Appendix. A.

Flow around a circular cylinder using principle of superposition

The uniform potential flow around circular cylinder can be obtained by superposition of uniform flow and doublet flow.

Complex potential function for uniform flow along $+x$ axis: $W_1 = Uz$

Complex potential function for doublet located at the origin of complex ζ-plane: $W_2 = \frac{|\mu|}{2\pi Z}$,

where, $|\mu|$ represents the strength of doublet.

The complex potential function for uniform flow around circular (W_U) cylinder can be written by the superposition of W_1 and W_2 as shown below,

$$W_U = W_1 + W_2 = Uz + \frac{|\mu|}{2\pi z}, \tag{A.1}$$

$$W_U = U\left(z + \frac{|\mu|}{2\pi U z}\right). \tag{A.2}$$

By substituting $a_r = \sqrt{\frac{|\mu|}{2\pi U}}$,

$$W_U = U\left(z + \frac{{a_r}^2}{z}\right), \tag{A.3}$$

where, a_r represents the radius of circular cylinder.

The velocity field around the cylinder in potential flow can be obtained by taking differentiation of complex potential (W_U) with respect to variable z as shown below,

$$V = u - iv = (V_r - iV_\theta)e^{-i\theta} = \frac{dW_U}{dz}, \tag{A.4}$$

where, u and v are velocity component along x and y direction respectively in Cartesian co-ordinate, and V_r and V_θ are the radial and tangential component of velocity respectively in polar co-ordinate system.

By substituting complex potential function W_U and taking differentiation,

$$V = U\left(1 - \frac{a_r^2}{z^2}\right). \tag{A.5}$$

By separating in real and imaginary part, the radial and tangential component of velocity can be obtained as,

$$V_r - iV_\theta = U\left[\cos\theta\left(1 - \frac{a_r^2}{r^2}\right) + i\ \sin\theta\left(1 + \frac{a_r^2}{r^2}\right)\right]. \tag{A.6}$$

At the surface of cylinder ($r = a_r$),

$$V_r = 0, V_\theta = -2U\sin\theta. \tag{A.7}$$

The pressure around the cylinder can be obtained by applying Bernoulli's principle between a point located at infinite distance away from the cylinder and a point around the cylinder as shown below,

$$P + \frac{1}{2}\varrho V^2 = P_\infty + \frac{1}{2}\varrho U^2, \tag{A.8}$$

Simplifying the above relation gives,

$$Cp = \frac{P - P_\infty}{\frac{1}{2}\varrho U^2} = 1 - \frac{V^2}{U^2} \tag{A.9}$$

The derived form of dimensionless pressure is known as pressure co-efficient (Cp). The pressure co-efficient on the surface of cylinder can be obtained by substituting the absolute velocity ($V = 2U\sin\theta$) into above relation,

$$Cp = 1 - \frac{V^2}{U^2} = 1 - 4\sin^2\theta. \tag{A.10}$$

Appendix. B.

Bilinear Mapping

The bilinear mapping conformally maps the annulus $q \leq |\zeta| \leq 1$ (the outer circle $\zeta_1$of unit radius and inner circle ζ_2 of radius q) from ζ-plane to infinite region with two non-overlapping circles ξ_1 and ξ_2 of unit radius and radius R_1 respectively at distance H apart in ξ-plane as shown in Figure C.1.

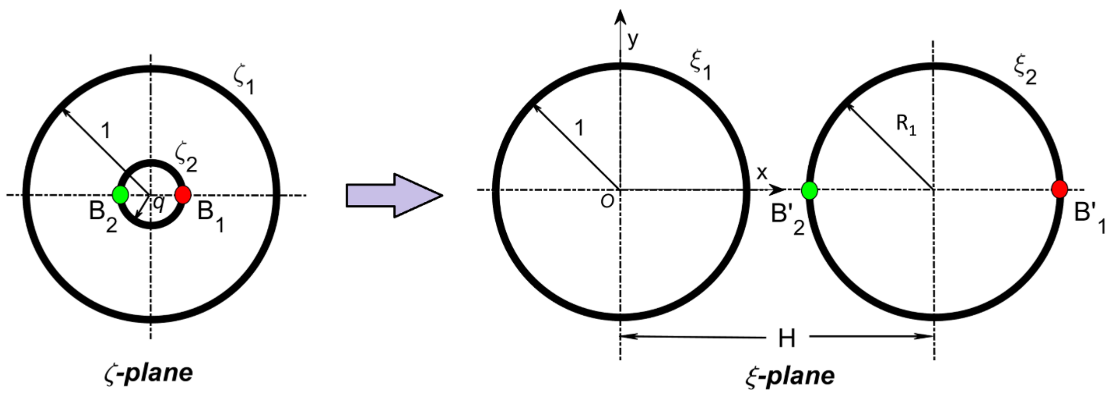

Figure C.1 Bilinear mapping from the annular area ($q \leq |\zeta| \leq 1$) from ζ-plane to infinite region with two non-overlapping circles ξ_1 and ξ_2 of unit radius and radius R_1 respectively at distance H apart in ξ-plane. Point B on the positive real axis in ζ-plane is transformed to point B' on the real axis in ξ-plane.

The relationship of radius (q) and constant (λ) with the center distance (H) can be obtained by mapping points $B_1 = q$ and $B_2 = -q$ on the smaller circle (ζ_2) in ζ-plane to the points B_1' and B_2' on the unit circle (ξ_2) in ξ-plane as written below,

$$B_1' = \frac{q - \lambda}{q\lambda - 1}, \qquad \text{(B.1)}$$

and,

$$B_2' = \frac{q + \lambda}{q\lambda + 1}. \qquad \text{(B.2)}$$

The point $B_1{}'$ is located at distance $H + R_1$ from the origin in ξ-plane, therefore,

$$H + R_1 = \frac{q - \lambda}{q\lambda - 1}. \tag{B.3}$$

By rearranging the terms in above equation, radius q can be written in terms of center distance H and λ as,

$$q = \frac{R_1 + H - \lambda}{\lambda(R_1 + H) - 1}. \tag{B.4}$$

The point $B_2{}'$ is located at distance $H - R_1$ from the origin in ξ-plane. Substituting $B_2' = H - R_1$ in Equation (B.2) yields,

$$H - R_1 = \frac{q + \lambda}{q\lambda + 1}, \tag{B.5}$$

After substituting $q = \frac{R_1+H-\lambda}{\lambda(R_1+H)-1}$ in above equation and simplifying we get,

$$\lambda^2(-H) + \lambda(H^2 - R_1^2 + 1) - H = 0, \tag{B.6}$$

For two non-overlapping unit circles ($H > 1 + R_1$) in ξ-plane, the solution of above equation yields constant (λ) as written below,

$$\lambda = \frac{-(H^2 - R_1^2 + 1) \pm \sqrt{(H^2 - (R_1 + 1)^2)(H^2 - (R_1 - 1)^2)}}{-2H}. \tag{B.7}$$