR^2 = PQ^2 + QR^2$ (Pythagoras Theorem, as $\angle Q = 90°$)

or $PR^2 = AB^2 + BC^2$ (By construction) (1)

But $AC^2 = AB^2 + BC^2$ (Given) (2)

So, AC = PR [From (1) and (2)] (3)

Now, in ΔABC and ΔPQR,

AB = PQ (By construction)

BC = QR (By construction)

AC = PR [Proved in (3) above]

So, ΔABC ≅ ΔPQR (SSS congruence)

Therefore, $\angle B = \angle Q$ (CPCT)

But $\angle Q = 90°$ (By construction)

So, $\angle B = 90°$

Various problems involving the real-life situations to be assigned to the learners to make the learned concepts clearer.

Recommended Teaching Methodologies:

- **Lecture method**
- **Demonstration method**
- **Illustration method**
- **Inductive method**
- **Blended learning method**

Notes:

UNIT – 6

SUM UP OF THE CHAPTER

1. Two figures having the same shape but not necessarily the same size is called similar figures.
2. All the congruent figures are similar, but the converse is not true.
3. Two polygons of the same number of sides are similar if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
6. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).
7. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
8. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
9. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).
10. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
11. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are like the whole triangle and to each other.
12. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras Theorem).
13. If in a triangle, a square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

UNIT – 7

NEP 2020 ASPECTS

NEP 2020 Aspects:

Below given are a few activities and projects which can be explained to learners to make them understand about few aspects of NEP-2020 to develop creative mathematical thinking ability in Mathematics Education.

This will indirectly encourage the learners to find interest in the subject and will enhance their concept clarity.

Art Integrated Project:

Make a **Lippan Art** wall décor using the concept of similar triangles.

Experiential learning activities:

- Construct a TENT, using wood/ bamboo sticks and clothes which can be used for a camping trip. Apply the concept of Basic Proportionality Theorem to fit a net (to put decorative lights) from the top of the roof, parallel to the base of the triangular shaped entrance.
- Know the function of the camera. When the light passes through the camera lens, two bow-tie shaped similar triangles are formed. Draw the figure to understand more about the functioning of camera as well as bow- tie shaped similar triangles.

An Interdisciplinary Aspect:

Subject: SOCIAL SCIENCE.

Egyptian Pyramids & Similar Triangles.

The ancient mathematician Thales used to apply the principles of similar triangles to calculate the heights of the pyramids in Egypt. Around 6000 years ago, he discovered that the heights of the shadows of the pyramids were in proportion with his own shadow. With this understanding he could measure the height of the great pyramid by using the properties of similar triangles.

UNIT – 8

ASSESSMENT METHODS

Every topic/ content presented in the class by the teacher and learned by the learner to be assessed on the very next period / next day, by using the warm-up activities mentioned in each unit.

After the completion of the chapter, a **Formative Assessment** to be conducted to know the learning status .On the basis of the performance of the learners, teachers need to make modifications in the teaching activities and in methodology, if required.

A **diagnostic test** can be conducted to know the specific problem area of learning pf different learners and accordingly required remedies to be provided.

Notes:

UNIT – 9

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