Chapter 5 Conclusion and Future Scope

5.1 Summery

Using a complex variable approach, the potential flow around single and multiple cylinders of various polygonal shapes are investigated. In this study, the complex potential function and the conformal maps are the main tools for the development of generalized mathematical solution for flow around single and multiple polygonal cylinders. A MATLAB code is prepared to get numerical results to investigate the effects of shapes, number of sides, corner radii, orientation angle, flow angle, and center-to-center distance on the flow around single and interacting polygonal cylinders. Some of the results obtained by using present method are compared with the results from existing literature and those obtained using ANSYS Fluent.

For single cylinder problem, the complex potentials are derived for vortex and doublet inside a circle using Milne-Thomson Circle theorem. The non-uniform potential flow around a circular cylinder is obtained by mapping the doublet (or vortex) from inside a circle (at $\zeta = \beta_\zeta$) to outside a circle at finite location ($Z = z_\beta$) using inverse transformation ($Z = 1/\zeta$). The uniform potential flow around a circular cylinder is obtained by mapping the doublet at infinite location ($z_\beta = \infty$) outside a circle (for the location $\beta_\zeta \cong 0$ inside a circle).

The potential flow over different shaped geometries such as elliptical, triangular, rectangular, pentagonal, hexagonal, octagonal and other N-sided polygons with varying corner radius is obtained using the hypotrochoidal transformation along with complex potential function for a doublet inside a circle. The hypotrochoidal transformation conformally map the area outside the circle to the area outside a polygonal geometry. Similar to the case of circular cylinders, the uniform and non-uniform flow conditions obtained arrived at by taking infinite and finite distance of doublet outside the polygonal cylinder, respectively. The equations are developed to obtain velocity and pressure around N-sided polygonal shaped cylinder with finite corner radius for uniform and non-uniform potential flow. The effect of geometrical parameters such as corner radius (ρ), number of sides (N), and orientation angle (γ) of polygonal shaped cylinders on flow parameters is also investigated.

The efforts are made to study three different cases of potential flow around multiple cylinders, (a) hydrodynamic interaction between two circular cylinders, (b) hydrodynamic interaction between polygonal and circular cylinder, and (c) hydrodynamic interaction between two polygonal cylinders. The bilinear transformation is employed to map the annular area between two concentric circles ($q \leq |\zeta| \leq 1$) in ζ-plane to the infinite region around two non-overlapping circles in ξ-plane for the study of hydrodynamic interaction between two circular cylinders. For the problem of interaction between cylinders in polygonal-circular and polygonal-polygonal pairs, the novel composite map is developed by the combination of bilinear and hypotrochoidal transformation. This composite mapping function maps the annular area ($q \leq |\zeta| \leq 1$) in ζ-plane onto the area around two circles in ξ-plane, which is further mapped to the area around two polygonal cylinders in Z-plane. The flow parameters are obtained by computing complex potential for doublet trapped in an annulus ($q \leq |\zeta| \leq 1$) along with composite conformal mapping. The complex potential is written in terms of Laurent series. Therefore, the convergence tests are performed and the numerical results are obtained to investigate the hydrodynamic interaction between cylinders for various geometrical parameters and flow conditions.

5.2 Conclusion

For the single cylinder problem, the area inside a circle is conformally mapped to the area outside the polygonal geometry using hypotrochoidal mapping and the numerical results are obtained. Following can be concluded for potential flow around single cylinder.

1) For the uniform potential flow (for $\beta \approx 0$) around circular cylinder ($\rho = 1$), the velocity and pressure co-efficients attain the extremum values $V_{max}/U = 2$ and $Cp = -3$ respectively at angular locations $\theta = \alpha + 90^0$ and $\alpha + 270^0$.

2) For uniform potential flow around single polygonal cylinder, the maximum velocities and minimum pressures are observed at the vertex (vertices) on the line joining it (them) to the center of cylinder, if the line is perpendicular to the flow.

3) It is found that, as normalised corner radius ρ decreases, maximum velocity and pressure drop increases at the corners of polygonal cylinders, and approaches equal to $V_{max}/U = 2$, $Cp_{min} = -3$ corresponding to a circular cylinder ($\rho = 1$).

4) At the cusp ($\rho = 0$) of hypocycloidal shaped cylinders, the maximum velocity and minimum pressure attain infinite value due to presence of singularity.

5) Number of sides (N) has significant effect on pressure co-efficient (Cp_{min}) and velocity (V_{max}/U). As $N \to \infty$, Cp_{min} and V_{max}/U approaches to -3 and 2 respectively.

6) Asymmetric velocity and pressure distribution are found around the circular and polygonal cylinders for the potential flow due to the presence of doublet at finite distance from the cylinder.

7) The corner radius (ρ), orientation angle (γ), number of sides (N) and flow angle (α) are important parameters that impact on flow parameters around the cylinder.

For the hydrodynamic interaction between two cylinders, the annular area between two concentric circles ($q \le |\zeta| \le 1$) in ζ-plane is mapped onto infinite region around two non-overlapping polygons in Z-plane by unique combination of bilinear and hypotrochoidal transformation. The flow parameters are obtained by computing complex potential in an annulus along with composite conformal transformation from an annular domain (D_ζ) to physical domain of interest (D_Z). Following are the major conclusions for potential flow around multiple cylinders

1) In uniform flow, the vicinity of one cylinder alters the flow parameters around the neighboring cylinder and the hydrodynamic interaction is observed.

2) The hydrodynamic interaction effects on the flow parameters intensifies with decrease of center-to-center distance between the cylinders for $H < 10$. For the center distance $H \gg 10$, no significant interaction effects are observed and the flow parameters around cylinders are found similar to the that around isolated cylinder in free stream flow.

3) The position of cylinders with flow has significant effect on interaction. For same center distance (H) between cylinders, the maximum interaction effects are observed in side-by-side configuration due to Venturi effect and minimum interaction effects are observed in tandem configuration of cylinders due to blocking of flow.

4) Not only center distance and flow angle, the shape of cylinders, orientations of cylinders and the corner radii are other important parameters that affect the hydrodynamic interaction between the cylinders.

5) For given configuration and centre distance (H), the hydrodynamic forces may be attractive or repulsive in nature based on the flow angle (α) that are found to be acting between the cylinders. The center distance between the cylinders, the shape of cylinders, planar orientations of cylinders and the corner radius are the deciding factors for the magnitude of hydrodynamic forces between cylinders.

The generalized solution presented in this thesis is capable to handle the problem of potential flow around any polygonal shaped cylinder with rounded corners and problem of hydrodynamic interaction between cylinders of different shapes ($N \in [2, \infty]$), orientations (γ), and corner radii (ρ) interacting at any center distance $H > 2$ in uniform potential flow at an angle (α). The commercial CFD software packages solve the problem numerically, and produces approximate results. Also, for every geometry we need to model and simulate the conditions. Therefore, it is time consuming and required significant man hours. The computation by present method is time efficient and the results are more accurate. In present method a code written in MATLAB is capable to give results for various polygonal shaped cylinders and its orientations.

5.3 Limitations of Present Work

The potential flow approach used in present research is based on the inviscid, incompressible and irrotational behaviour of fluid flow. Therefore, the present study does not involve the effects of viscous nature of fluid such as boundary layer formation, flow separation, unsteady wake behind cylinder, etc. The potential flow separation models are beyond the scope of present method. Since, the complex variable approach is employed, the problem is limited to the two-dimensional flow only.

5.4 Future Scope

The solution for potential flow around single and multiple cylinders of polygonal shapes presented in this thesis using complex variable approach is time efficient and accurate. The present research can be extended for the problems of expanding and rotating cylinders, and polygonal cylinders with vibratory motion in potential flow. The present method can modify to investigate the hydrodynamic mass of polygonal cylinders. Some of the viscous flow effects can be included in present method by considering vortex shedding from the cusps of the polygonal cylinder using Brown-Michael vortex relation along with complex potential and conformal mapping.