

Chapter 4 Results and Discussion

4.1 Preliminary Remark

The generalized analytical solution for potential flow around single and multiple cylinders of circular and polygonal shapes is presented in previous section using complex variable approach. A code is developed in MATLAB based on the mathematical formulation to obtain the results of flow parameters around cylinders of various shapes and to investigate hydrodynamic interaction between cylinders. The effects of geometrical parameters such as corner radii, numbers of side, shapes and orientation angles of polygonal shaped geometries, as well as the effects of center-to-center distance between the cylinders and flow directions on hydrodynamic parameters are addressed.

4.2 Stepwise Procedure to Obtain the Results

The MATLAB code is prepared to obtain numerical results, where the input parameters are shape, size, corner radius, numbers of sides and orientation angles of polygonal cylinders, center-to-center distance between the cylinders and flow directions. The output parameters of programs are velocity field and pressure co-efficient around the cylinders, and the hydrodynamic forces acting on the cylinders while interacting with other cylinders.

The step-by-step procedure to obtain output for single cylinder is as follows:

1. The shape of polygonal cylinder is obtained using hypotrochoidal mapping (Equation (3.15)), where R_d , N , ρ and γ are inputs. The parameters R_d decides the size of polygonal cylinder, N decides number of sides of cylinder, ρ decides the corner radius, and γ decides the orientation of polygonal cylinder.
2. The parameter h in hypotrochoidal mapping is calculated based on the selection of radius of directing circle R_d , corner radius ρ and number of sides N using Equation (4.2).
3. Choose the free stream velocity (U) and flow angle (α) to set the global flow condition around the cylinders.

4. The circular domain D_ζ is formed in ζ -plane by the input radius of circle $r = 1$ and $\theta = 0$ to 2π in $\zeta = re^{i\theta}$.
5. Input proper location of doublet (β) inside the circle in Equation (3.9) to obtain uniform and non-uniform flow conditions around circular/polygonal cylinder.
6. The area inside the circle (ζ -plane) is mapped to the area outside the circle (ξ -plane) by using inverse transform $Z = 1/\zeta$ to obtain flow around single circular cylinder.
7. For flow around polygonal cylinder, the area around circle in ξ -plane is mapped to the area outside a polygonal geometry in F -plane by computing hypotrochoidal transformation (Equation (3.15)).
8. The complex potential function is calculated using Equation (3.9).
9. The velocity around polygonal shaped cylinder is obtained by taking differentiation of complex potential function (Equation (3.9) and mapping function (Equation (3.15))).
10. The pressure co-efficient around cylinder is calculated using Bernoulli's theorem (Equation (3.12)).

The stepwise procedure to obtain flow parameters around the pair of polygonal cylinders is as follows:

1. The geometry of two interacting cylinders is decided based on hypotrochoidal mapping, where input parameters are N_j (number of sides), ρ_j (corner radii), and γ_j (orientation angle).
2. The mapping constants h_j ($j = 1, 2$) are calculated based on the selection of corner radii ρ_j and number of sides N_j using Equation (4.2).
3. Enter center-to-center distance H between two cylinders.
4. Calculation of constant λ and radius q of smaller circle of an annulus using Equation (3.25) and (3.26) respectively.
5. Enter flow parameter α , that decides the tandem and side-by-side configuration of interacting cylinders.
6. Input the density of fluid $\rho = 1$, and free stream velocity $U = 1$.
7. The annulus domain D_ζ is formed in ζ -plane by using parameters $q \leq r \leq 1$ and $0 \leq \theta \leq 2\pi$ into the relation $\zeta = [q, 1]e^{i\theta}$.

8. Choose the doublet location inside an annulus (ζ -plane) at pole ($\beta = 1/\lambda$) to obtain uniform potential flow around pair of cylinders in Z -plane.
9. Fix number of terms n and n_1 based on convergence test. Computation of series parameter ‘ A ’ in Equation (3.30) using values of q and n_1 .
10. Calculate P -function in terms of Laurent series inside an annulus ($q \leq \zeta \leq 1$) in ζ -plane using Equation (3.29).
11. The area inside an annulus is mapped to area around two polygonal geometries by computing composite conformal map in Equation (3.52).
12. Calculate constant ‘ a ’ by evaluating composite mapping function at pole $\zeta = 1/\lambda$.
13. The complex potential function (Equation (3.54)) is computed inside an annulus ($q \leq \zeta \leq 1$) corresponding to the uniform potential flow around pair of cylinders.
14. The velocity around interacting cylinders in physical Z -planes is calculated by taking differentiation of complex potential and conformal mapping together (Equation (3.55)).
15. Calculate the pressure co-efficient around interacting polygonal cylinders using Equation (3.56).
16. Calculate the hydrodynamic forces by integrating pressure field around the cylinders by applying Blasius theorem (Equation (3.58)).

The graphical representations of the procedure described above to obtain flow parameters around single cylinder and two interacting cylinders in potential flow are shown in Figure 4.1 and Figure 4.2 respectively.

4.3 Potential Flow around Single Circular/Polygonal Cylinder

The results of velocity and pressure around circular cylinder for uniform and non-uniform potential flows are obtained by mapping doublet inside a circle (for $\zeta = \beta$) to outside a circle at infinite and finite locations respectively using inverse map. The solution of potential flow around circular cylinder is further mapped around polygonal shaped cylinder using hypotrochoidal mapping and the flow parameters are obtained and presented. The effect of geometrical parameters such as corner radius and number of sides of polygonal shaped geometries on flow parameters and its comparison with existing literature is presented and discussed.

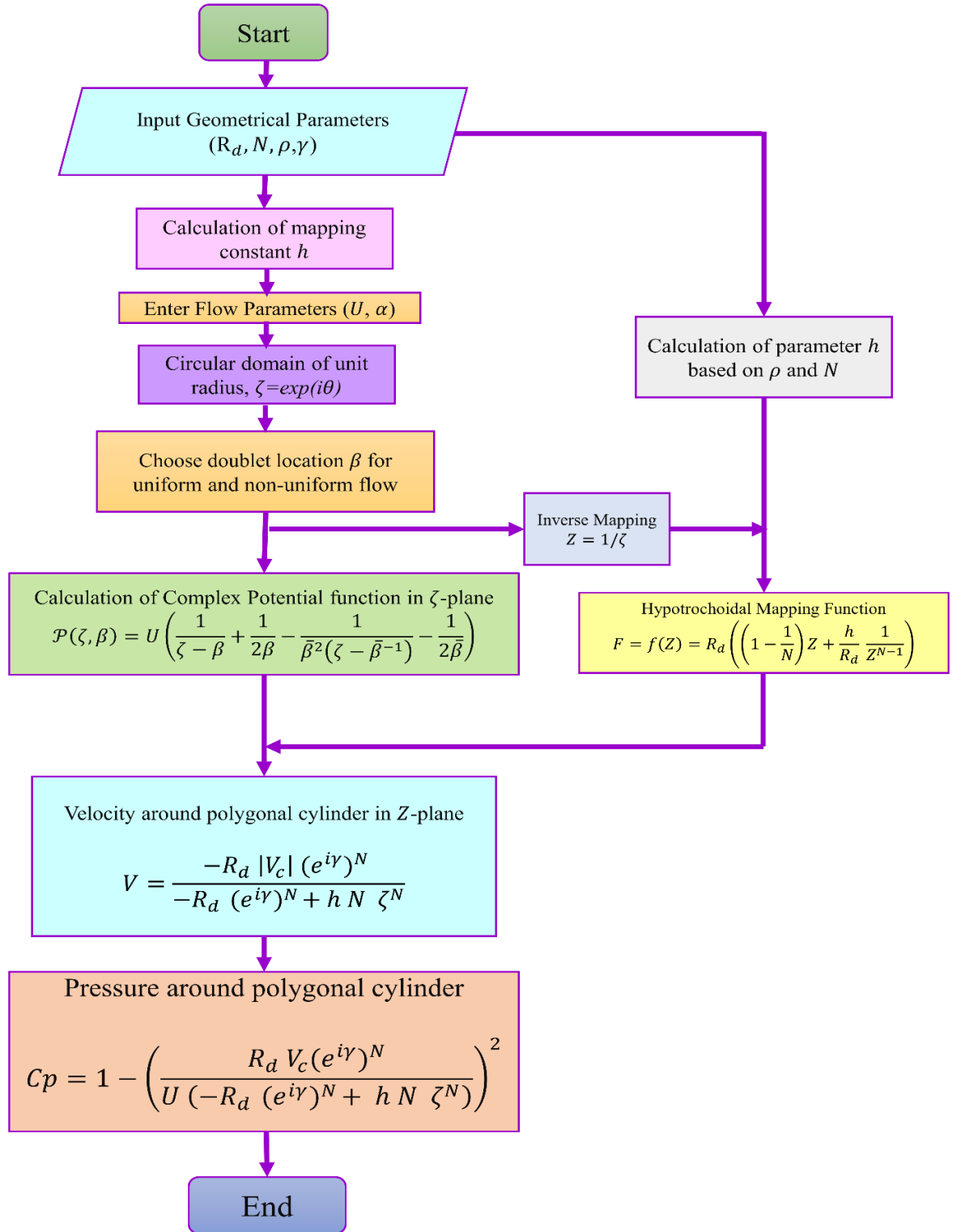


Figure 4.1 Methodology to obtain the flow parameters around single polygonal cylinders in potential flow.

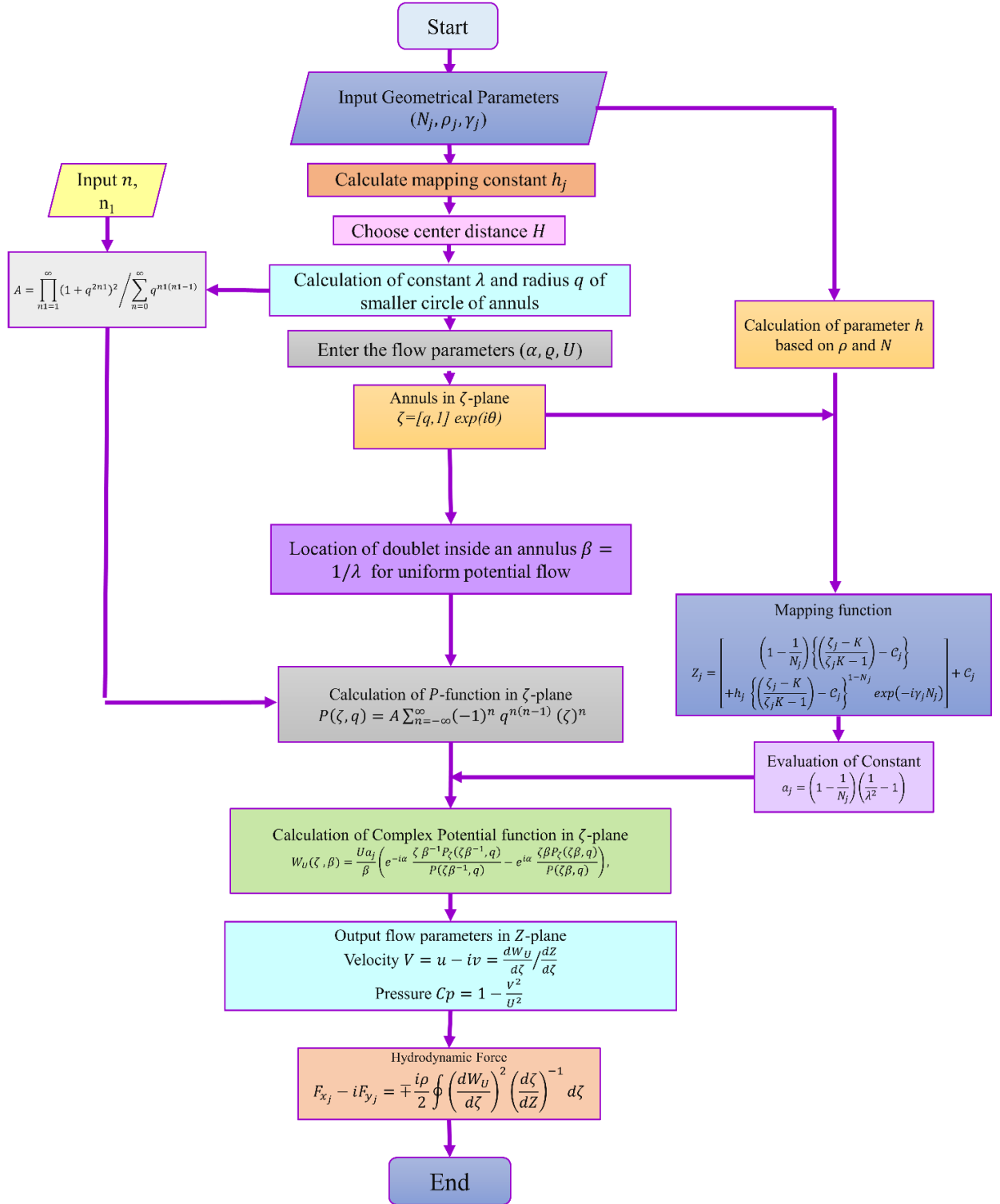


Figure 4.2 Methodology to obtain the flow parameters around two interacting polygonal cylinders.

Some of the results of velocity and pressure distribution around polygonal shaped cylinders are compared with the results obtained from commercial software package (ANSYS). In ANSYS, polygonal shaped geometries are obtained by importing geometric key points extracted using the hypotrochoidal mapping function through the MATLAB program. To model fluid flow, a two-dimensional rectangular fluid domain is generated around the cylinder. The inlet boundary condition at the fluid domain is defined as uniform velocity ($U = 1 \text{ m/s}$) and the outlet boundary condition is defined as zero-gauge pressure. The cylinder surfaces are considered as non-penetrating wall. The structured mesh grid containing quadrilateral elements is used and the fine mesh grid is generated near the cylinder geometry by blocking the domain around the geometry and sizing the elements at the edges of cylinder as shown in Figure 4.3. The flow parameters around the cylinder are checked for grid independence and based on that number of elements are selected for different geometries in simulation, e.g. 22766 number of elements are selected for triangular geometry ($N = 3, h = 0.11$ and $\rho = 0.26$) in fluid domain. The inviscid flow model is selected for modelling potential flow, and the SIMPLE algorithm and Second-Order Upwind scheme is used for the solution of problem.

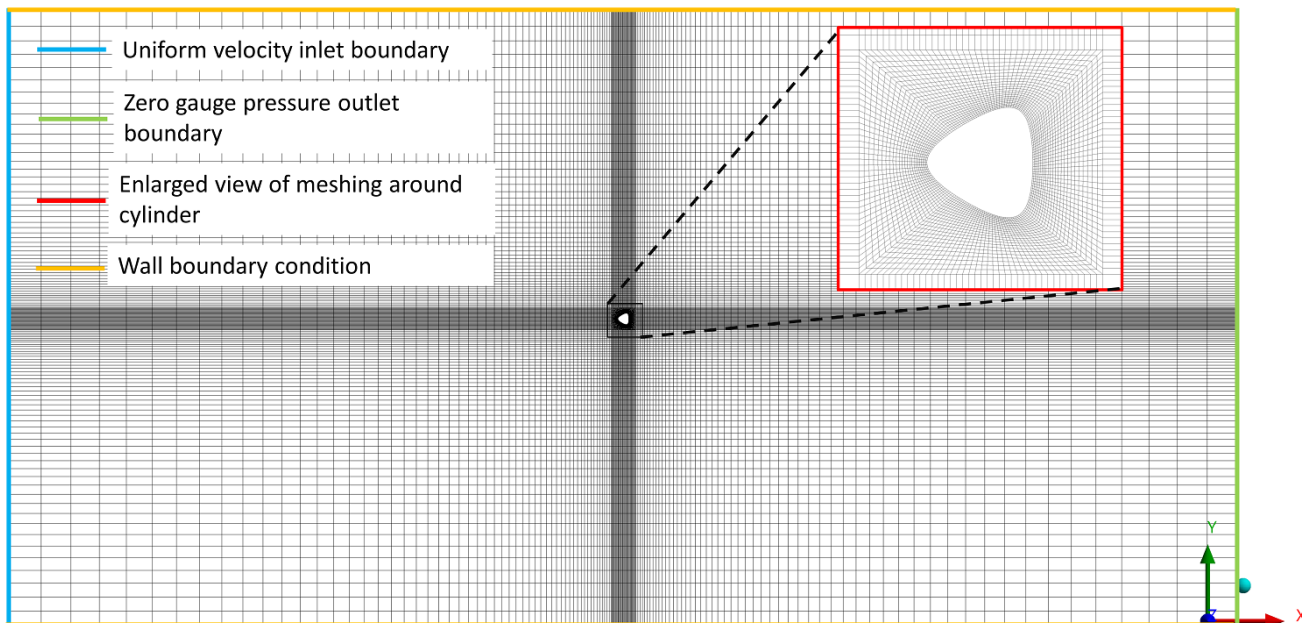


Figure 4.3 Computational domain, meshing, and boundary conditions for flow around single cylinder.

4.3.1 Uniform Potential Flow around a Cylinder

The velocity and pressure distribution around the cylinder for uniform potential flow is arrived at by using Equation (3.37), by taking ' β ' suitably close to zero. The formula presented for velocity and pressure co-efficient (Equation (3.39) and Equation (3.44)) can be directly employed at any point around polygonal shaped geometry (for uniform potential flow $\beta \approx 0$).

Streamline potential flow around a circular cylinder ($R_d = 1$, $h = 0$, $\forall N \geq 2$) and square shaped cylinder ($R_d = 1$, $N = 4$ and $h = 0.1$) using complex potential function and the streamline potential flow obtained using ANSYS are shown in Figure 4.4. The streamlines obtained using complex variable approach (Present method) is quite identical to that obtained through ANSYS. The velocity distribution and pressure co-efficient around circular cylinder is shown in Figure 4.5. The maximum surface velocity obtained is equal to two times the free stream velocity and the value of pressure co-efficient is -3 at 90° and 270° (the angular location θ on cylinder is measured from $+x$ axis direction in counter-clockwise direction) as shown in Figure 4.5(b) and Figure 4.5(e). The maximum velocity and minimum pressure co-efficient are calculated using ANSYS as 1.950 and -2.962 on the surface of circular cylinder (refer Figure 4.5(c) and Figure 4.5(f)).

The results for potential flow around circular cylinder obtained using present method are also compared with existing literature [26,174,175] and found to be in close confirmation (refer Table 4.1). The magnitude and location of flow parameters are found equal to that obtained using principle of superposition (Equation (A.7) and Equation (A.10) from Appendix A).

The velocity and pressure co-efficient contours around elliptical, triangular ($N = 3$) and square ($N = 4$) shaped cylinders for $h = 0.1$ and $R_d = 1$ are shown in Figure 4.6 to Figure 4.7, respectively. The velocity and pressure co-efficient values obtained by complex variable methods are in very good agreement with the results obtained from ANSYS. The maximum velocity and minimum pressure co-efficient obtained using the present method and ANSYS are compared in Table 4.2. The results of maximum surface velocity and pressure co-efficient for polygonal shaped geometries for $N = 2$ to 15, and ∞ for different corner radius are presented in Table 4.3.

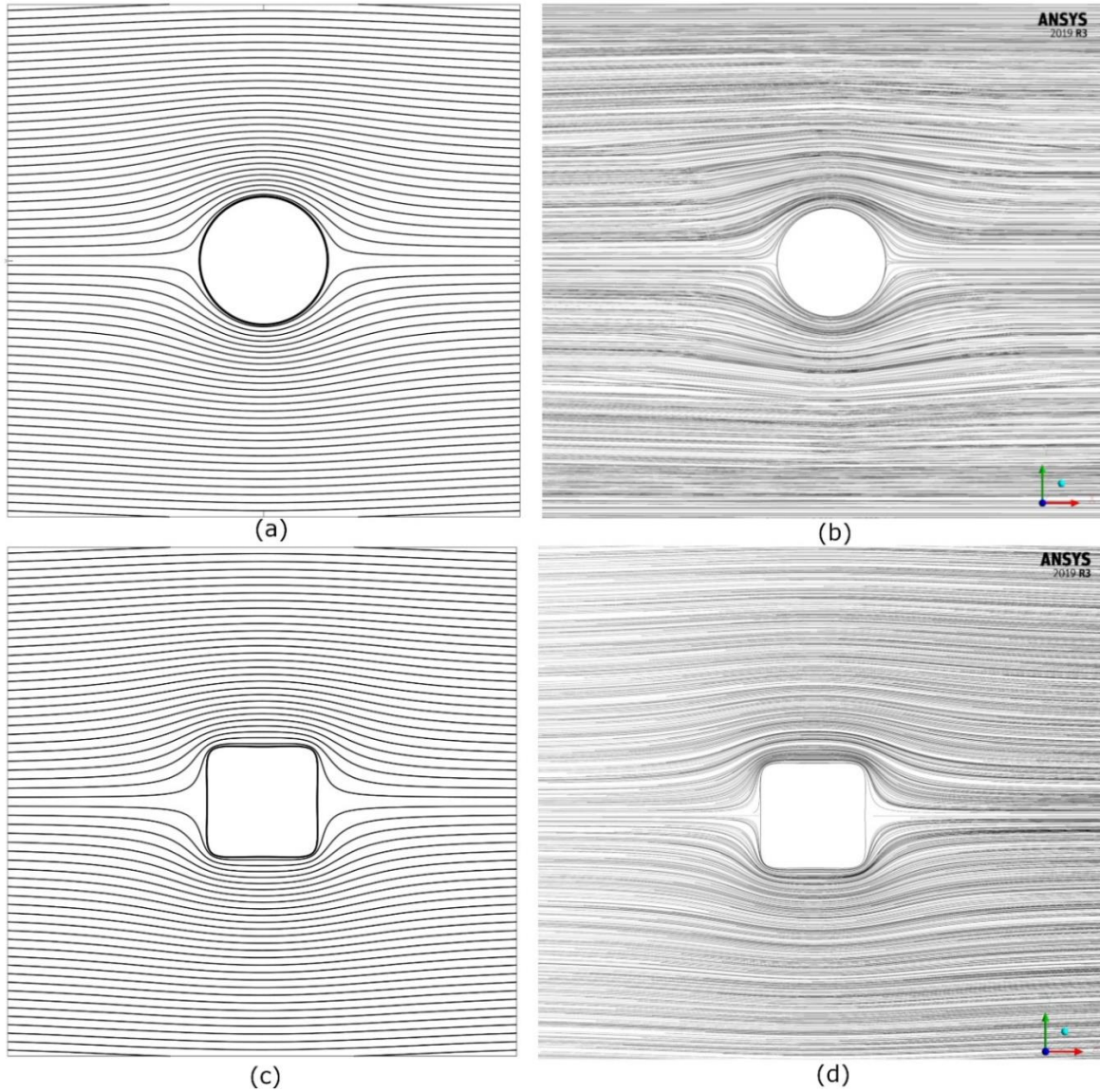


Figure 4.4 Streamlines of Potential flow (a) around circular cylinder obtained using complex potential function along with hypotrochoidal mapping function for $R_d = 1, h = 0, \forall N \geq 2$ (b) around circular cylinder obtained using inviscid flow model in ANSYS (c) around square shaped cylinder obtained using complex potential function along with hypotrochoidal mapping function for $R_d = 1, h = 0.1, N = 4$ (d) around square shaped cylinder obtained using inviscid flow model in ANSYS, hypotrochoidal geometry in ANSYS is generated using key points from MATLAB program.

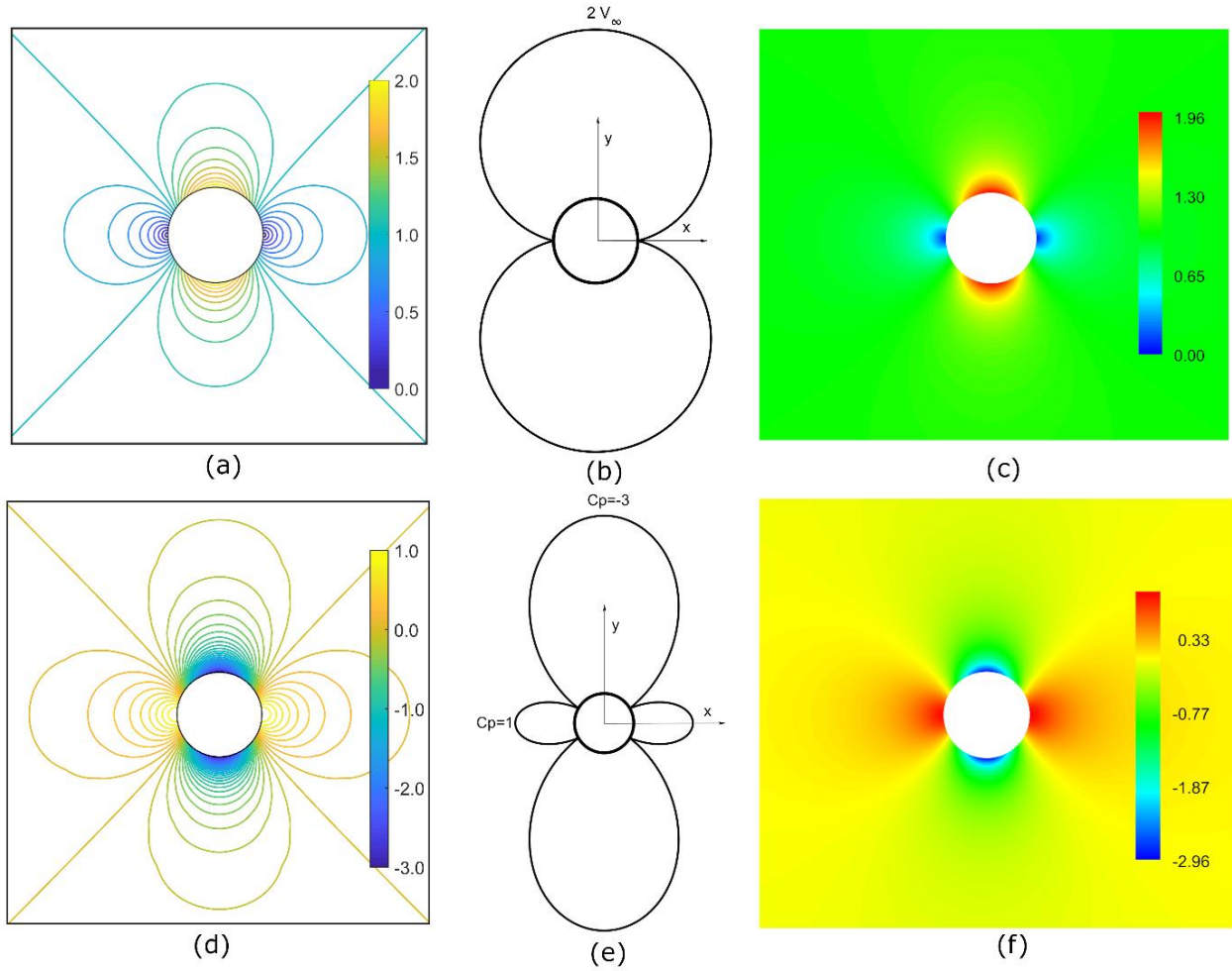


Figure 4.5 Flow parameters around circular cylinder for $R_d = 1$, $h = 0$, $\forall N \geq 2$, using hypotrochoidal mapping function (a) velocity contour obtained using complex potential function (b) velocity profile obtained using complex potential function (c) velocity contour obtained using ANSYS (d) pressure co-efficient contour obtained using complex potential function (e) pressure co-efficient profile obtained using complex potential function (f) pressure co-efficient contour obtained using ANSYS.

4.3.2 Effect of Corner Radius on Flow Parameters in Potential Flow

The corner radius of polygonal geometry is one of the important parameters that can bring significant changes in velocity and pressure field around polygonal cylinders. Corner radius of polygonal shaped cylinders depends on radius of directing circle (R_d), radius of rolling circle (R_r) and distance of curve tracing point from center of rolling circle (h).

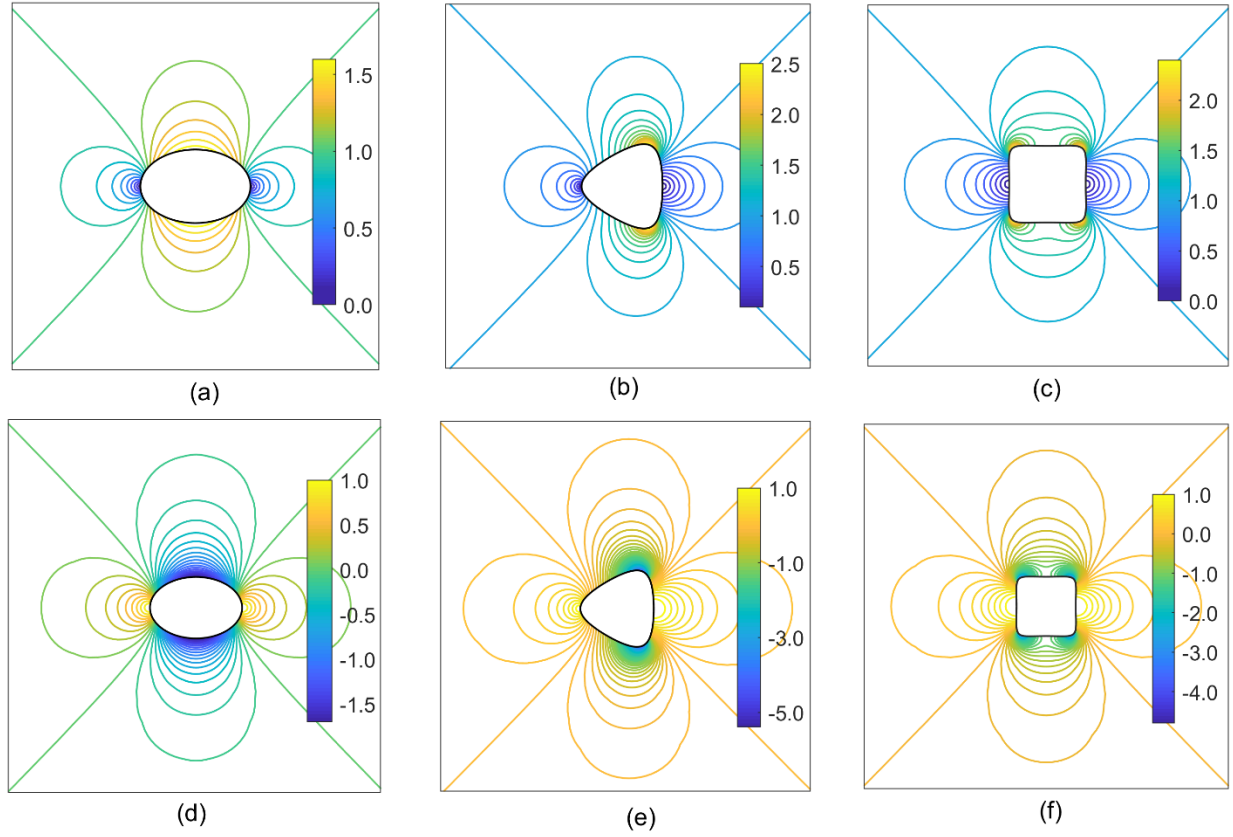


Figure 4.6 Potential flow contours obtained using complex potential function along with hypotrochoidal mapping for $R_d = 1$, $h = 0.1$, (a) velocity contour for $N = 2$ (b) velocity contour for $N = 3$ (c) velocity contour for $N = 4$ (d) pressure co-efficient contour for $N = 2$ (e) pressure co-efficient contour for $N = 3$ (f) pressure co-efficient contours for $N = 4$.

Table 4.1 Results of maximum surface velocity (V_{max}/U in non-dimensional form) on the surface of flat plate, elliptical and circular cylinder are presented, the flow direction is parallel to the flat plat and major axis of elliptical cylinder, and x/y is the ratio of major axis to minor axis of given geometries. Results obtained by present methods are compared with results from existing literature.

Geometry	x/y	Present Method		Milne-Thomson [26]		Chen et al. [78]		White [174]	
		V_{max}/U	Cp_{min}	V_{max}/U	Cp_{min}	V_{max}/U	Cp_{min}	V_{max}/U	Cp_{min}
→ —	∞	1	0	1	0	1	0	1	0
→ 	1.33	1.77	-2.13	1.75	-2.06	1.75	-2.06	1.74	-2.03
→ 	1	2	-3	2	-3	2	-3	2	-3

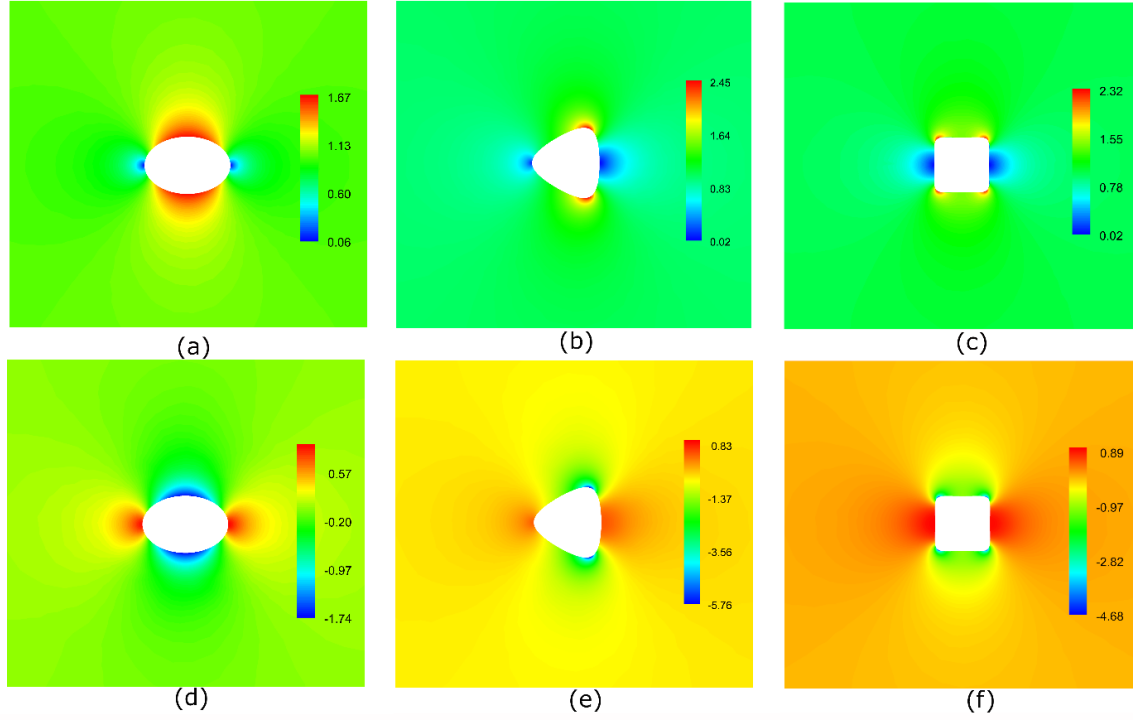


Figure 4.7 Potential flow contours obtained using inviscid flow model in ANSYS (a) velocity contour for $N = 2$ (b) velocity contour for $N = 3$ (c) velocity contour for $N = 4$ (d) pressure co-efficient contour for $N = 2$ (e) pressure co-efficient contour for $N = 3$ (f) pressure co-efficient contours for $N = 4$. Hypotrochoidal geometry in ANSYS is generated using key points from MATLAB program.

Table 4.2 Results of maximum velocity (V_{max}/U in non-dimensional form) and minimum pressure co-efficient C_{pmin} on the surface of polygonal geometries are presented, results obtained by present method are compared with results from ANSYS for $N=2, 3, 4$ at $h=0.1$.

N	Shape & Orientation	V_{max}/U		C_{pmin}	
		Present Method	ANSYS	Present Method	ANSYS
2	→ 	2	1.950	-3	-2.962
2	→ 	1.667	1.668	-1.778	-1.740
3	→ 	2.535	2.454	-5.428	-5.760
4	→ 	2.419	2.408	-4.850	-4.680

The general equation of radius of curvature at the corner is given as follow,

$$\rho_1 = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} / \frac{d^2y}{dx^2}. \quad (4.1)$$

The corner radius of polygonal shaped geometries is computed at one of the corners by taking derivative of parametric equations (Equation (3.13)). Here, all geometries are symmetric about the x-axis and one of the corners of all geometries lies on the x-axis as shown in Figure 3.3. However, the geometry can be rotated to any angle to find flow parameters. All geometries using Hypotrochoidal mapping function are inscribed in the circle of radius $R_d - R_r + h$.

The normalised form of corner radius (ρ) depends on normalised distance h/R_r as shown in equation given below,

$$\rho = \left[\frac{(1 - h/R_r)^2}{(1 + h/R_r (N - 1))} \right]. \quad (4.2)$$

The effect of varying corner radius ρ on the flow parameters distribution around the polygonal shaped cylinders are presented here. For given N normalised corner radius ρ can be varied from 0 to 1 and geometries ranging from hypocycloids with N cusps to circle can be obtained. The results for $\rho = 0$ are not included in surface velocity and pressure plots as flow parameters at cusps tends to infinite.

The effect of corner radius on the surface velocity and pressure distribution on an elliptical shaped cylinder for $\rho = 1$ to $\rho = 0.26$ with a major axis parallel (Figure 4.8 (a) and (b)) and perpendicular to the flow direction (Figure 4.8 (c) and (d)) is explained. The radii are measured at the end of major axis.

As the corner radius decreases from $\rho = 1$, the shape of the cylinder changes from circular to elliptical. Maximum surface velocity on a cylinder surface at 90° and 270° decreases from the two times of free stream velocity due to increase of radius of curvature at a given location (refer Figure 4.8 (a)). The minimum C_p value increases from -3 at 90° and 270° as shown in Figure 4.8 (b).

For the case of zero corner radius a flat plate is obtained, for which the maximum velocity is equal to free stream velocity and the minimum pressure co-efficient reaches to zero value.

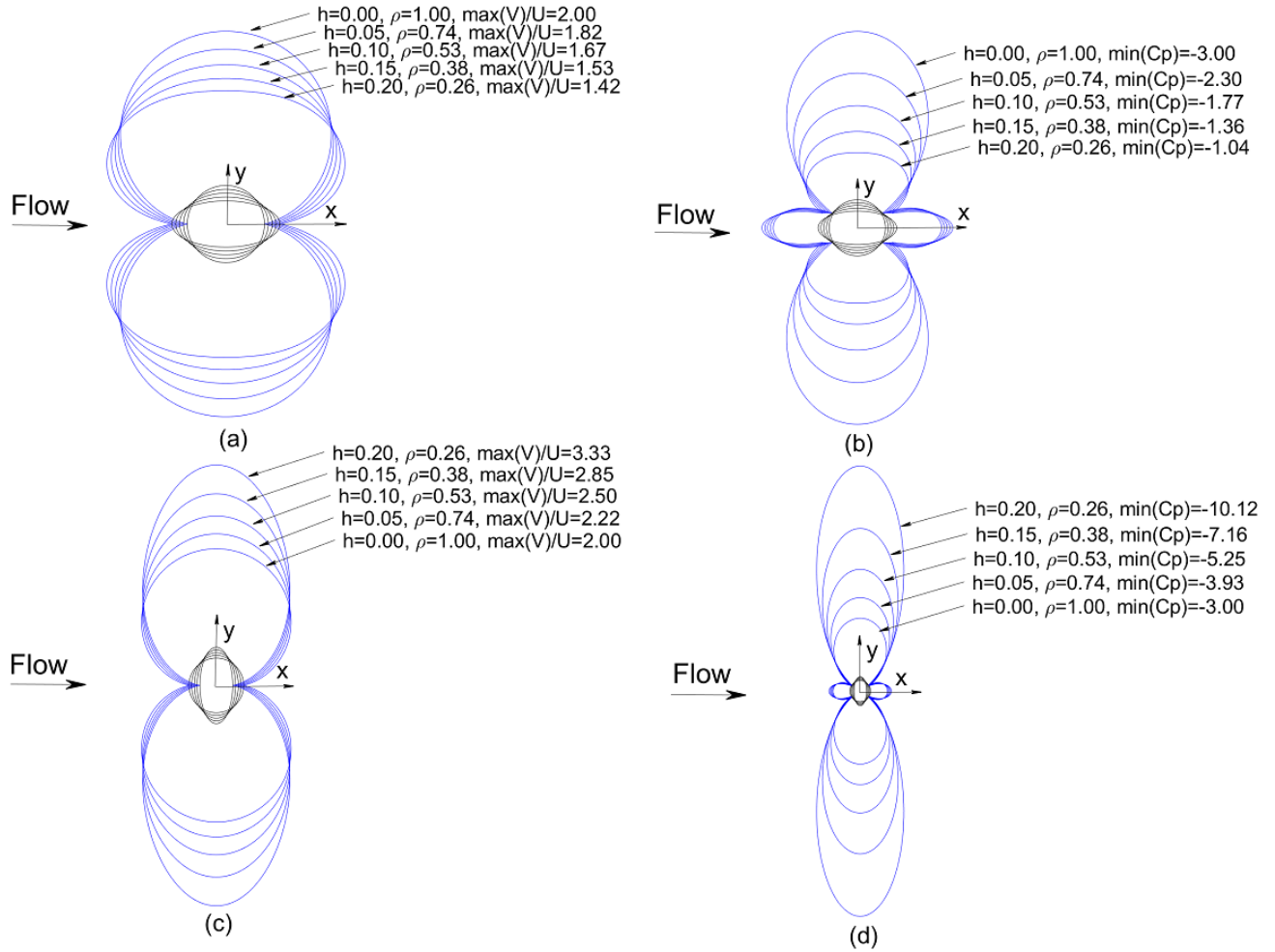


Figure 4.8 Flow parameters obtained using complex potential function along with hypotrochoidal mapping for $N = 2$, $R_d = 1$, and corner radius $\rho = 1$ to $\rho = 0.26$, (a) velocity distribution on the surface of elliptic cylinder with major axis parallel to the flow (b) pressure co-efficient distribution on the surface of elliptic cylinder with major axis parallel to the flow (c) velocity distribution on the surface of elliptic cylinder with major axis normal to the flow, (d) pressure co-efficient on the surface of elliptic cylinder with major axis normal to the flow.

In the case of elliptic cylinder with major axis perpendicular to the flow direction as shown in Figure 4.8 (c) and (d), the maximum velocity increases above the two times of free stream velocity due to increased radius of curvature at 0° and 180° , and the pressure coefficient C_p decreases below the value of -3.

The circular geometry become a flat plate at zero corner radius (singular point), and velocity and pressure co-efficient at the endpoints of flat plate perpendicular to the flow approaches to positive infinity and negative infinity respectively. From Figure 4.8, it can be established that not only the shape of the cylinder, but its orientation with respect to flow decides the flow parameters near it (refer Table 4.1).

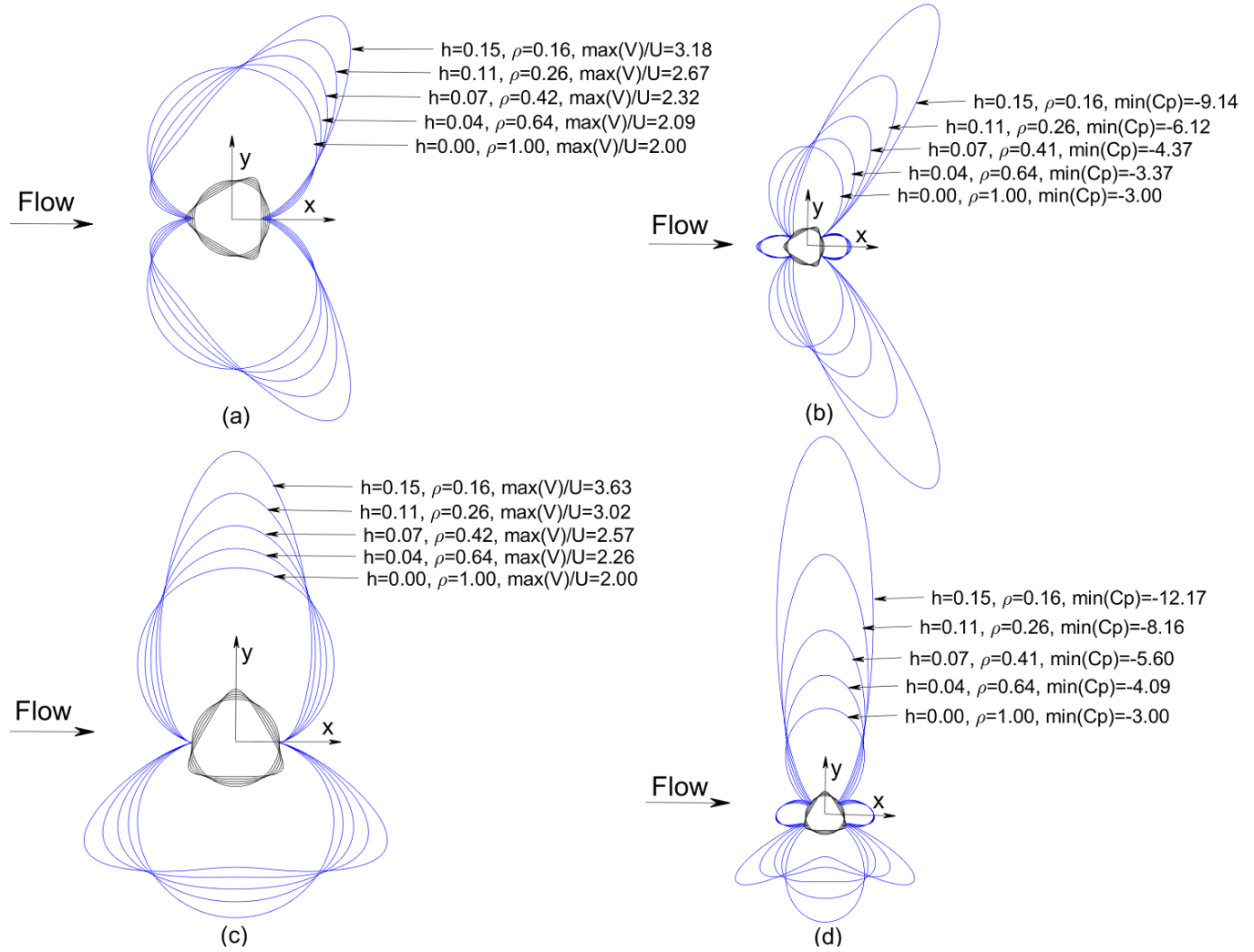


Figure 4.9 Flow parameters obtained using complex potential function along with hypotrochoidal mapping for $N = 3$, $R_d = 1$, and corner radius $\rho = 1$ to $\rho = 0.16$, (a) velocity distribution on the surface of triangular cylinder with vertex facing the flow (b) pressure co-efficient distribution on the surface of triangular cylinder with vertex facing the flow (c) velocity distribution on the surface of triangular cylinder rotated by 90° , (d) pressure co-efficient distribution on the surface of triangular cylinder rotated by 90° .

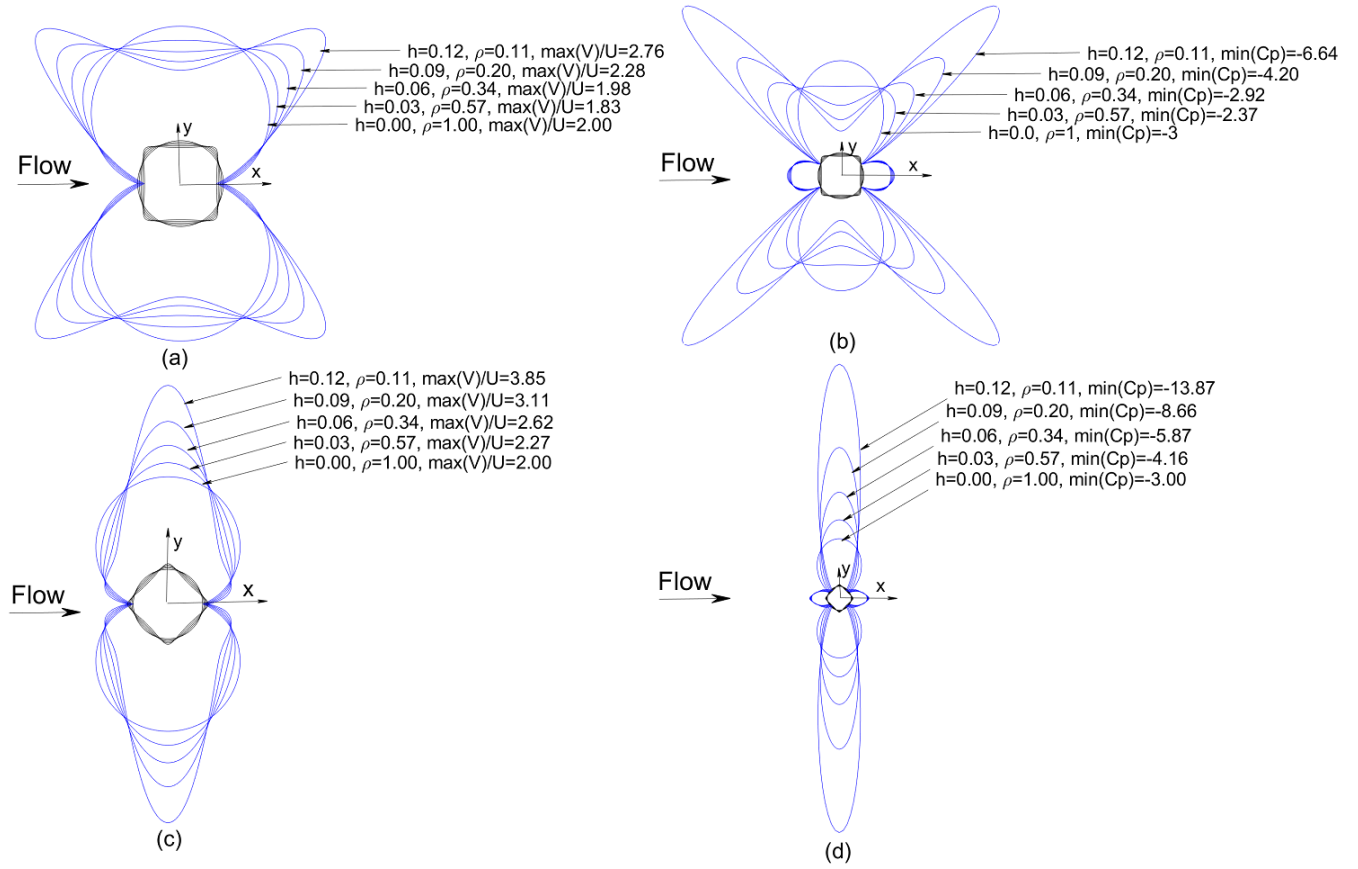


Figure 4.10 Flow parameters obtained using complex potential function along with hypotrochoidal mapping for $N = 4$, $R_d = 1$, and corner radius $\rho = 1$ to $\rho = 0.11$, (a) velocity distribution on the surface of square cylinder with side facing the flow (b) pressure co-efficient distribution on the surface of square cylinder with side facing the flow (c) velocity distribution on the surface of square cylinder rotated by 45° with vertex facing the flow, (d) pressure co-efficient distribution on the surface of square cylinder rotated by 45° with vertex facing the flow.

The velocity and pressure distribution on the surface of polygonal shaped cylinders for $N = 3, 4$ with varying normalized corner radius $\rho = 1$ to $\rho = 0.16$ and $\rho = 1$ to $\rho = 0.11$ respectively are shown in Figure 4.9 and Figure 4.10. The results for the corner radii $\rho < 0.11$ are not shown in figures as the flow parameters attain large values at the corners and inclusion of that in figures reduces overall readability of plots. The maximum velocity and minimum pressure drop on the polygon surface attained at corners excluding corners where stagnation points are located. The velocity at the corners reaches above the two times of free stream velocity and the minimum pressure co-efficient C_p drops below the value of -3 corresponding to the circular cylinder.

For the polygons with number of sides $N \geq 3$, the effect of corner radius on maximum surface velocity is shown in Figure 4.11. The maximum surface velocity for all polygonal shaped cylinders reduces from infinity to 2.0 (two times of free stream velocity) as ρ increases from 0 to 1. The pressure co-efficient C_p for all values of $N \geq 3$ converges to -3 at $\rho = 1$ and becomes negative infinite at $\rho = 0$ (Refer Figure 4.11).

The pressure drops for hexagon, octagon, dodecagon, and polygons with $N = 36$, $N = 100$, $N = 500$ is presented in Figure 4.12 for the finite corner radius $\rho = 0.1$. Magnitude of pressure drop at each finite radius corner decreases with increasing number of sides of polygon. The range of pressure drop below -3 becomes narrower and finally becomes similar to the pressure drop corresponding to circular cylinder at $N = \infty$. Results of pressure drops shown in Figure 13 are comparable with that shown in Figure 2 (page number 125) from Tian and Wu [128]. Tian and Wu [128] has obtained pressure drop results for polygonal shaped geometries with zero corner radius and straight edges by using Schwarz-Christoffel mapping function. The results are in very good agreement.

For flow direction along the x -axis, the minimum pressure drop is found at the corners located at or near to 0° and 180° , pressure drop increases at corners on the cylinder surface while moving from 0° to 90° and found maximum at corner located at or near to 90° and 270° as shown in Figure 4.13.

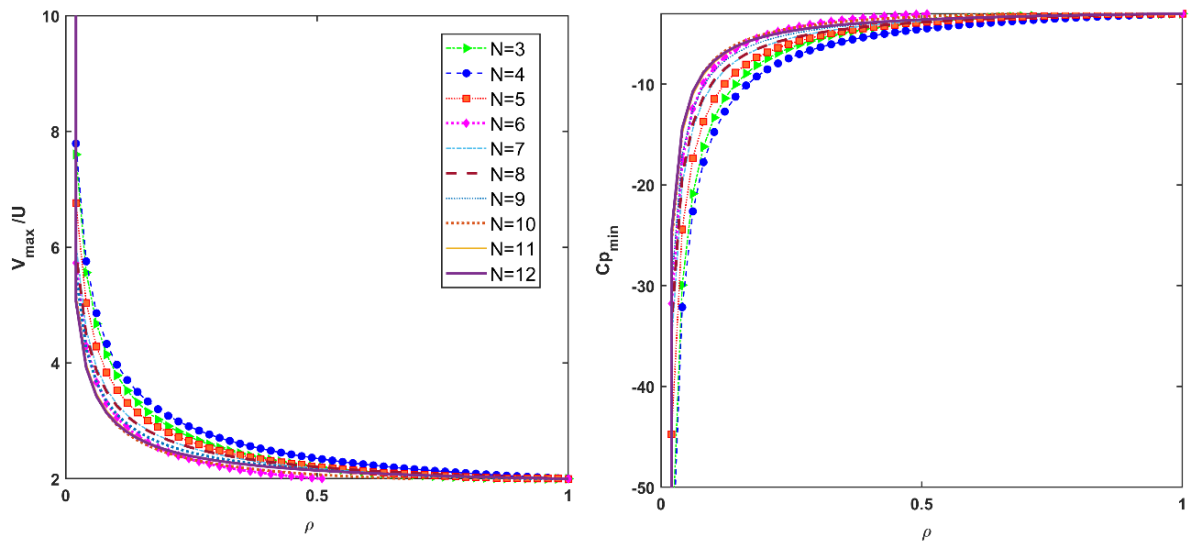


Figure 4.11 variation of maximum surface velocity V_{max}/U and minimum pressure co-efficient $C_{p_{min}}$ with normalised corner radius $\rho = 0$ to $\rho = 1$ for different number of sides of polygon.

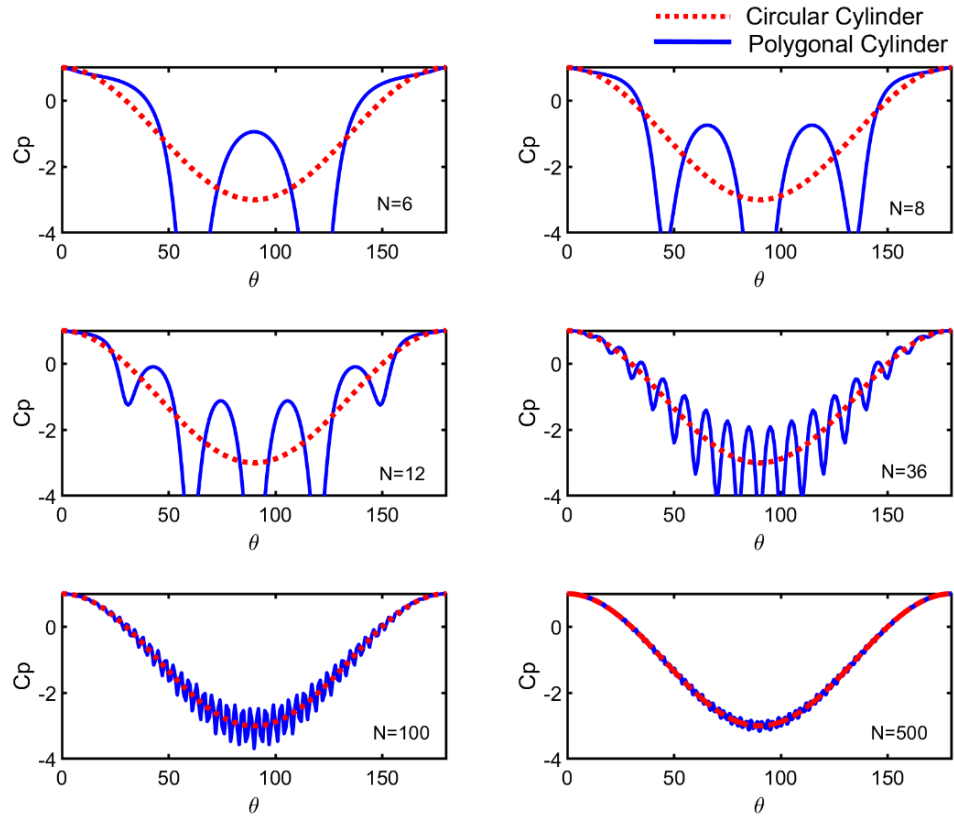


Figure 4.12 Effect of number of sides of polygonal cylinder on flow parameters, pressure coefficient C_p is shown by dotted line for unit circular cylinder and for inscribed regular polygonal cylinder C_p is shown by continues line.

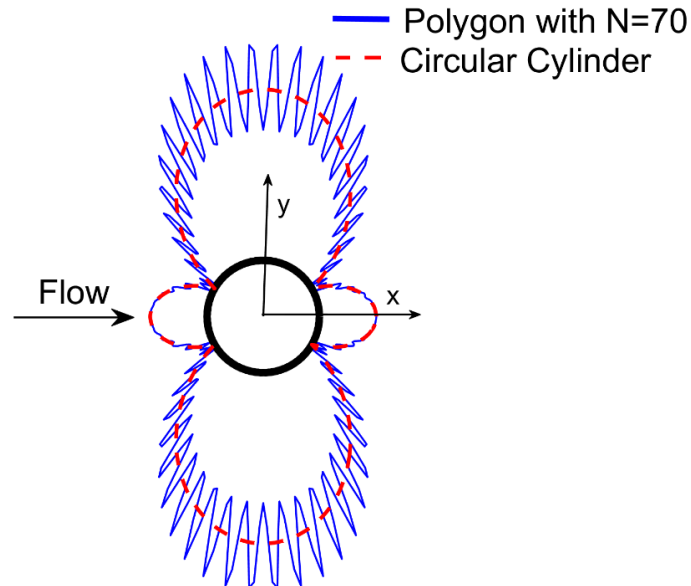
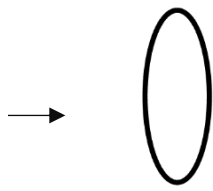


Figure 4.13 Distribution of pressure co-efficient C_p on the surface of circular cylinder is shown by dotted line and on the surface of polygonal cylinder with number of sides $N = 70$ is shown by continues line.

Table 4.3 Results of the effect of geometry orientation on maximum surface velocity (V_{max}/U in non-dimensional form) and minimum pressure co-efficient $C_{p_{min}}$ is presented. Elliptic cylinder is taken as a geometry for $N = 2$ and varying corner radius ($0 < \rho < 0.74$).

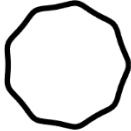
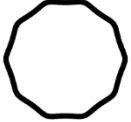
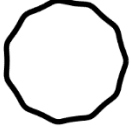
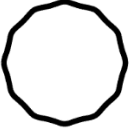
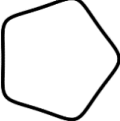
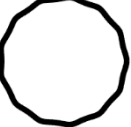
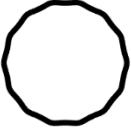
Shape ($N = 2$) orientation with respect to the flow along the x -axis	ρ	V_{max}/U at 90° and 270°	$C_{p_{min}}$ at 90° and 270°
	0	1	0
	0.26	1.42	-1.06
	0.38	1.53	-1.36
	0.53	1.67	-1.77
	0.74	1.82	-2.30
	0	$+\infty$	$-\infty$
	0.26	3.33	-10.12
	0.38	2.85	-7.16
	0.53	2.50	-5.25
	0.74	2.22	-3.93

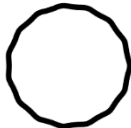
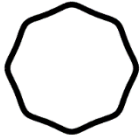
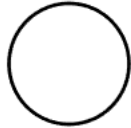
4.3.3 Potential Flow due to presence of Singularity (Doublet or Vortex) at Finite Distance from the Cylinder

The presence of singularity such as vortex or doublet close to the cylinder gives rise to non-uniform flow around cylinder. For any value of $\beta \neq 0$, the point vortex and doublet can be mapped at finite distance from the cylinder by using complex potential function given in Equation (3.31) and Equation (3.32) respectively (refer Figure 4.14).

The velocity distribution on surface of circular, elliptical (for $\gamma = 0^\circ$ and $\gamma = 90^\circ$), triangular (for $\gamma = 180^\circ$ and $\gamma = 90^\circ$), and square (for $\gamma = 0^\circ$ and $\gamma = 90^\circ$) shaped cylinders in non-uniform potential flow for various value of $\beta \neq 0$ are shown in Figure 4.15 to Figure 4.17. Asymmetric velocity distribution is found on the surface of circular and polygonal shaped cylinders, which shows the effect of presence of doublet nearby the cylinder. As the value of $\beta \rightarrow 0$, the symmetrical distribution of flow parameters similar to the uniform potential flow are obtained.

Table 4.4 Maximum surface velocity (V_{max}/U) and minimum pressure co-efficient Cp_{min} on the surface of polygonal shaped cylinders for corner radius $0.28 \leq \rho \leq 0.82$ and number of sides $N=2$ to $15, \infty$ are presented using present method.

N	Geometry orientation for flow along +x- axis	ρ	V_{max}/U	Cp_{min}	N	Geometry orientation for flow along +x- axis	ρ	V_{max}/U	Cp_{min}
2		0.82	1.88	-2.52	9		0.82	2.02	-3.09
		0.64	1.75	-2.06			0.64	2.08	-3.33
		0.46	1.61	-1.59			0.46	2.18	-3.77
		0.28	1.45	-1.1			0.28	2.39	-4.73
2		0.82	2.14	-3.58	10		0.82	1.98	-2.9
		0.64	2.34	-4.46			0.64	2.01	-3.05
		0.46	2.64	-5.99			0.46	2.1	-3.4
		0.28	3.22	-9.36			0.28	2.28	-4.21
3		0.82	2.02	-3.08	11		0.82	2.22	3.93
		0.64	2.09	-3.38			0.64	2.50	-5.25
		0.46	2.25	-4.08			0.46	2.85	-7.16
		0.28	2.61	-5.82			0.28	3.33	-10.1
4		0.82	1.92	-2.69	12		0.82	2.03	-3.13
		0.64	1.84	-2.39			0.64	2.08	-3.34
		0.46	1.87	-2.50			0.46	2.17	-3.7
		0.28	2.07	-3.28			0.28	2.34	-4.48
5		0.82	2.02	-3.1	13		0.82	2.02	-3.08
		0.64	2.1	-3.41			0.64	2.06	-3.26
		0.46	2.25	-4.05			0.46	2.14	-3.58
		0.28	2.55	-5.5			0.28	2.3	-4.3
6		0.82	1.94	-2.77	14		0.82	1.99	-2.97
		0.64	1.94	-2.78			0.64	2.03	-3.11
		0.46	2.04	-3.15			0.46	2.10	-3.39
		0.28	2.27	-4.13			0.28	2.24	-4.03

7		0.82	2.02	-3.1	15		0.82	2.02	-3.07
		0.64	2.09	-3.37			0.64	2.06	-3.23
		0.46	2.21	-3.9			0.46	2.13	-3.52
		0.28	2.46	-5.07			0.28	2.27	-4.16
8		0.82	2.05	-3.19	∞		$0 \leq \rho \leq 1$ 2 -3		
		0.64	2.12	-3.49					
		0.46	2.24	-4.00					
		0.28	2.47	-5.11					

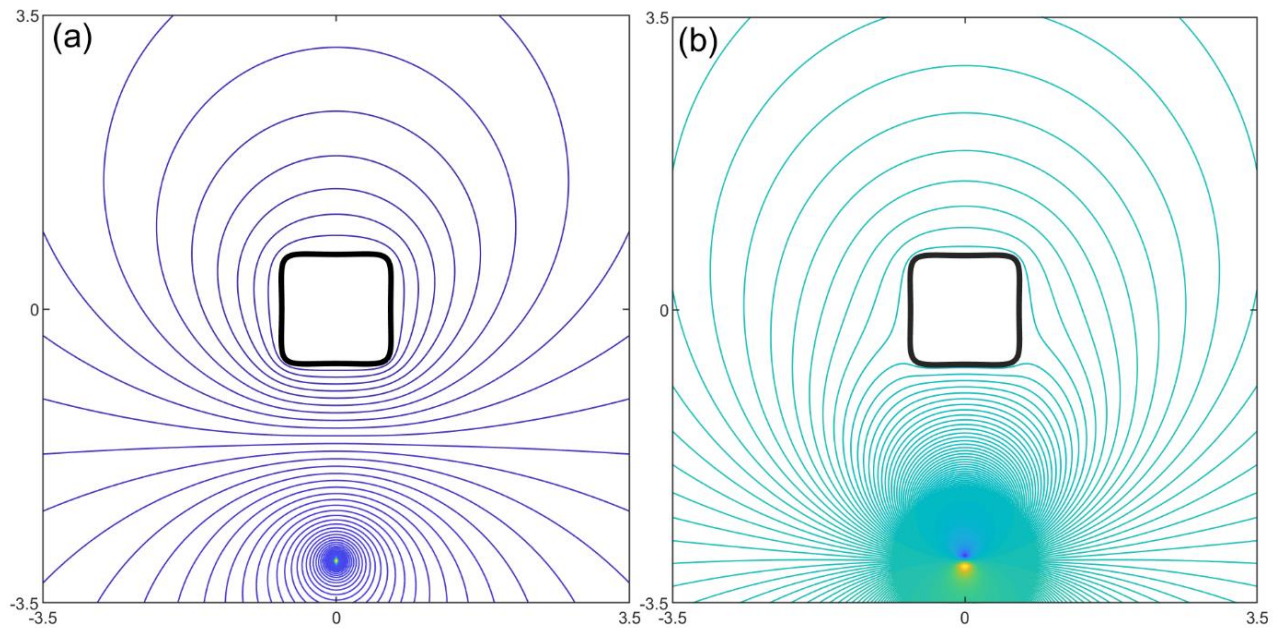


Figure 4.14 Non-uniform Potential flow (a) streamlines around square shaped cylinder for $R_d = 1, h = 0.1$ due to presence of point vortex, for $\beta = +0.25i$ (b) streamlines around square shaped cylinder due to presence of doublet, for $\beta = +0.25i$

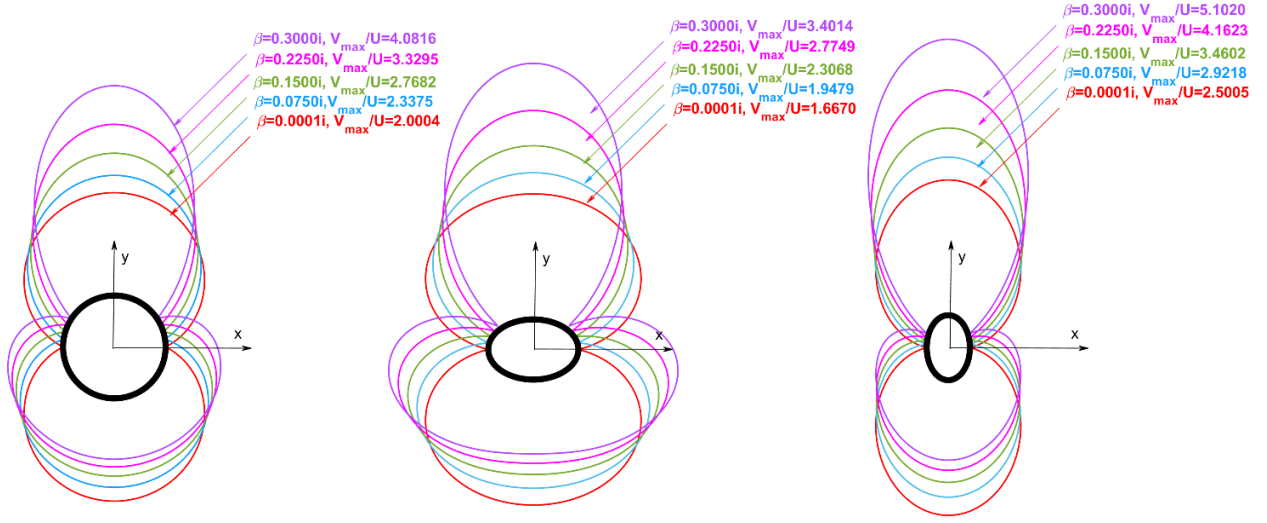


Figure 4.15 Non-uniform Potential flow, velocity distribution on surface of (left) circular cylinder ($R_d = 1, h = 0, \forall N \geq 2$) (middle) elliptical cylinder with flow along major axis ($R_d = 1, N = 2, h = 0.1, \gamma = 0^\circ$), (right) elliptical cylinder with flow normal to major axis ($R_d = 1, N = 2, h = 0.1, \gamma = 90^\circ$), for varying doublet location β on positive imaginary axis.

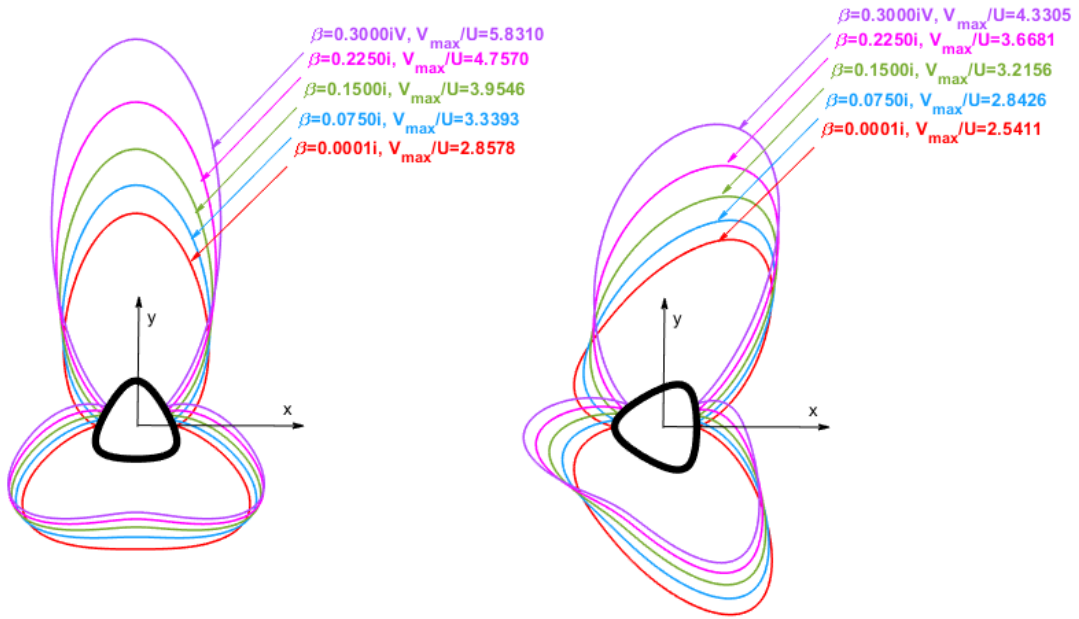


Figure 4.16 Non-uniform Potential flow, velocity distribution on surface of (left) triangular cylinder ($R_d = 1, N = 3, h = 0.1, \gamma = 90^\circ$), (right) triangular cylinder ($R_d = 1, N = 3, h = 0.1, \gamma = 180^\circ$), for varying doublet location β on positive imaginary axis.

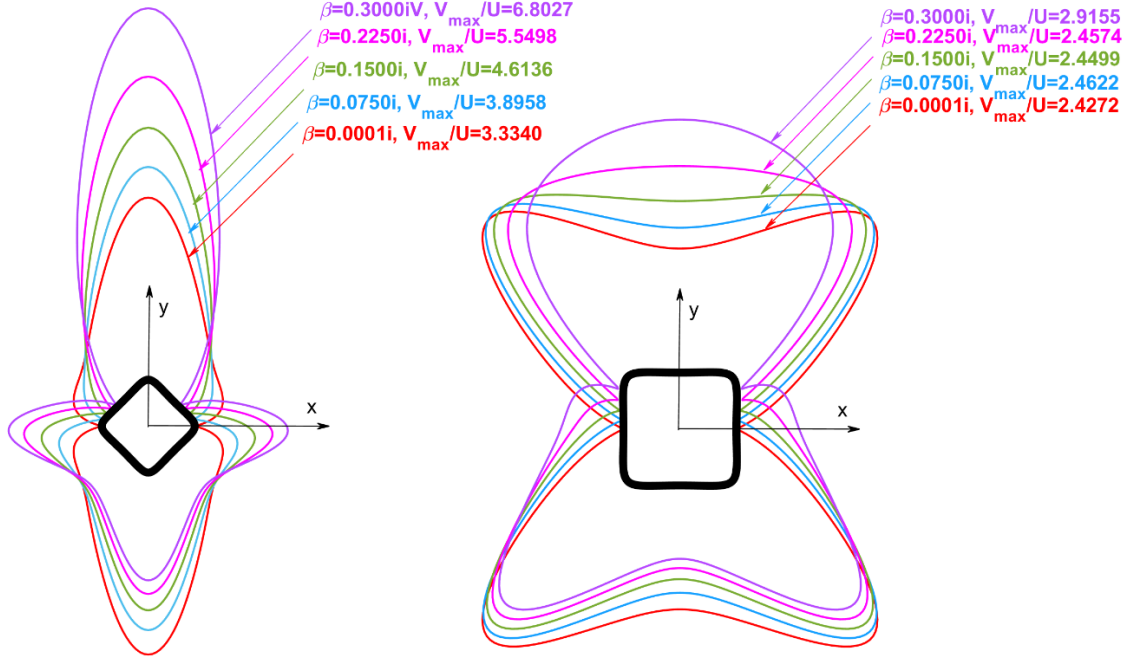


Figure 4.17 Non-uniform Potential flow, velocity distribution on surface of (left) triangular cylinder ($R_d = 1, N = 4, h = 0.1, \gamma = 90^\circ$), (right) triangular cylinder ($R_d = 1, N = 4, h = 0.1, \gamma = 0^\circ$), for varying doublet location β on positive imaginary axis.

4.4 Hydrodynamic Interaction between Two Circular Cylinders

The hydrodynamic interaction between two-circular cylinders of diameter ratio D/d in uniform potential flow is discussed in this section. The center distance between the circular cylinder (H/D), diameter ratio (D/d), flow angle (α), fluid density (ρ), and free stream velocity (U) are various independent parameters that influence the hydrodynamic interaction.

Some of the results obtained using present method are compared with the results obtained through ANSYS and available from existing literature for validation purpose. The geometry of two-dimensional cylinders in ANSYS Design Modeler is generated using key-points extracted from the MATLAB program of present method and the two-dimensional rectangular fluid domain is generated around the pair of circular cylinders. The fluid domain around geometry with structured mesh grid containing quadrilateral elements and edge sizing at the periphery of cylinders are shown in Figure 4.18.

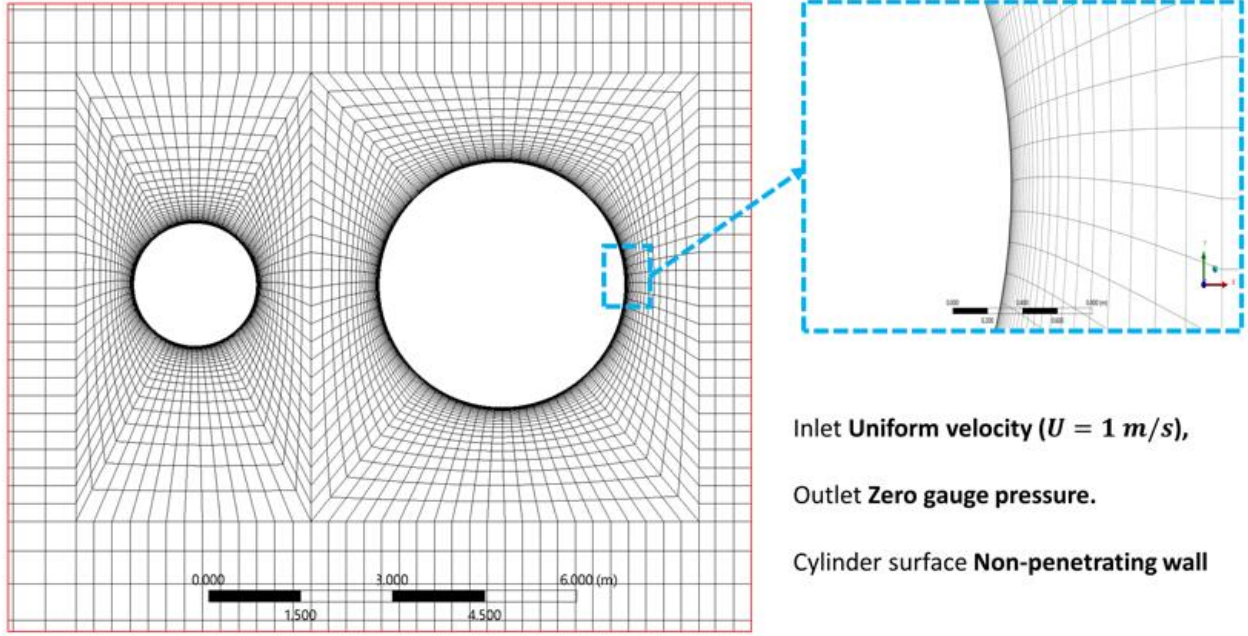


Figure 4.18 Computational domain for the flow around pair of circular cylinders in uniform potential flow.

4.4.1 Convergence Test

For two-cylinder problem, the output parameters (velocity, pressure and hydrodynamic forces) are obtained by calculating complex potential function in an annulus in ζ -plane. The complex potential function and the output parameters are dependent on P -function. The P -function is represented as Laurent series, therefore, convergence test of P -function is performed to ensure accuracy of the results. For $q = 0.4$, and $\theta = 356^\circ$ to 360° , the convergence of P -function is shown in Figure 4.19. The function converges after $n = 5$ terms.

The output parameters are found to be converge for $n \geq 5$ terms. However, to ensure accuracy the output results are obtained for $n \geq 15$ terms. The sample convergence plots of output parameters on the surface of triangular cylinder interacting with diamond shaped cylinder for $\theta = \pi/4$, $q = 0.25$, $H = 2.5, 3$ and 3.5 , $\gamma_1 = \gamma_2 = \pi$ and corner radii $\rho_1 = \rho_2 = 0.1$ are shown in Figure 4.20.

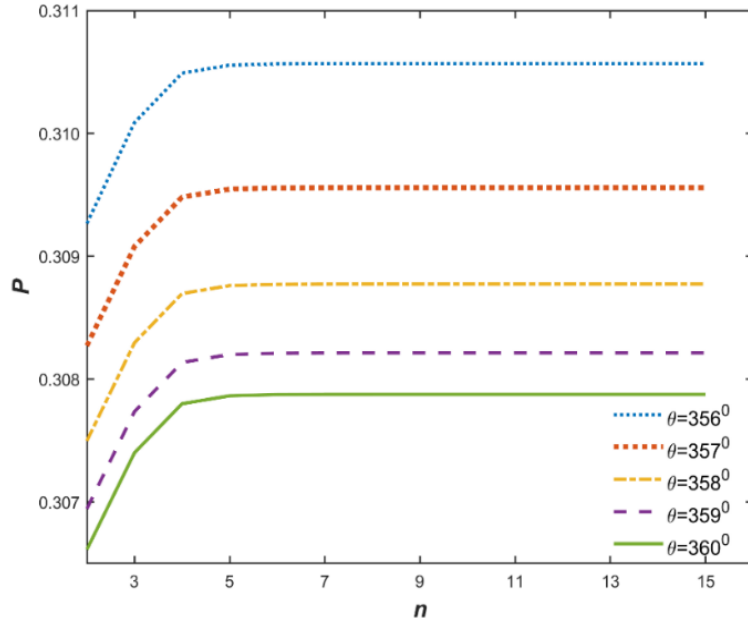


Figure 4.19 Convergence of P -function (Laurent Series representation), evaluated for $q = 0.4$, and $\theta = 356^\circ$ to 360° inside an annulus region.

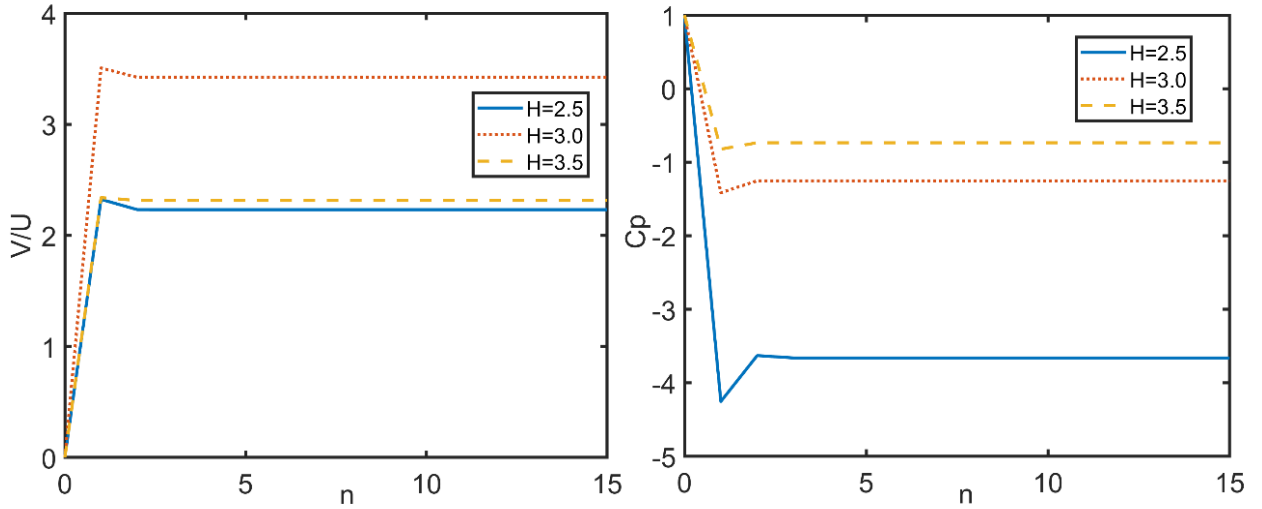


Figure 4.20 Convergence of velocity (V/U) and pressure co-efficient (C_p) on the surface of triangle cylinder ($N_1 = 3, \rho_1 = 0.1, \gamma_1 = \pi$) interacting with downstream diamond shaped cylinder ($N_2 = 4, \rho_2 = 0.1, \gamma_2 = \pi$). The velocity and pressure coefficients are evaluated for input parameters $\theta = 45^\circ$ and for center-to-center distances $H = 2.5, 3$ and 3.5 .

4.4.2 Velocity and Pressure distribution

The velocity and pressure field around the pair of two circular cylinders of different diameter ratio (D/d) is investigated using mathematical formulation presented in Section 3.3. The diameter of one cylinder (Cylinder-1) is kept constant ($d = 2$), while the diameter of another cylinder (Cylinder-2) is changed ($D \geq 2$) for different center-to-center distances. The angular location (θ_1) on Cylinder-1 is measured from $-x$ axis in clockwise direction (Figure 3.5), whereas the angular location (θ_2) on the surface of Cylinder-2 and the flow angle (α) are measured from $+x$ axis in counter clockwise direction. The flow along the line connecting the centers of cylinders ($\alpha = 0^\circ$) is considered as tandem configuration, and the flow normal to the line connecting the centers of cylinders ($\alpha = 90^\circ$) is considered as side-by-side configuration in present study.

The velocity contours around the pair of equal circular cylinders for the flow along the line connecting the centers of cylinders and for the flow normal to the line connecting the centers of cylinders are presented in Figure 4.21 and Figure 4.24. For the flow along the line connecting the centers of cylinders (Figure 4.21), the maximum velocity $V_{max}/U = 1.864$ is found using the present method for center distance $H = 1.5d$, which is in close confirmation to the maximum velocity $V_{max}/U = 1.863$ obtained using ANSYS for the same flow direction and center distance. For $H = 10d$, using present method and ANSYS the maximum velocities are found as $V_{max}/U = 1.995$ and 1.980 respectively (Figure 4.22). It is observed from the velocity contours that for smaller center distance ($H = 1.5d$), the flow parameter around the cylinders is significantly affected by the vicinity of each other, whereas at larger center distance ($H = 10d$) the flow parameter weakens significantly. The velocity contour shown in Figure 4.22 for the center distance $H = 10d$ is found identical to that for single isolated circular cylinder in uniform potential flow shown in Figure 4.5.

The velocity contours around two equal circular cylinders in side-by-side arrangement for $H = 1.5d$ and $10d$ using present method and ANSYS are shown in Figure 4.23 and Figure 4.24 respectively. The results using present method are found in very good agreement with those obtained through ANSYS.

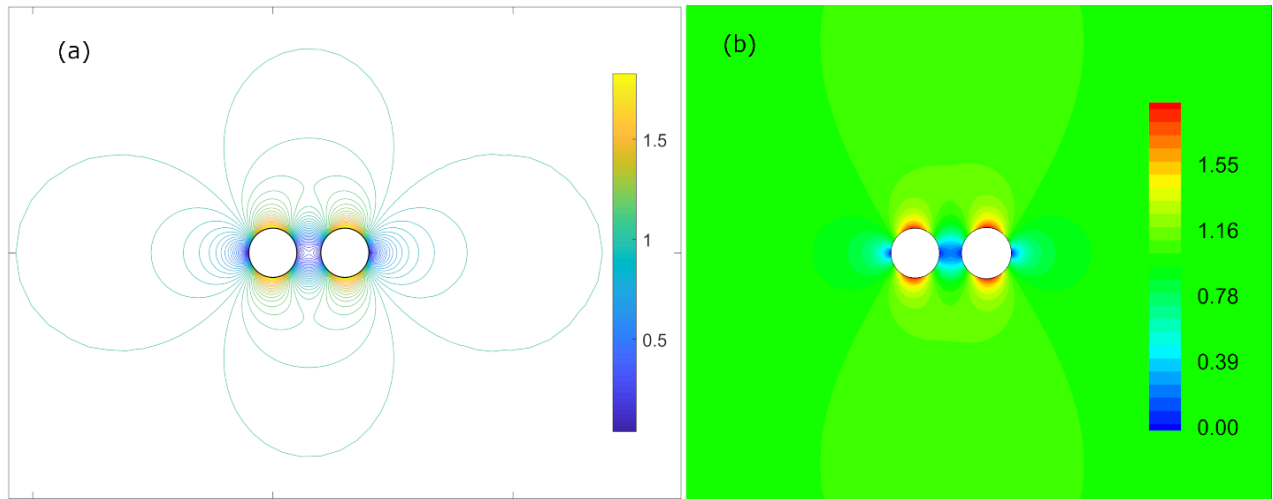


Figure 4.21 Velocity contours around two circular cylinders of diameter ratio $D/d = 1$ with center distance $H = 1.5d$ in uniform potential flow along the line connecting the centers of cylinders using (a) present method and (b) ANSYS.

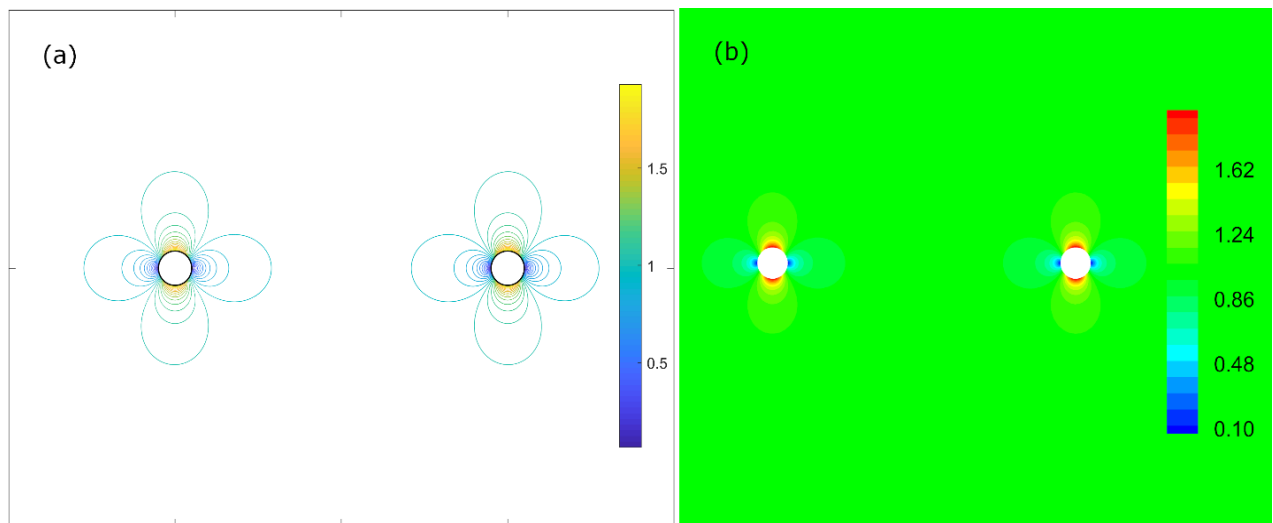


Figure 4.22 Velocity contours around two circular cylinders of diameter ratio $D/d = 1$ with center distance $H = 10d$ in uniform potential flow along the line connecting the centers of cylinders using (a) present method and (b) ANSYS.

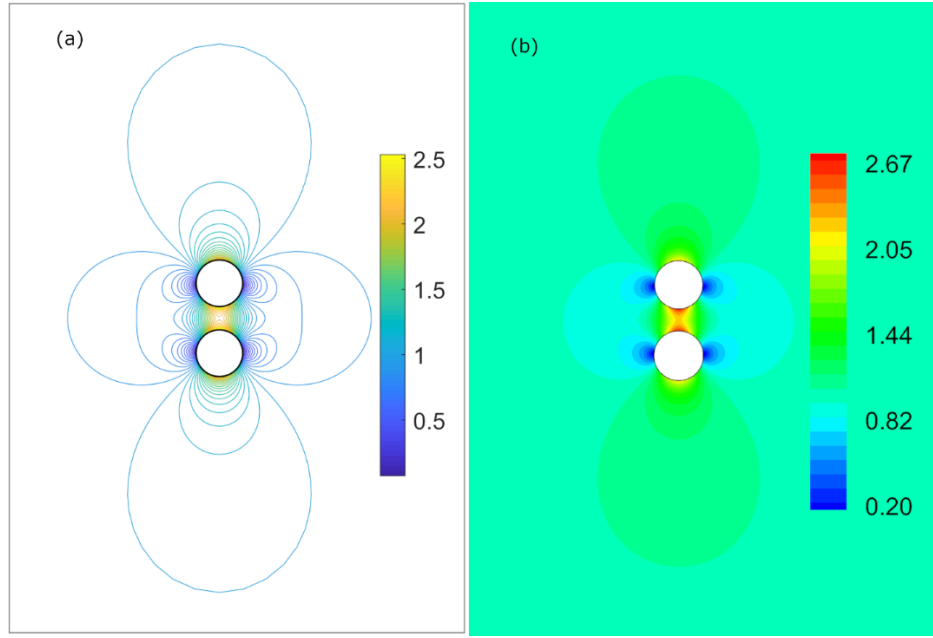


Figure 4.23 Velocity contours around two circular cylinders of diameter ratio $D/d = 1$ with center distance $H = 1.5d$ in uniform potential flow normal to the line connecting the centers of cylinders using (a) present method and (b) ANSYS.

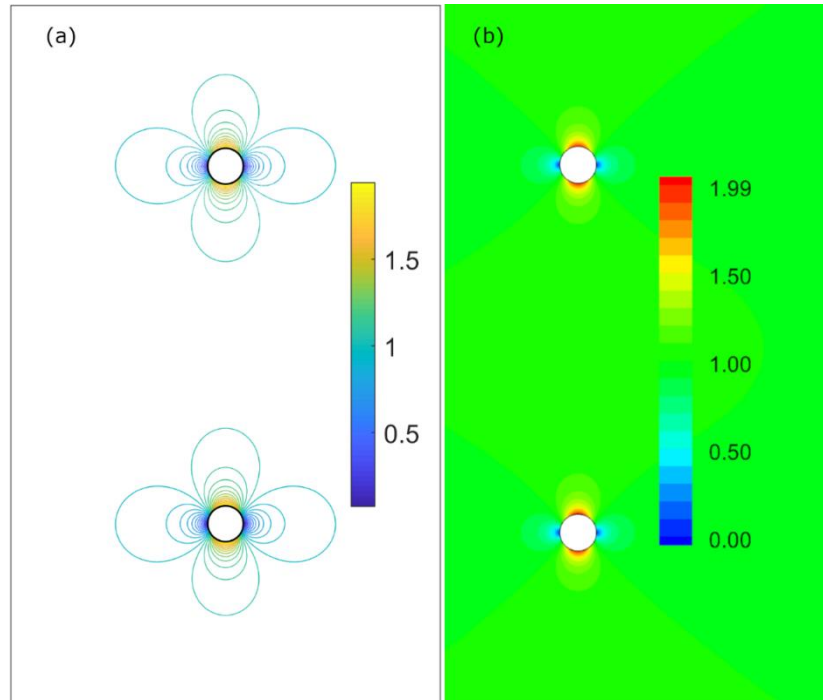


Figure 4.24 Velocity contours around two circular cylinders of diameter ratio $D/d = 1$ with center distance $H = 10d$ in uniform potential flow normal to the line connecting the centers of cylinders using (a) present method and (b) ANSYS.

For two interacting circular cylinders in uniform potential flow, the dimensionless pressure (P^*) is obtained as,

$$P^* = \frac{1}{2}(Cp(\theta) - Cp(0)), \quad (4.1)$$

where, $Cp(\theta)$ and $Cp(0)$ are pressure coefficient at angular location θ^0 and 0^0 respectively on the cylinder surface. The numerical values of normalized pressure (P^*) on the surface of circular cylinder interacting with another circular cylinder of same diameter for the center distances $H/d = 1.25, 1.5, 1.75, 2, 3$ and ∞ , and flow angle $\alpha = \pi/2$ (flow direction normal to the line connecting the centers of cylinders) are obtained. The comparison of these results with the results obtained from ANSYS and that available from literature [77] is shown in Figure 4.25. It is observed that the minimum dimensionless pressure at $\theta = 180^0$ on the cylinder surface decreases with decrease of center distance (H/d). The results are found to be in close confirmation with results from ANSYS and available from literature [77].

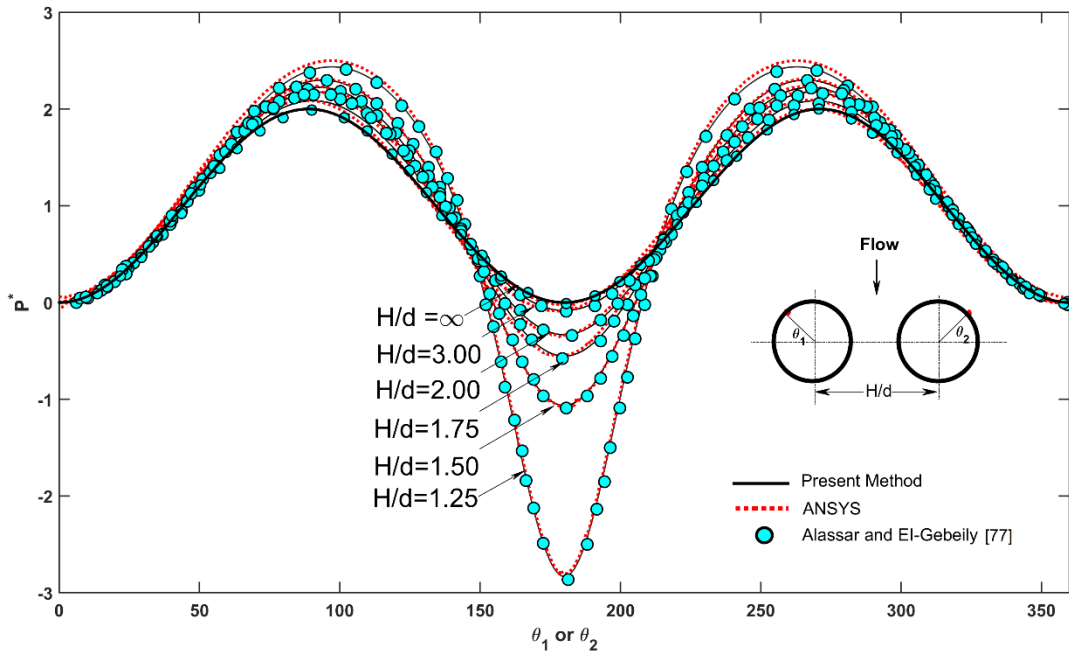


Figure 4.25 The dimensionless pressure P^* on the surface of two equal circular cylinders evaluated for the flow direction normal to the line connecting the centres of cylinders and for different center distances $H/d = 1.25, 1.5, 1.75, 2.0, 3.0$ and ∞ .

Table 4.5 The comparison of results of dimensionless pressure P^* obtained using present method with the results obtained through ANSYS and results from existing literature on the surface of two equal diameter circular cylinders at $\theta_1 = \theta_2 = 180^\circ$ for center distance $H/d = 1.25, 1.5, 1.75, 2.0, 3.0$ and ∞ , and the flow direction normal to the line connecting the centres of cylinders.

Distance between the centers of cylinders (H)	Dimensionless Pressure $P^* = \frac{1}{2}(Cp(\theta) - Cp(0))$		
	Present method	ANSYS	Alassar and EI-Gebeily [77]
$H = 1.25$	-2.832	-2.796	-2.834
$H = 1.5$	-1.065	-1.070	-1.067
$H = 1.75$	-0.553	-0.554	-0.545
$H = 2.0$	-0.333	-0.334	-0.343
$H = 3.0$	-0.083	-0.083	-0.083
$H = \infty$	0.00	-0.018	0.00

4.4.2.1 Effect of center distance

The velocity and pressure profile on the surface of circular Cylinder-1 ($d = 2$) interacting with circular Cylinder-2 in tandem arrangement are presented in Figure 4.25 for the diameter ratio $D/d = 1$ and center-to-center distance $H/d = 1.5, 2, 2.5, 3$ and 100 . The maximum velocities $V_{max}/U = 1.8640, 1.9049, 1.9326, 1.9505$ and 2 , and minimum pressure co-efficient $Cp_{min} = -2.4768, -2.6288, -2.7350, -2.8046$ and -3 are observed on the surface of Cylinder-1 for the center distances $H/d = 1.5, 2.0, 2.5, 3.0$ and 100 respectively. It is observed that with the decrease of center distance the maximum velocity on the surface of cylinder decreases and minimum pressure co-efficient increases due to obstruction of flow between cylinders. The velocity and pressure profile around two equal circular cylinders in side-by-side arrangement interacting at center distances $H/d = 1.5, 2, 2.5, 3$ and 100 in uniform potential flow are shown in Figure 4.26. It is found that, the velocity between the cylinders increases and the pressure decreases with decrease of center distances due to Venturi effect that flow experiences while passing through the space between the cylinders.

The variation of extremum values of velocity and pressure co-efficient on the surface of two interacting circular cylinders of diameter ratio $1 \leq D/d \leq 6$ for increasing center-to-center distance $2 < H/d < 25$ are shown in Figure 4.28 and Figure 4.29. The strong hydrodynamic interaction between the cylinders is observed for $H/d < 10$. In tandem arrangement of cylinders the values of flow parameters are found lower than that of individual cylinders in free stream flow

and in side-by-side arrangement of cylinders the flow parameters gain extremum values higher than that for single cylinder. It is found that as center-to-center distance between the cylinders increases, the extremum values of flow parameters (velocity and pressure) increases in tandem arrangement and decreases in side-by-side arrangement. For large center distance ($H/d \geq 10$), the interaction effect dies out and flow parameters around the cylinders behaves similar to that around single cylinder in uniform flow ($V_{max}/U = 2$, and $C_p = -3$). It is also to be noted that the hydrodynamic interactions not only get affected by the vicinity but also by the diameter ratio of cylinders.

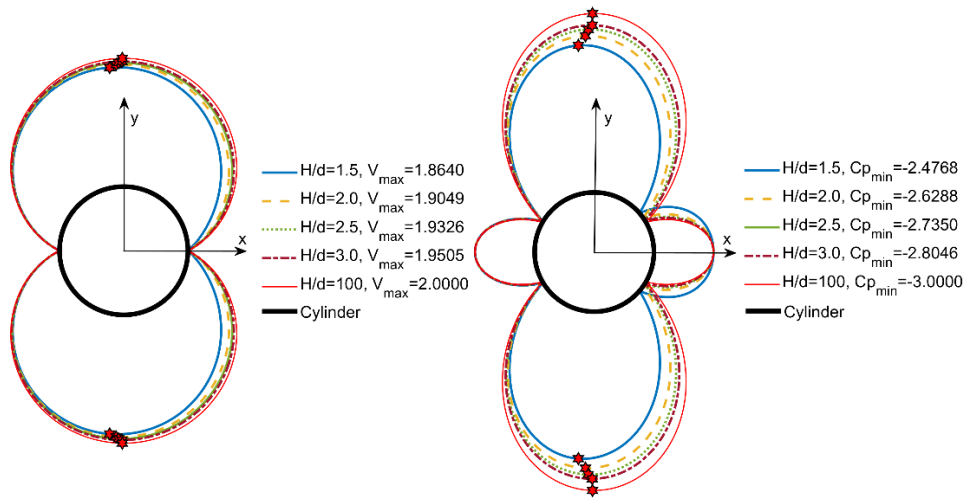


Figure 4.26 Velocity (left) and pressure (right) profile on the surface of circular Cylinder 1 ($d=2$) interacting with circular Cylinder 2 (D) for different center-to-center distances (H/d) in tandem configuration.

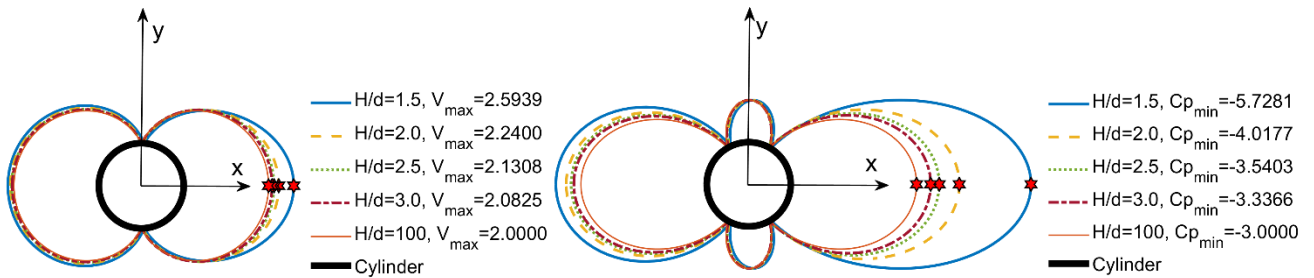


Figure 4.27 Velocity (left) and pressure (right) profile on the surface of circular Cylinder 1 ($d=2$) interacting with circular Cylinder 2 (D) for different center-to-center distances (H/d) in side-by-side configuration.

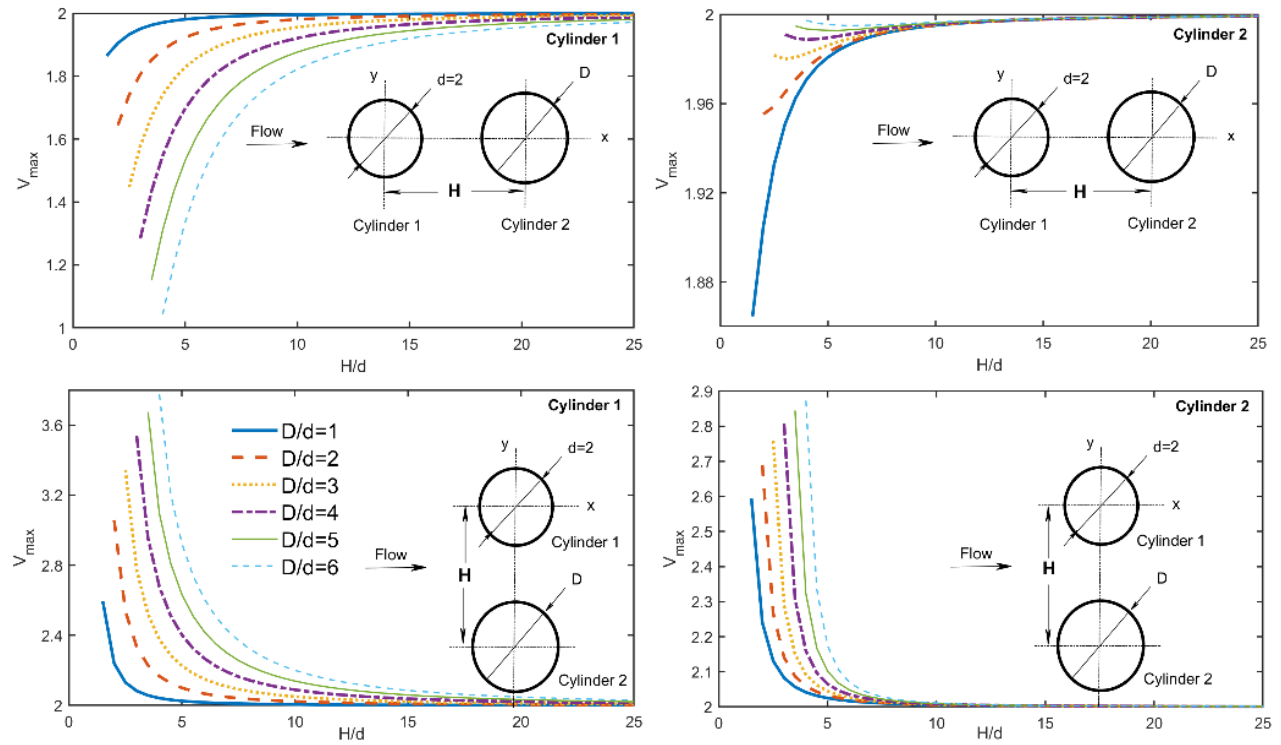


Figure 4.28 Maximum velocity on the surface of circular cylinder at different diameter ratio, for uniform potential flow along the center line and normal to the center line.

Table 4.6 Maximum Velocity (v_{max}/U) and minimum pressure co-efficient (Cp_{min}) on the surface of two equal interacting circular cylinders ($D/d = 1$) in tandem and side-by-side arrangement in uniform potential flow for different center-to-center distance $H/d = 1.5, 2.0, 2.5, 3.0$ and 100 .

H/d	Cylinder-1						Cylinder-2					
	Tandem Arrangement			Side by Side			Tandem Arrangement			Side by Side		
	V_{max}/U	Cp_{min}	θ_1	V_{max}/U	Cp_{min}	θ_1	V_{max}/U	Cp_{min}	θ_2	V_{max}/U	Cp_{min}	θ_2
1.5	1.8646	-2.4768	$\pm 85.62^0$	2.5939	-5.7281	180^0	1.8646	-2.4768	$\pm 85.62^0$	2.5939	-5.7281	180^0
2.0	1.9049	-2.6288	$\pm 87.67^0$	2.2400	-4.0177	180^0	1.9049	-2.6288	$\pm 87.67^0$	2.2400	-4.0177	180^0
2.5	1.9326	-2.7350	$\pm 88.44^0$	2.1308	-3.5403	180^0	1.9326	-2.7350	$\pm 88.44^0$	2.1308	-3.5403	180^0
3.0	1.9505	-2.8046	$\pm 89.50^0$	2.0825	-3.3366	180^0	1.9505	-2.8046	$\pm 89.50^0$	2.0825	-3.3366	180^0
100	2.0000	-3.0000	$\pm 90.00^0$	2.0000	-3.0000	180^0	2.0000	-3.0000	$\pm 90.00^0$	2.0000	-3.0000	180^0

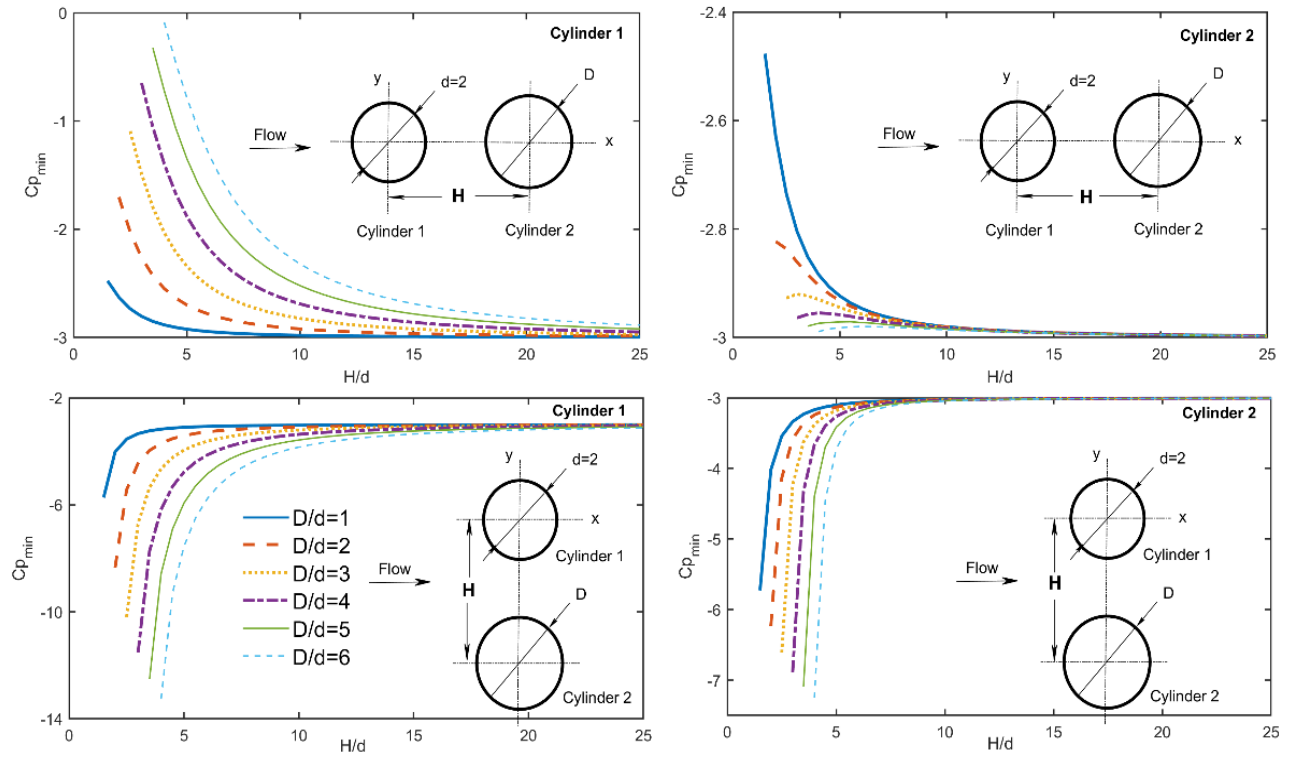


Figure 4.29 Minimum pressure co-efficient on the surface of circular cylinder at different diameter ratio, for uniform potential flow along the center line and normal to the center line.

4.4.2.2 Effect of diameter ratio

The velocity and pressure contours around the pair of circular cylinders of diameter ratio $D/d = 2$ interacting at center distance $H = 2.5d$ in tandem and side by side configuration are shown in Figure 4.30 and Figure 4.31 respectively plotted using present method and ANSYS. For center distance $H = 2.5d$, the values of maximum velocities obtained using present method in both tandem and side by side-by-side arrangement are $V_{max}/U = 1.959$ and 2.536 respectively, and values of minimum pressure co-efficients are $Cp_{min} = -2.837$ and -5.43 respectively, which are in close confirmation to the maximum velocities ($V_{max}/U = 1.949$ and 2.510), and minimum pressure co-efficient ($Cp_{min} = -2.827$ and -5.295) obtained using ANSYS for same geometrical configuration.

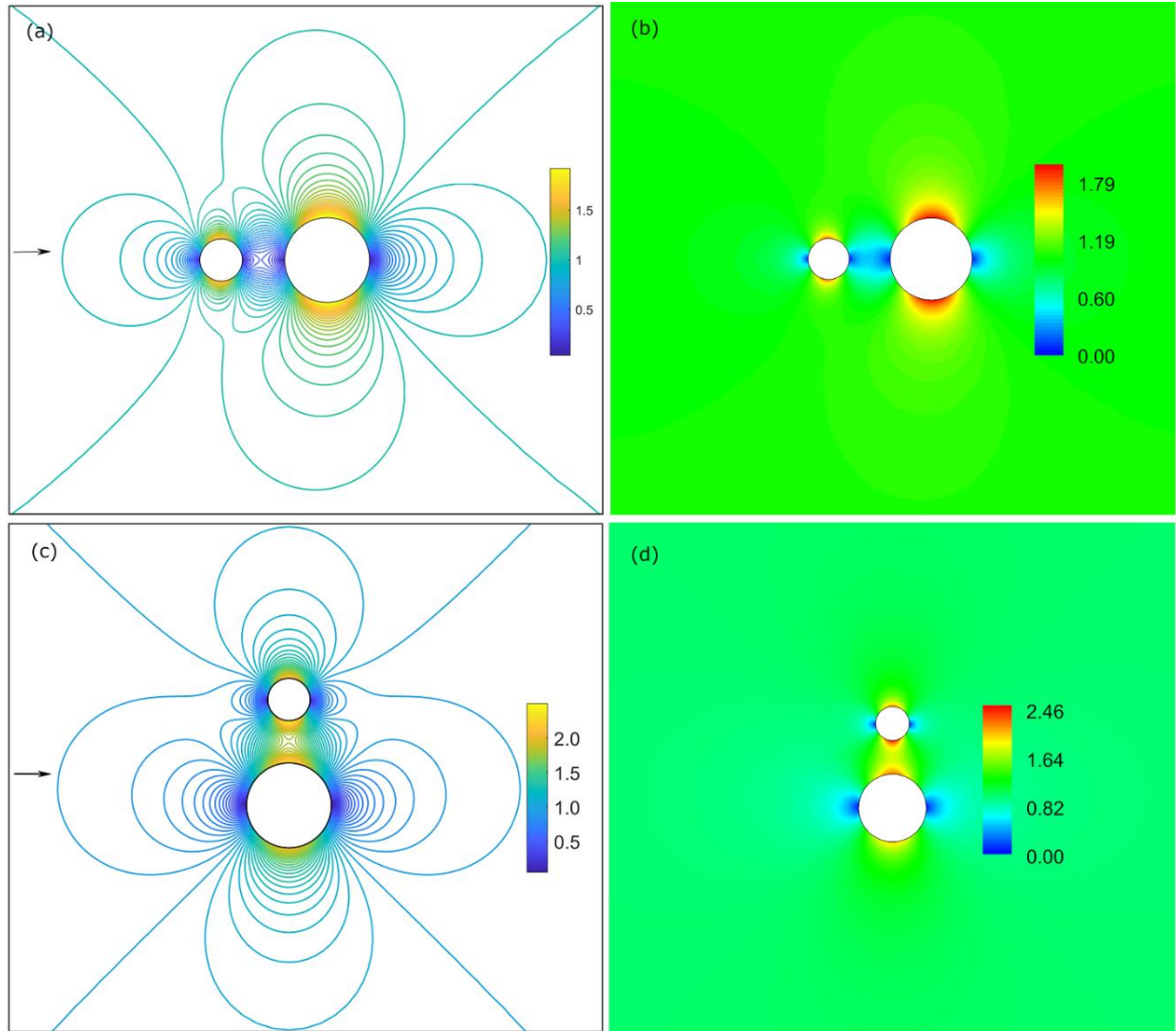


Figure 4.30 Velocity contours around two circular cylinders of diameter ratio $D/d = 2$ with center distance $H = 2.5d$ in uniform potential flow using present method (a) Tandem (c) side-by-side and using ANSYS (b) Tandem (d) Side-by-side.

For the tandem configuration ($\alpha = 0^\circ$), the dimensionless pressure on the surface of fixed diameter circular cylinder ($d = 2$) interacting with the cylinder of variable diameter ($D = 1d, 2d, 3d$ and $4d$) is obtained and compared with the results from Alassar and Ei-Gebeily ([77]) and obtained using ANSYS as shown in Figure 4.32. The maximum value of dimensionless pressure decreases with increase of diameter ratio (D/d). The results obtained using present method closely agree with the results from available literature [77] and ANSYS.

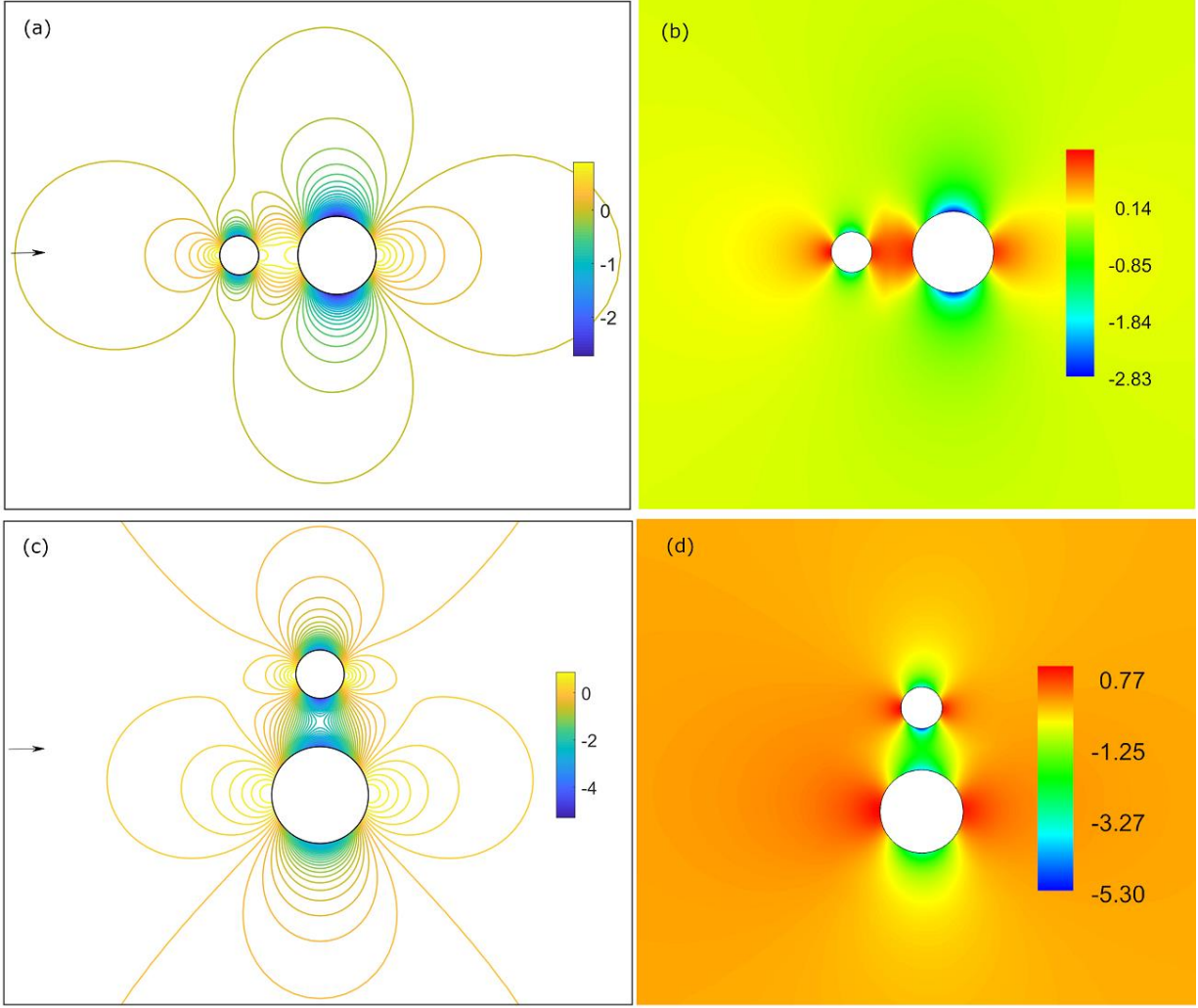


Figure 4.31 Pressure co-efficient contours around two circular cylinders of diameter ratio $D/d = 2$ with center distance $H = 2.5d$ in uniform potential using present method (a) Tandem (c) side-by-side and using ANSYS (b) Tandem (d) Side-by-side.

The velocity and pressure profile on the surface of fixed diameter ($d = 2$) Cylinder-1 interacting with variable diameter (D) Cylinder-2 at distance $H - ((D + d)/2) = 1$ for flow angle $\alpha = 0^\circ$ and 90° , and diameter ratio ranging between $1 \leq D/d \leq 5$ are shown in Figure 4.33 and Figure 4.34 respectively. For $\alpha = 0^\circ$, with increase in diameter ratio, the maximum velocity ($V_{max}/U = 1.8646, 1.6439, 1.4464, 1.2839, 1.1512$ and 1.0421) decreases and the minimum pressure co-efficient ($Cp_{min} = -2.4768, -1.7023, -1.0922, -0.6484, -0.3252$ and -0.0859) increases on the surface of Cylinder-1 due to blocking of flow between cylinders. For $\alpha = 90^\circ$, the maximum velocity increases and minimum pressure decreases on the surface of Cylinder-1 due to venturi

effect. However, on the surface of larger diameter cylinder (Cylinder-2) the extremum values of flow parameters increase with increase of diameter ratio in both tandem ($\alpha = 0^\circ$) and side-by-side ($\alpha = 90^\circ$) configurations (refer Figure 4.35).

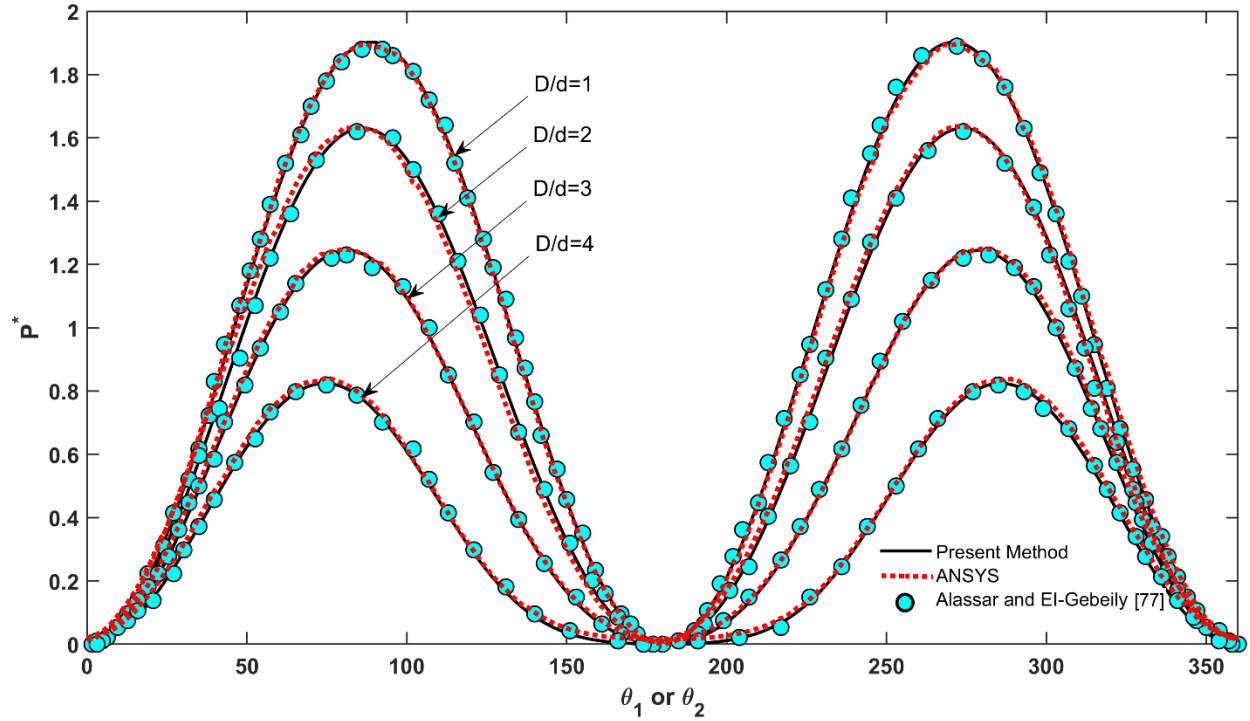


Figure 4.32 The dimensionless pressure P^* on the surface of two circular cylinders of diameter ratio $D/d = 1, 2, 3$ and 4 evaluated for the flow direction along the line connecting the centres of cylinders and for center distances $H/d = 3.0$.

The location of extremum values of flow parameters (shown by markers in Figure 4.33 and Figure 4.34) on the surface of cylinder changes with diameter ratio in tandem configuration, while in side-by-side configuration the location of extremum values of flow parameters ($\theta_1 = \theta_2 = 180^\circ$) remains unaltered. The extremum values of flow parameters and their locations on the cylinders surface for different diameter ratio are presented in Table 4.7.

From the results of effects of center distance and diameter ratio on flow parameters, it is observed that the significant hydrodynamic interaction effects are experienced by smaller cylinder with increase of diameter ratio and decrease of center distance. However, for given center distance and flow angle the interaction effects weaken on larger cylinder with increase of diameter ratio.

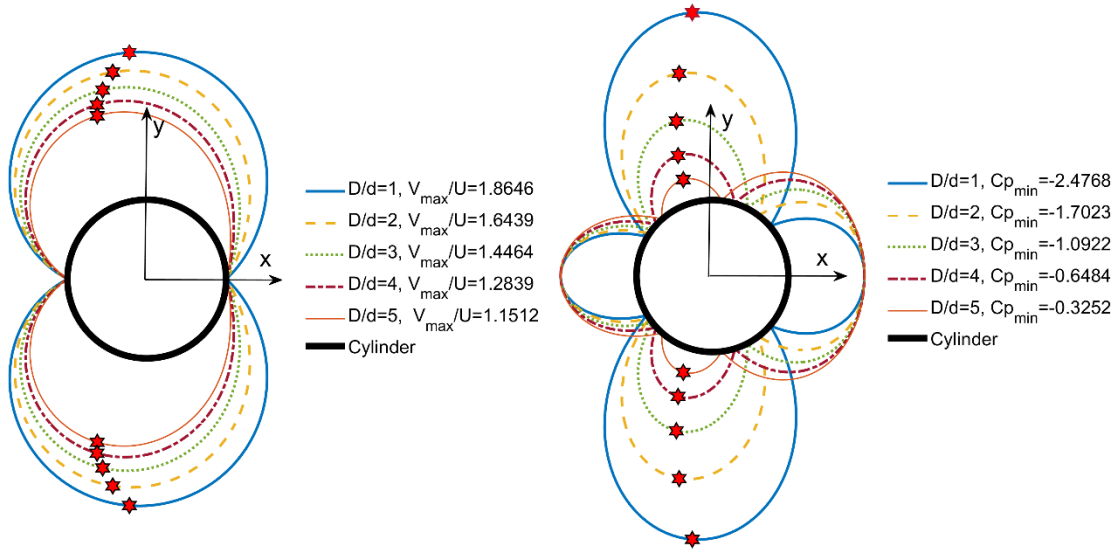


Figure 4.33 Velocity (left) and pressure (right) profile on the surface of circular Cylinder 1 (d=2) interacting with circular Cylinder 2 (D) for different diameter ratio.

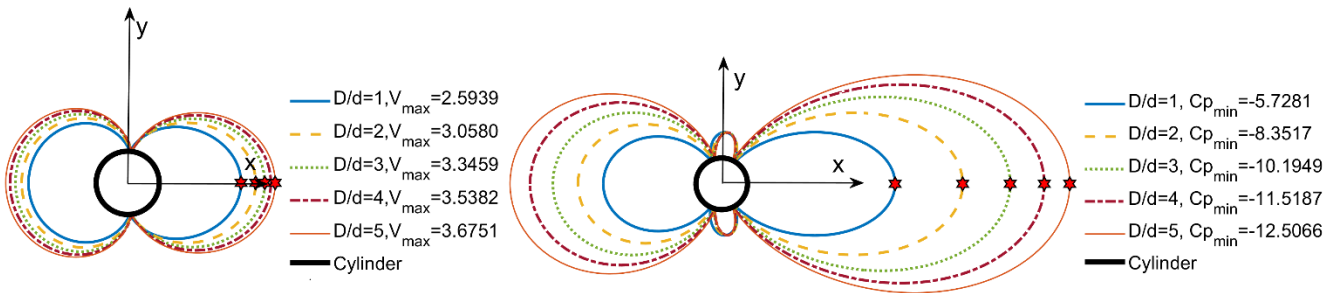


Figure 4.34 Velocity (left) and pressure (right) profile on the surface of circular Cylinder 1 (d=2) interacting with circular Cylinder 2 (D) for different diameter ratio.

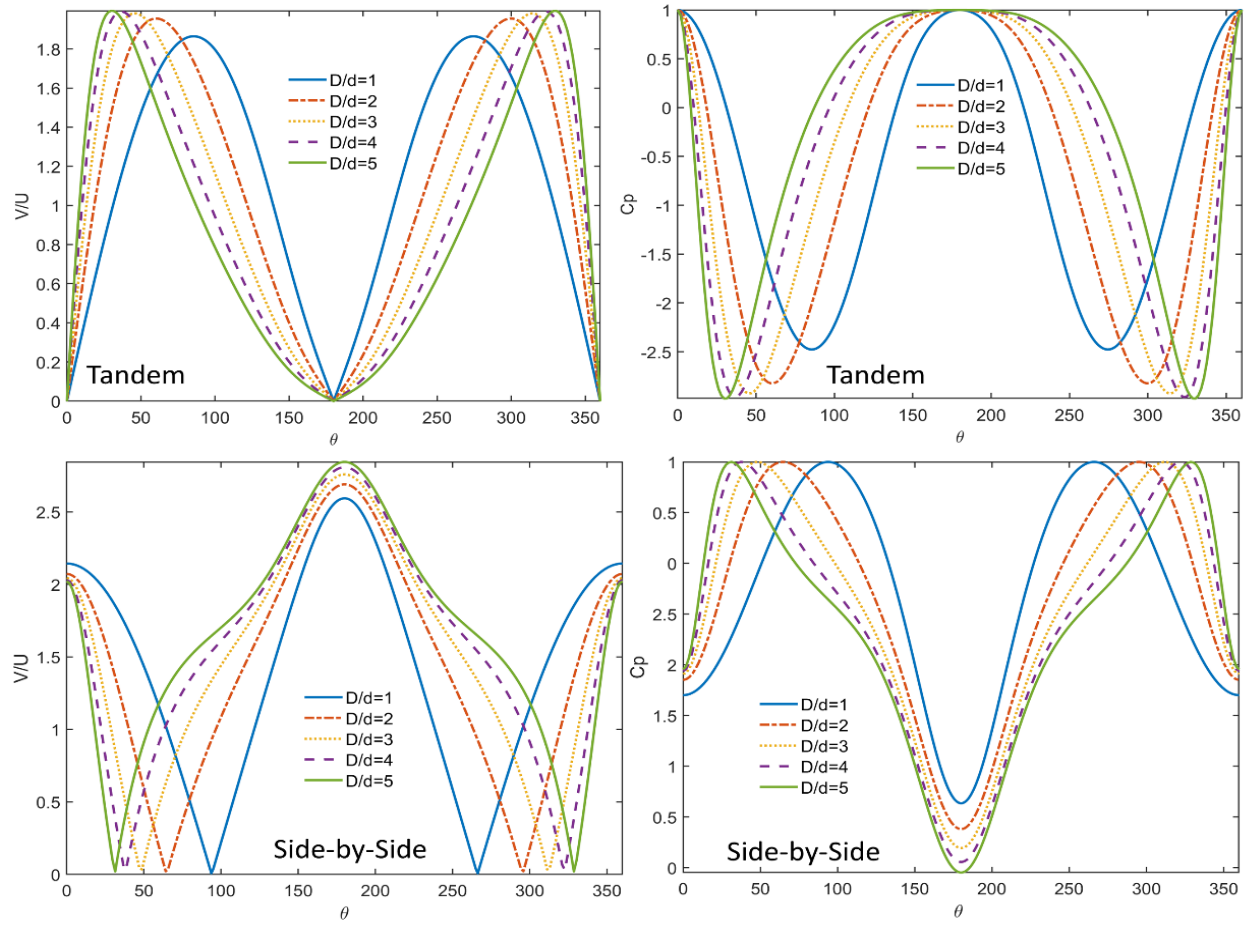


Figure 4.35 Velocity and pressure distribution on the surface of circular Cylinder-2 of variable diameter ($1 \leq D/d \leq 5$) interacting with Cylinder-1 of fixed diameter $d = 2$ at distance $H - ((D + d)/2) = 1$ in tandem and side-by-side arrangement in uniform potential.

Table 4.7 Maximum Velocity (V_{max}/U) and minimum pressure co-efficient (Cp_{min}) on the surface of two interacting circular cylinder in tandem and side-by-side arrangement in uniform potential flow for different diameter ratio $1 \leq D/d \leq 6$.

D/d	Smaller Cylinder (Cylinder-1)						Larger cylinder (Cylinder-2)					
	Tandem Arrangement			Side by Side			Tandem Arrangement			Side by Side		
	V_{max}/U	Cp_{min}	θ_1	V_{max}/U	Cp_{min}	θ_1	V_{max}/U	Cp_{min}	θ_2	V_{max}/U	Cp_{min}	θ_2
1	1.8646	-2.4768	$\pm 85.62^\circ$	2.5939	-5.7281	180°	1.8646	-2.4768	$\pm 85.61^\circ$	2.5939	-5.7281	180°
2	1.6439	-1.7023	$\pm 80.65^\circ$	3.0580	-8.3517	180°	1.9552	-2.8229	$\pm 60.88^\circ$	2.6902	-6.2373	180°
3	1.4464	-1.0922	$\pm 76.86^\circ$	3.3459	-10.1949	180°	1.9817	-2.9272	$\pm 45.32^\circ$	2.7587	-6.6105	180°
4	1.2839	-0.6484	$\pm 74.05^\circ$	3.5382	-11.5187	180°	1.9909	-2.9638	$\pm 37.31^\circ$	2.808	-6.885	180°
5	1.1512	-0.3252	$\pm 73.13^\circ$	3.6751	-12.5066	180°	1.9947	-2.9790	$\pm 29.64^\circ$	2.8448	-7.0931	180°

4.4.2.3 Effect of Flow angle

For two circular cylinders located at infinite distance away in uniform potential flow, the flow parameters (velocity and pressure) attain extremum values ($V_{max}/U = 2$ and $Cp_{min} = -3$) on the surface of cylinder at two distinct locations ($\theta = \pm 90^\circ - \alpha$) for all values of flow angle (α). For finite distance between two circular cylinders, the change in flow angle leads significant changes in magnitude and location of flow parameters. The velocity and pressure profiles on the surface of circular Cylinder-1 ($d = 2$) interacting with equal diameter Cylinder-2 ($D = 2$) at distance $H - ((D + d)/2) = 1$ for different flow angles $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° are shown in Figure 4.36. For flow angle $\alpha = 0^\circ$, the velocity and pressure attain extremum values $V_{max}/U = 1.8646$ and $Cp_{min} = -2.4767$ respectively at two points ($\theta_1 = \pm 85.6123^\circ$) on the surface of Cylinder-1. However, for flow angle $\alpha = 30^\circ, 45^\circ, 60^\circ$ and 90° , the velocity attains maximum values ($V_{max}/U = 2.0139, 2.0708, 2.2836$ and 2.5939) and pressure co-efficient attains minimum values ($Cp_{min} = -3.0558, -3.2882, -4.2148$ and -5.7281) at single angular location $\theta_1 = 58.2806^\circ, 43.0366^\circ, 187.6475^\circ$ and 180° respectively).

The variation of extremum values of velocity and pressure co-efficient with respect to flow angle ($0^\circ \leq \alpha \leq 180^\circ$) for different diameter ratio ($1 \leq D/d \leq 4$) and distance between cylinders $H - ((D + d)/2) = 1$ are shown in Figure 4.37. For finite center distance and flow angle $\alpha = 0^\circ$, the magnitudes of flow parameters are found less than that for single cylinder due to blocking of flow between cylinders. As the flow angle increases ($0^\circ < \alpha < 90^\circ$), the blocking effect weakens and flow passes through converging-diverging types geometry formed between the gap through pair of cylinders and experiences Venturi effect. Hereafter, the maximum values of velocity on cylinder surface increases and minimum values of pressure co-efficient decreases with flow angle. At certain flow angle, the magnitude of flow parameters abruptly changes and attain extremum values for $\alpha = 90^\circ$ (side-by-side configuration). For $90^\circ < \alpha < 180^\circ$, the values of flow parameters further start decreasing due to blocking of flow between cylinders.

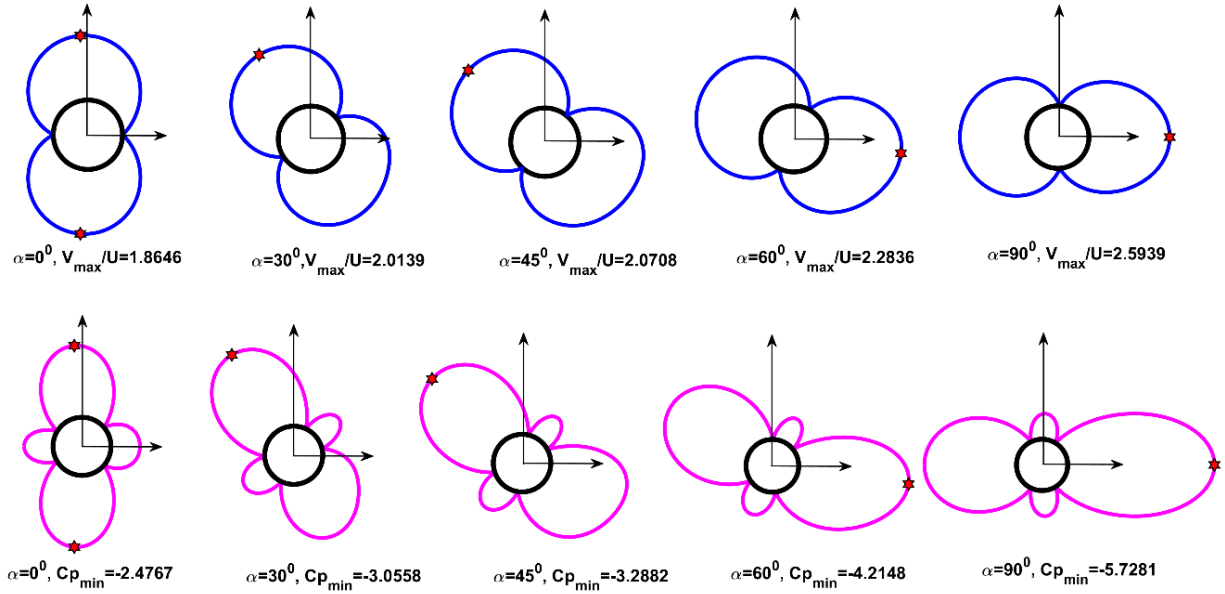


Figure 4.36 Velocity and pressure profile on the surface of circular Cylinder-1 ($d = 2$) interacting with another Cylinder-2 ($D/d = 1$) at distance $H - ((D + d)/2) = 1$ in uniform potential flow for different flow angles.

4.4.3 Hydrodynamic Forces on Cylinders

4.4.3.1 Effect of Center Distance

As discussed above, the velocity of flow between two-cylinders decreases for the angle of attack $\alpha = 0^\circ$ (flow along the line connecting the centers of cylinders), which in turn increases the pressure between the cylinders. The increased pressure between two-cylinders results in repelling force between them along the flow direction. Similarly, for the angle of attack $\alpha = 90^\circ$ (flow normal to the line connecting the centers of cylinders), the velocity of flow between the cylinders increases and the magnitude of pressure decreases leading to the attracting forces normal to the flow direction. The force co-efficients acting on two equal circular cylinders in tandem and side by side configurations are shown in Figure 4.38.

In tandem configuration, the forces of equal magnitude acts on Cylinder-1 and Cylinder-2 in $-x$ and $+x$ axis directions respectively while in side-by-side configuration, the forces of equal magnitude acts in $-y$ and $+y$ axis directions. The magnitude of forces in side-by-side configuration are significantly larger because of larger velocity and pressure drop. The magnitude of the forces decreases with increase of center distance H/D and approaches to zero at $H \rightarrow \infty$. Figure 4.39 shows the force co-efficients acting on circular Cylinder 1 in side by side configuration

of equal circular cylinders. The force co-efficients estimated using present method for various center distances are in close agreement with the results presented by Dalton and Helfinstine ([90]) and those obtained using ANSYS.

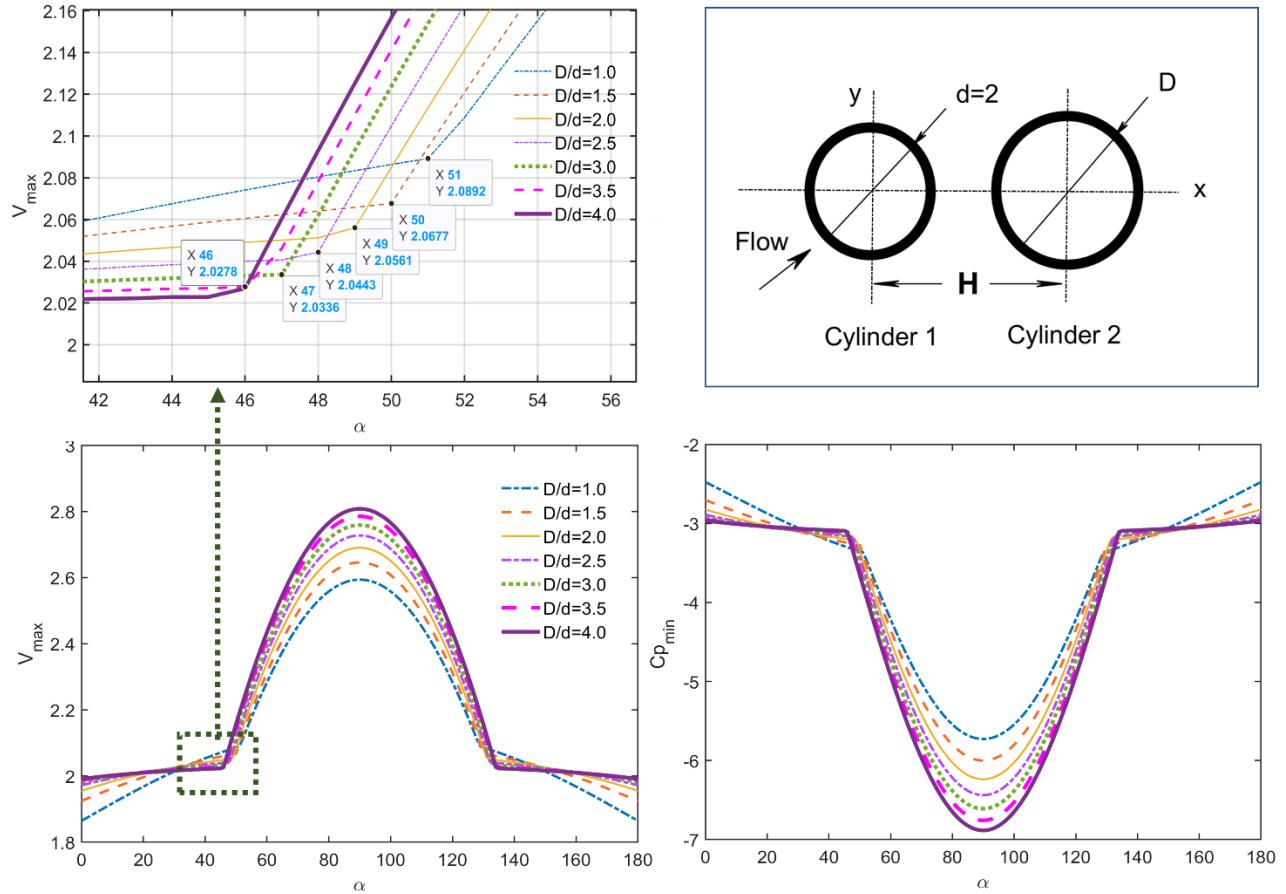


Figure 4.37 Extremum values of velocity (left) and pressure co-efficient (right) around pair of circular cylinders in uniform potential flow for flow angle $0^\circ \leq \alpha \leq 180^\circ$ and distance between the cylinder $H - ((D + d)/2) = 1$.

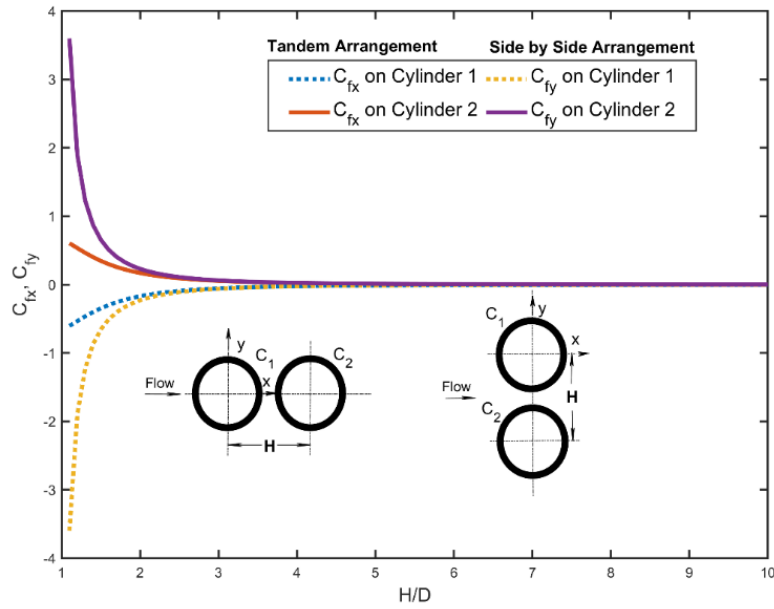


Figure 4.38 Force co-efficient C_{fx} and C_{fy} acting on the circular cylinders of equal diameters in tandem and side-by-side arrangement for center distance H/D . The flow direction is considered along the line joining the centers of the cylinders in tandem arrangement and normal to the center line in side by side arrangement.

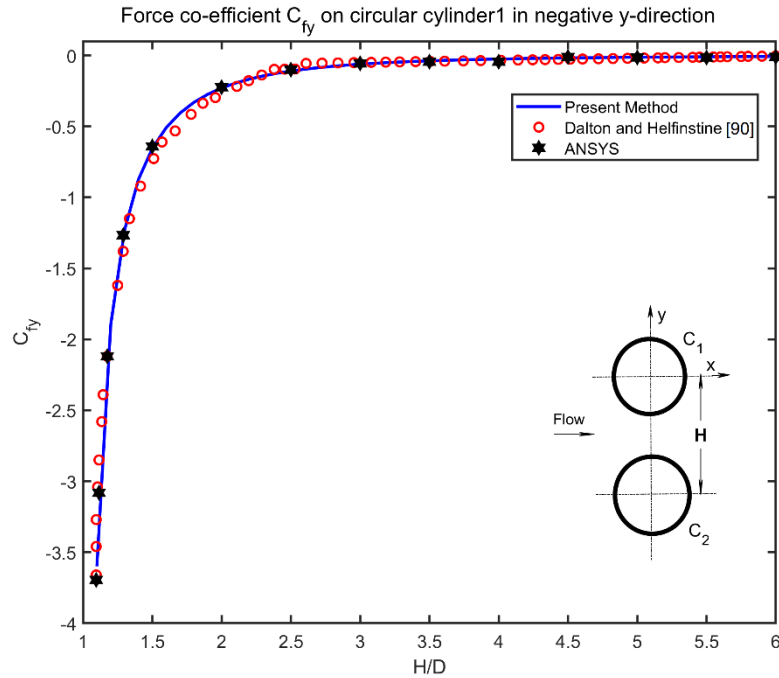


Figure 4.39 Comparison of results of force co-efficients acting on Cylinder-1 in side by side configuration of two equal circular cylinders with the results available from literature and those obtained using ANSYS for flow direction normal to the line connecting the centers of cylinders.

4.4.3.2 Effect of diameter ratio

The hydrodynamic force acting on each cylinder in tandem and side by side arrangement is shown in Figure 4.40 for center distance ($0 \leq H/d \leq 10$) and diameter ratio ($D/d = 0.5, 1.0$ and 2.0). In tandem arrangement of cylinder, the repelling force between the cylinders decreases with increase of center distance and vanished at center distance $H > 10d$ and each cylinder behaves as an individual cylinder. Also, the effect of diameter ratio on such forces is shown, the effect of diameter ratio is found significant at close proximity of cylinders ($H \leq 5d$). In side by side configuration of cylinders, the forces of attractive nature are found acting between the cylinders and the magnitude of these forces are found significantly larger than in tandem arrangement. The maximum value of hydrodynamic forces acting on smaller cylinder in both tandem and side-by-side configuration increases in magnitude with increase of diameter ratio (D/d). The maximum values of hydrodynamic forces acting on the Cylinder-2 of variable diameter (D) decreases in magnitude with increase of diameter ratio (D/d).

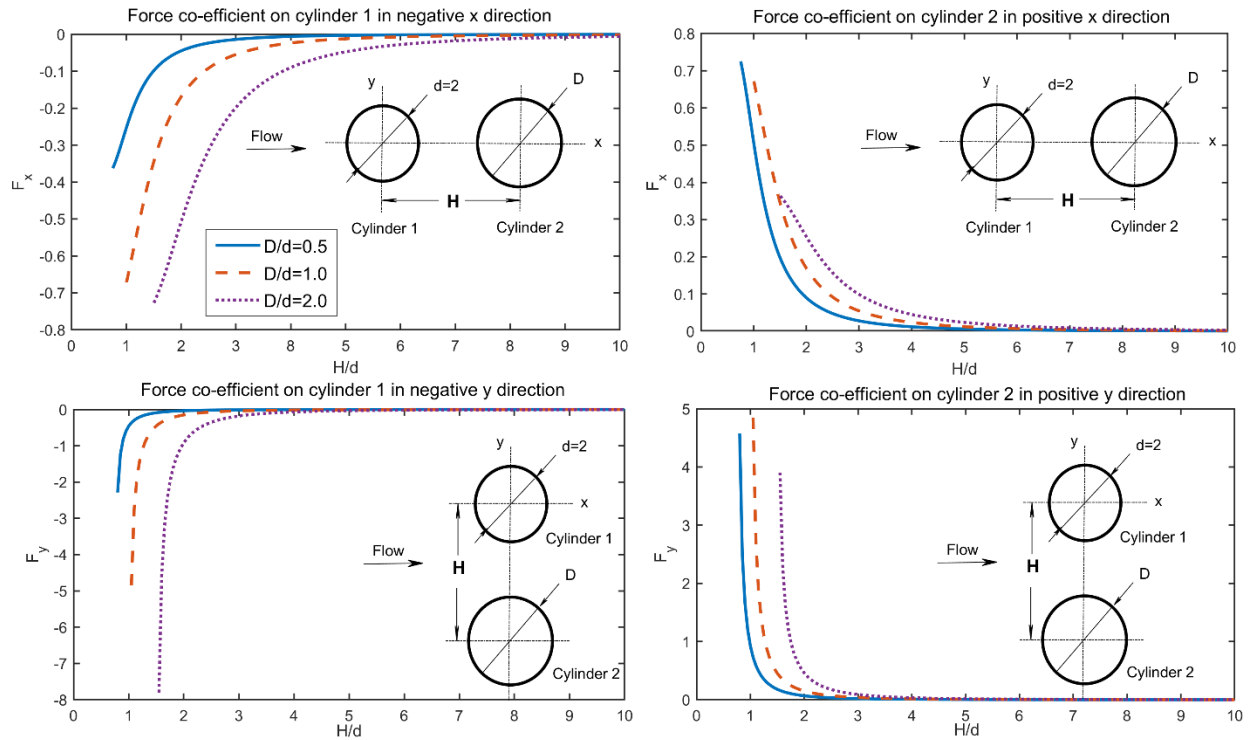


Figure 4.40 Hydrodynamic force co-efficient on the circular cylinder at different diameter ratio, for uniform potential flow along the center line and normal to the center line.

4.4.3.3 Effect of flow angle

To show the effect of angle of attack on hydrodynamic forces, the hydrodynamic forces acting along the line connecting the centers of cylinders are investigated against the angle of attack. The angle of attack is measured with respect to the line connecting the centers of cylinders and varied from $0^\circ \leq \alpha \leq 180^\circ$. The magnitude of forces increases with angle of attack α and become maximum at $\alpha = 90^\circ$. It is also observed from the sign of force co-efficient that the repulsive nature of forces between the cylinders at angle of attack $\alpha = 0^\circ$ and 180° changes to the attractive type in nature at certain flow angle depending on diameter ratio (refer Figure 4.41).

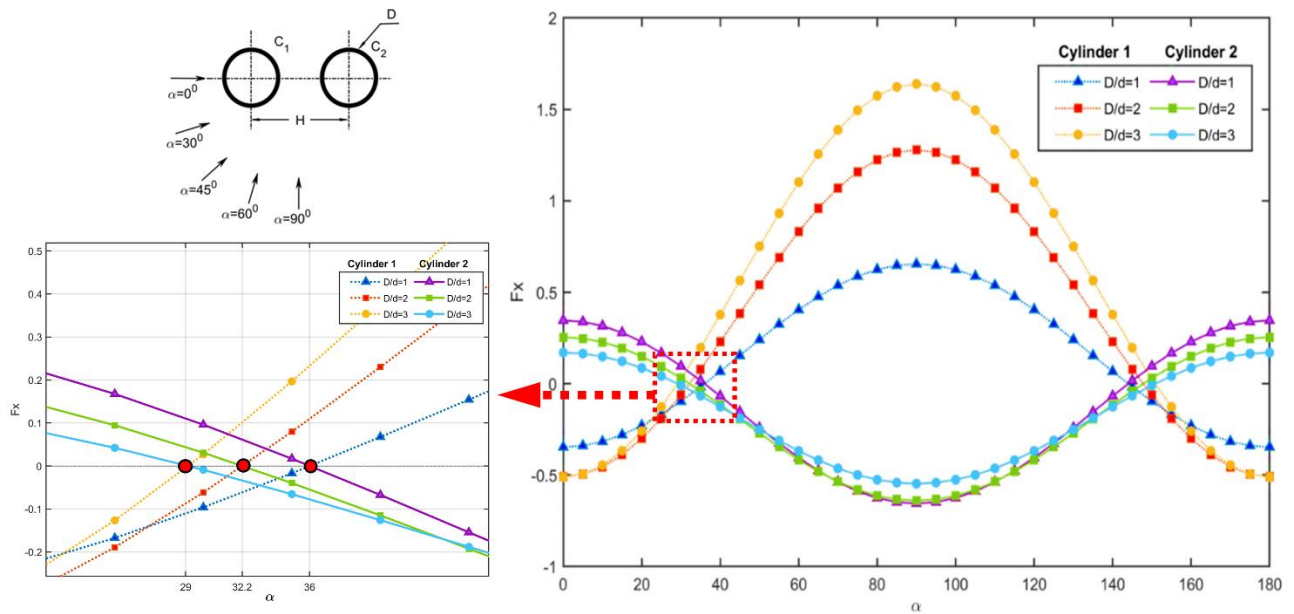


Figure 4.41 Hydrodynamic force acting on cylinder along the center line for angle of attack $0^\circ \leq \alpha \leq 180^\circ$ at diameter ratio $D/d = 1, 2$ and 3 .

4.5 Hydrodynamic Interaction between Circular and Polygonal Cylinder

The effect of various geometrical and flow parameters on the hydrodynamic interaction between polygonal and circular cylinder in uniform potential flow is discussed in this section. The center distance between polygonal and circular cylinder (H), angle of attack (α), fluid density (ρ), free stream velocity (U) and, number of sides (N), corner radius (ρ, h), and orientation (γ) of polygonal cylinder are various independent parameters that influence the hydrodynamic interaction.

Some of the results obtained using present method are compared with the results obtained through ANSYS for validation purpose. The geometry of two-dimensional cylinders in ANSYS is generated using key-points extracted from the MATLAB program of present method and the two-dimensional rectangular fluid domain is generated around the pair of polygonal-circular cylinders. The geometry, meshing and boundary conditions applied on the fluid domain in ANSYS is shown in Figure 4.42. The structured mesh grid containing quadrilateral elements is used and the fine mesh grid is generated near the cylinder by blocking the domain around the geometry and sizing the elements at the edges of cylinders.

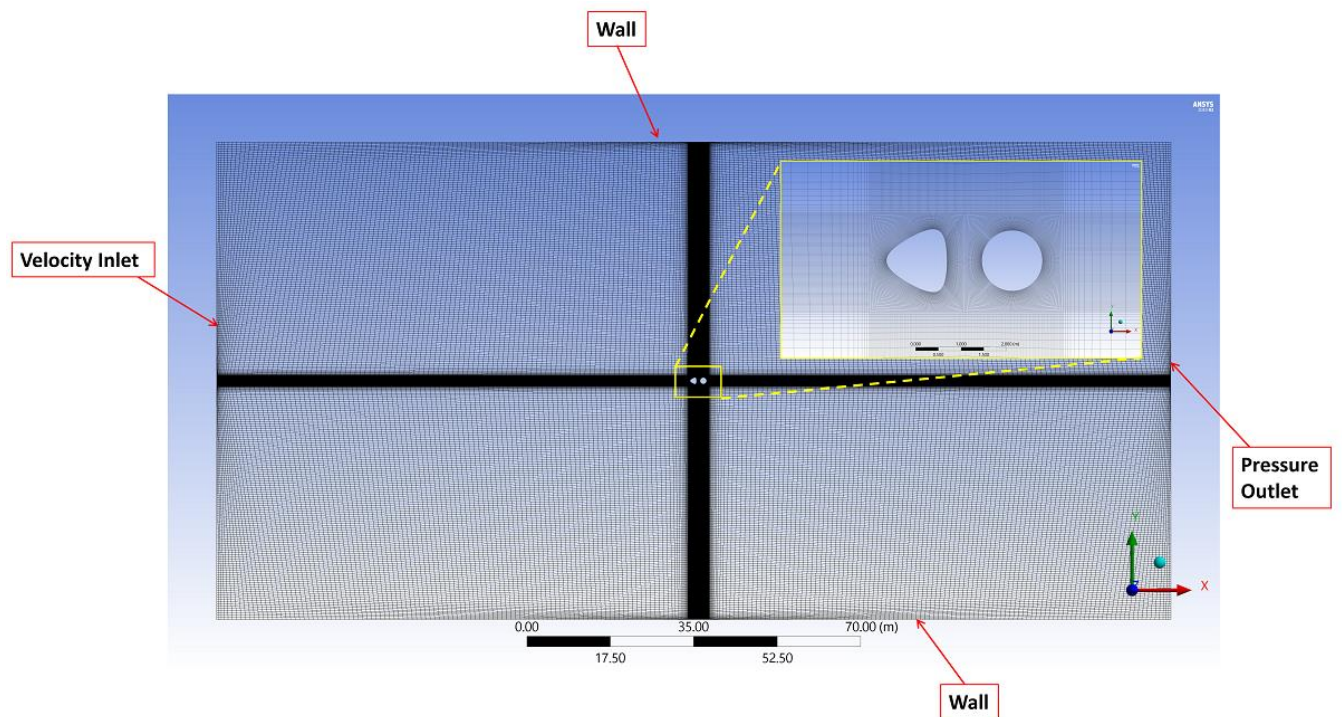


Figure 4.42 Computational domain for the flow around polygonal-circular cylinder pairs in uniform potential flow.

4.5.1 Velocity and Pressure Distribution

The velocity and pressure are important parameters to be studied around the polygonal and circular cylinders to investigate the hydrodynamic interaction between them. The velocity and pressure field around the pair of N -sided polygonal cylinder and unit radius circular cylinder are obtained for various center distances and geometrical orientations.

The velocity contours around circular-circular, triangular-circular, square-circular, and 45° rotated square-circular cylinder pairs for $H = 1.5D$ and $H = 10D$ for tandem configuration obtained using present method and ANSYS are shown in Figure 4.43 to Figure 4.47. The contour shows the velocity variation around the pairs of polygonal and circular cylinders. The maximum velocities found using the present method for center distance $H = 1.5D$ are $V_{max}/U = 1.864$ (circular-circular), 2.225 (triangular-circular), 2.324 (square-circular), and 3.096 (45° rotated square-circular) as shown in Figure 4.43. These results are in close confirmation to the maximum velocities obtained using ANSYS for the same flow and geometrical conditions ($V_{max}/U = 1.863$ (Figure 4.44 (a)), 2.207 (Figure 4.44 (b)), 2.262 (Figure 4.44 (c)) and 3.012 (Figure 4.44 (d))). For $H = 10D$, using present method and ANSYS the maximum velocities are found as $V_{max}/U = 1.995$ and 1.980 ($N = \infty$), 2.529 and 2.4354 ($N = 3, \gamma = 180^\circ$), 2.356 and 2.2853 ($N = 4, \gamma = 45^\circ$) and, 3.323 and 3.183 ($N = 4, \gamma = 0^\circ$) respectively.

For the center distance $H = 1.5D$, the flow pattern around one cylinder affects the flow pattern around other cylinders significantly and it diminishes as cylinders go apart (Refer Figure 4.45 and Figure 4.46). It is also to be noted that the hydrodynamic interactions not only get affected by the vicinity but also by the shape of the cylinder, orientation with respect to flow and corner radii of cylinders.

The velocity and pressure contours, for side-by-side configuration, for $H = 1.5D$ using present method are shown in Figure 4.47 and Figure 4.48. Due to Venturi effect, the magnitude of maximum velocity and pressure drop is found significantly larger in side by side configuration as compare to tandem configuration. The area between two cylinders forms geometry similar to convergent-divergent nozzle which decreases pressure and increases velocity of flow passing through a gap between the cylinders.

The maximum velocities (V_{max}/U) and minimum pressure co-efficient (Cp_{min}) for side by side configurations for $H = 1.5D$ and $H = 10D$, and the results for other configurations are presented in Table 1. The results obtained using present method are found in very good agreement with those obtained through ANSYS. The percentage difference between them is observed below 3% for almost all the cases except a few where in it falls below 5%.

Table 4.8 Maximum velocities V_{max}/U and minimum pressure co-efficient Cp_{min} around polygonal-circular cylinder pair in tandem and side-by-side configuration placed in uniform potential flow. The results obtained using present method are compared with results obtained using ANSYS. The polygonal shaped geometry in ANSYS is generated using key point extracted from MATLAB program (the polygonal geometries are traced for $h = 0.1$)

Two-cylinder Arrangement	V_{max}/U				Cp_{min}			
	$H/D = 1.5$		$H/D = 10$		$H/D = 1.5$		$H/D = 10$	
	Present Method	ANSYS	Present Method	ANSYS	Present Method	ANSYS	Present Method	ANSYS
  	1.864	1.8629	1.995	1.980	-2.4745	-2.4739	-2.9601	-2.9405
  	2.225	2.2074	2.529	2.4354	-3.95	-3.9311	-5.206	-5.0519
  	2.324	2.2616	2.356	2.2853	-4.403	-4.2924	-4.52	-4.5227
  	3.096	3.0122	3.323	3.183	-8.5852	-8.2481	-10.04	-9.5382
  	2.594	2.6728	2.000	1.9894	-5.7280	-6.1525	-2.952	-2.9769
  	2.9906	2.9859	2.499	2.4257	-8.216	-8.0986	-5.283	-5.1786
  	2.5990	2.5663	2.425	2.299	-5.7550	-5.7744	-4.638	-4.5259
  	4.2424	4.2217	3.2013	3.2059	-17.498	-17.267	-9.369	-9.6621

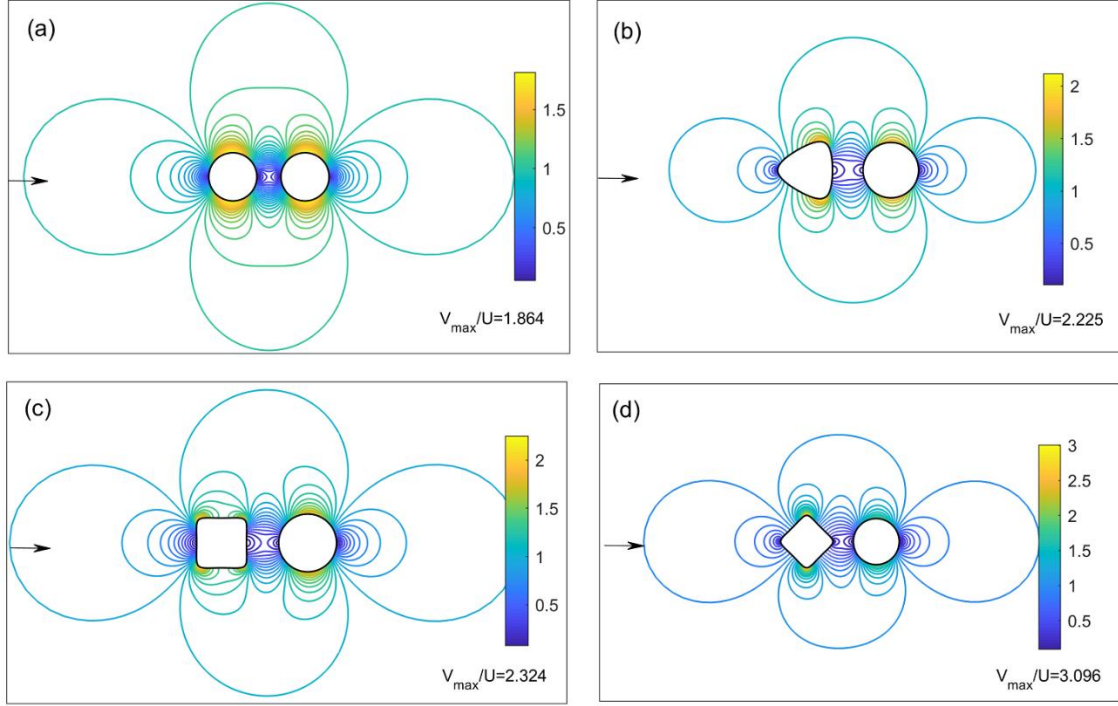


Figure 4.43 Velocity contours around two-cylinders in tandem arrangement with center distance $H = 1.5D$ in uniform potential flow along positive x -direction using complex potential function for the cases of (a) circular-circular cylinder pair ($N = \infty$ or $h = 0$) (b) triangular -circular cylinder pair ($N = 3, \gamma = 180^\circ$, and $h = 0.1$) (c) square-circular cylinder pair ($N = 4, \gamma = 45^\circ$, and $h = 0.1$) (d) 45° rotated square-circular cylinder pair ($N = 4, \gamma = 0^\circ$, and $h = 0.1$)

4.5.1.1 Effect of Center Distance

It is evident from above results, that for $H = 10D$, the interaction between polygonal and circular cylinder is found weaker as compared to that for $H = 1.5D$. The impact of presence of each cylinder in the flow field remains up to some distance, and for very large center distance the disturbances in the flow field caused by an individual cylinder does not affects flow pattern around the other cylinder. At large center distance ($H \rightarrow \infty$) there is no hydrodynamic interaction found between the cylinders and they behave as an individual cylinder.

For $H > 10D$, the velocity and pressure contours can be compared to that of single polygonal cylinder in uniform potential flow. In Figure 4.49, the velocity and pressure profile around single triangular cylinder for $N = 3, \gamma = 180^\circ, h = 0.15$, and $\alpha = 0^\circ$ in uniform potential flow is plotted compared with velocity and pressure profile around triangular cylinder for the same geometrical

parameters located at the center distance $H = 1.5D, 2.0D, 2.5D, 3D$ and $100D$ from circular cylinder in uniform flow. For center distance $H = 100D$ the velocity and pressure profile are found identical to that for single triangular cylinder due to negligible interaction effect. For the small center distances, the velocity and pressure profile alter as shown in Figure 4.49(a) and (b), and the extremum value of flow parameters changes significantly due to strong interaction effects. The maximum value of velocities $V_{max}/U = 2.76, 2.9336, 3.0249, 3.0758$ and 3.1849 , and the minimum pressure co-efficients $Cp_{min} = -6.6178, -7.6059, -8.1501, -8.4603$ and -9.1434 are found on the surface of triangular cylinder interacting with circular cylinder for the center distances $H = 1.5D, 2.0D, 2.5D, 3.0D$ and $100D$ respectively.

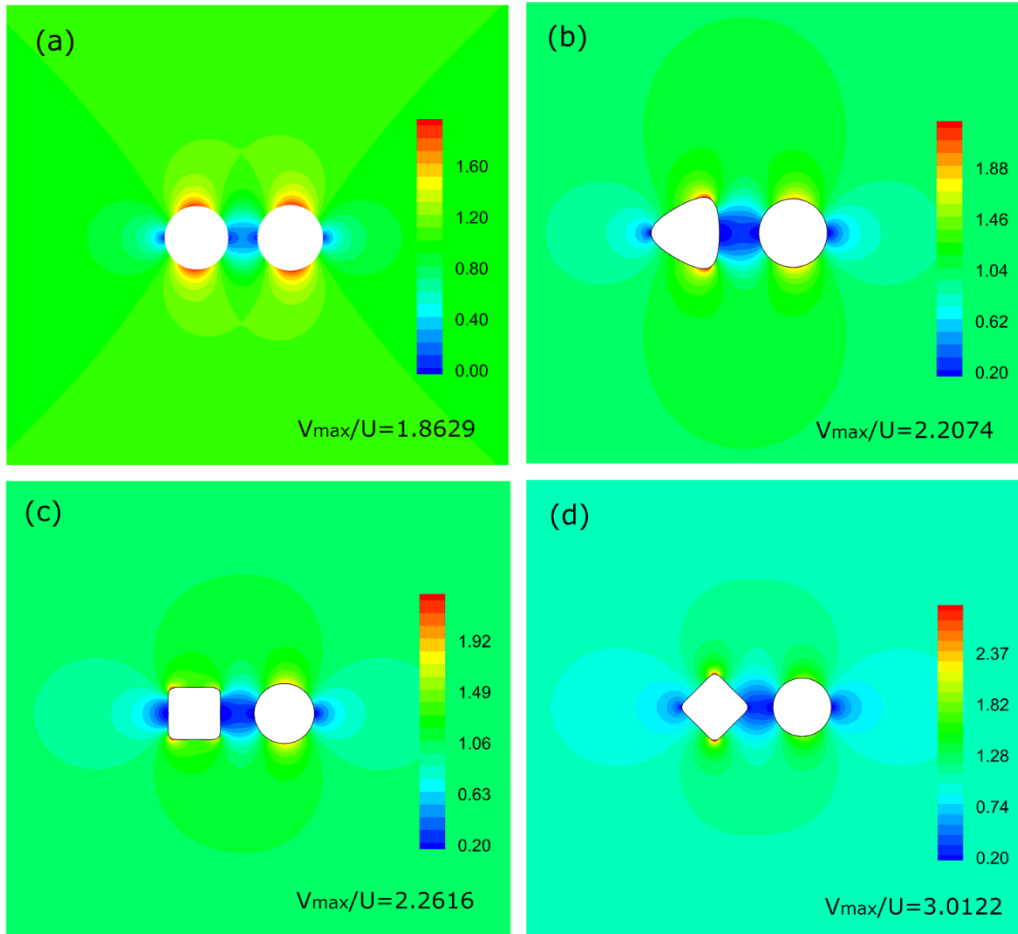


Figure 4.44 Velocity contours around two-cylinders in tandem arrangement with center distance $H = 1.5D$ in uniform potential flow along positive x -direction using ANSYS for the cases of (a) circular-circular cylinder pair ($N = \infty$ or $h = 0$) (b) triangular-circular cylinder pair ($N = 3, \gamma = 180^\circ$, and $h = 0.1$) (c) square -circular cylinder pair ($N = 4, \gamma = 45^\circ$, and $h = 0.1$) (d) 45° rotated square -circular cylinder pair ($N = 4, \gamma = 0^\circ$, and $h = 0.1$)

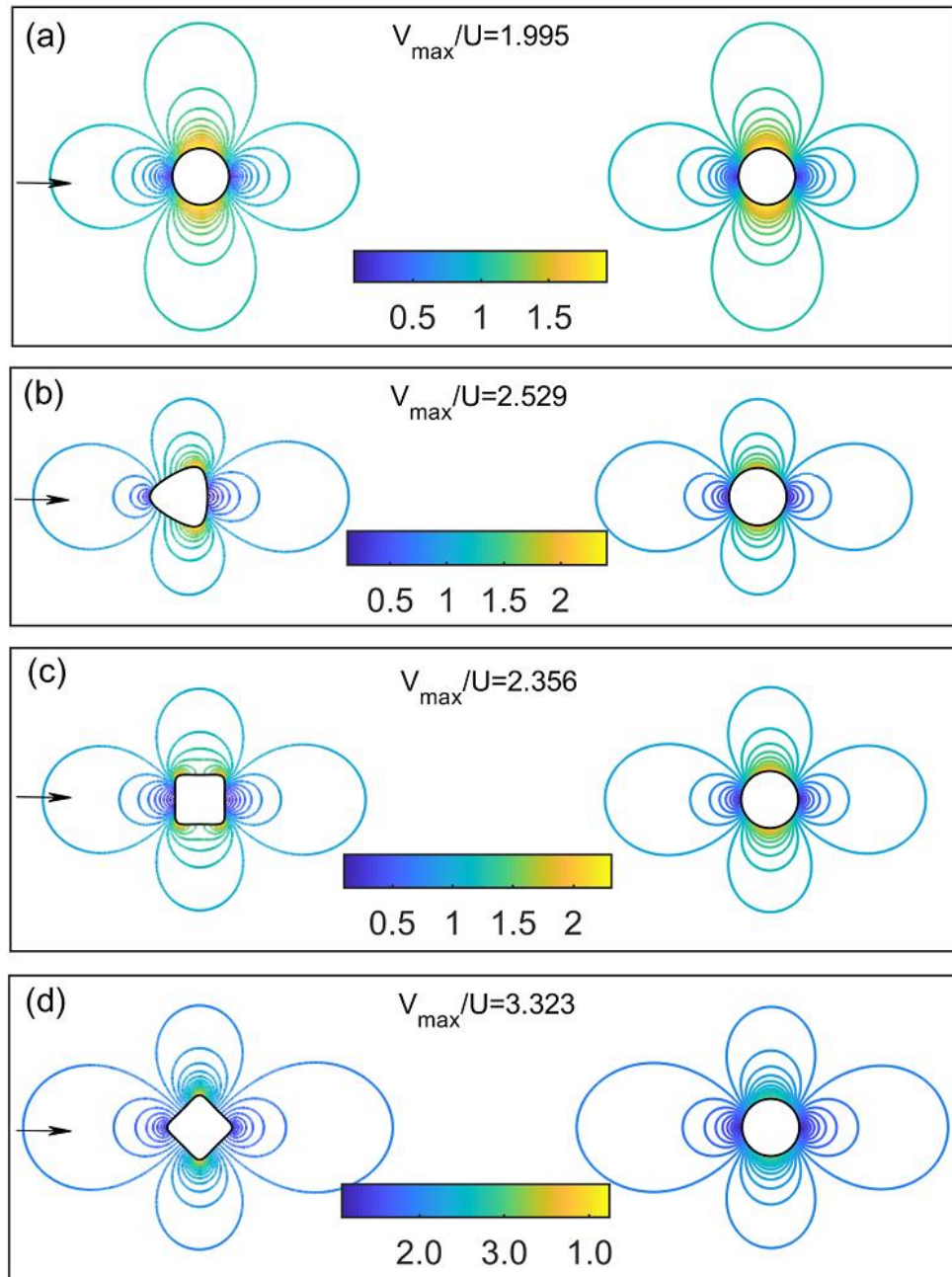


Figure 4.45 Velocity contours around two-cylinders in tandem arrangement with center distance $H = 10d$ in uniform potential flow along positive x -direction using complex potential function for the cases of (a) circular-circular cylinder pair ($N = \infty$ or $h = 0$) (b) triangular -circular cylinder pair ($N = 3, \gamma = 180^\circ$, and $h = 0.1$) (c) square -circular cylinder pair ($N = 4, \gamma = 45^\circ$, and $h = 0.1$) (d) 45° rotated square -circular cylinder pair ($N = 4, \gamma = 0^\circ$, and $h = 0.1$)

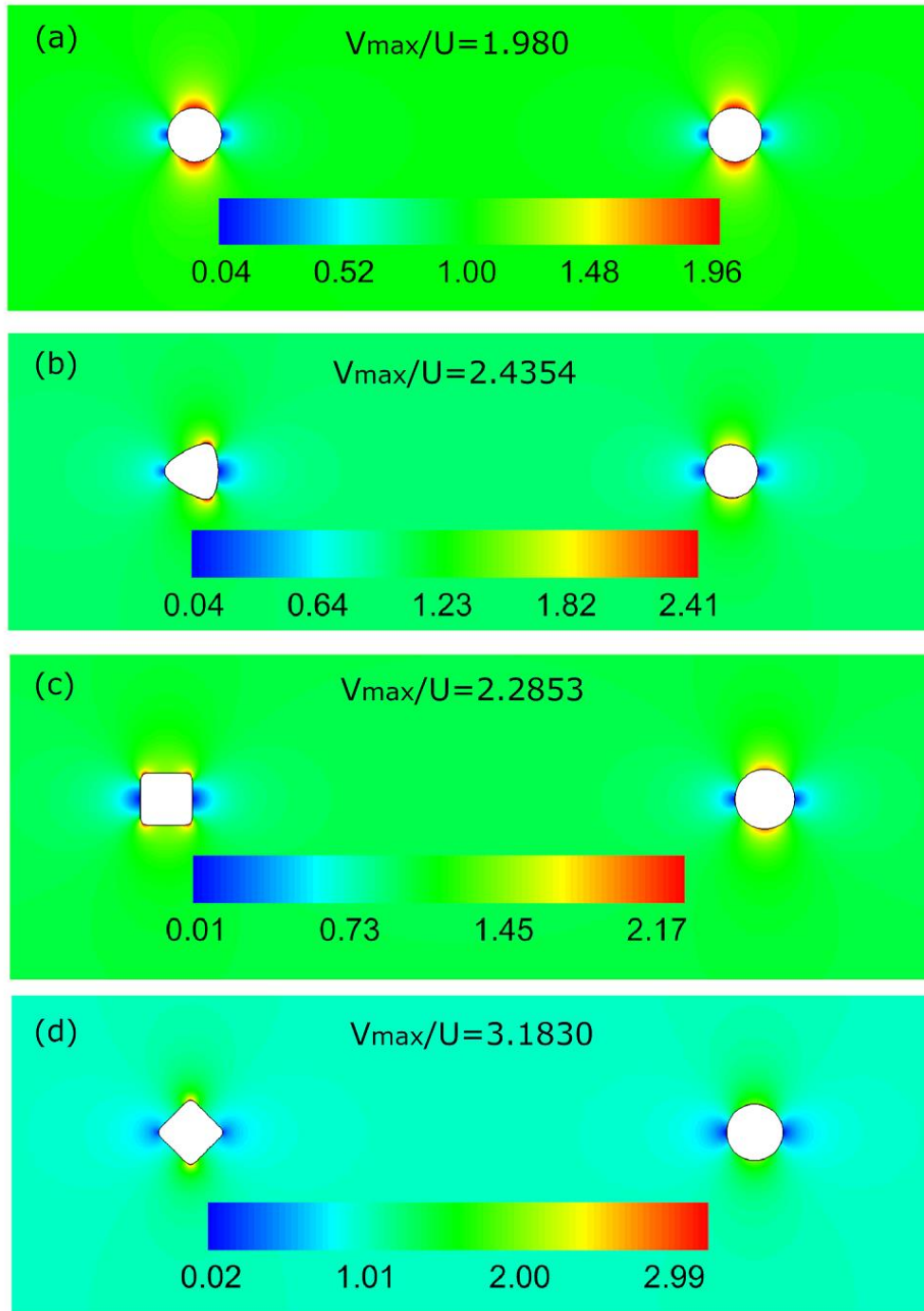


Figure 4.46 Velocity contours around two-cylinders in tandem arrangement with center distance $H = 10d$ in uniform potential flow along positive x -direction using ANSYS for the cases of (a) circular-circular cylinder pair ($N = \infty$ or $h = 0$) (b) triangular -circular cylinder pair ($N = 3, \gamma = 180^\circ$, and $h = 0.1$) (c) square -circular cylinder pair ($N = 4, \gamma = 45^\circ$, and $h = 0.1$) (d) 45° rotated square -circular cylinder pair ($N = 4, \gamma = 0^\circ$, and $h = 0.1$)

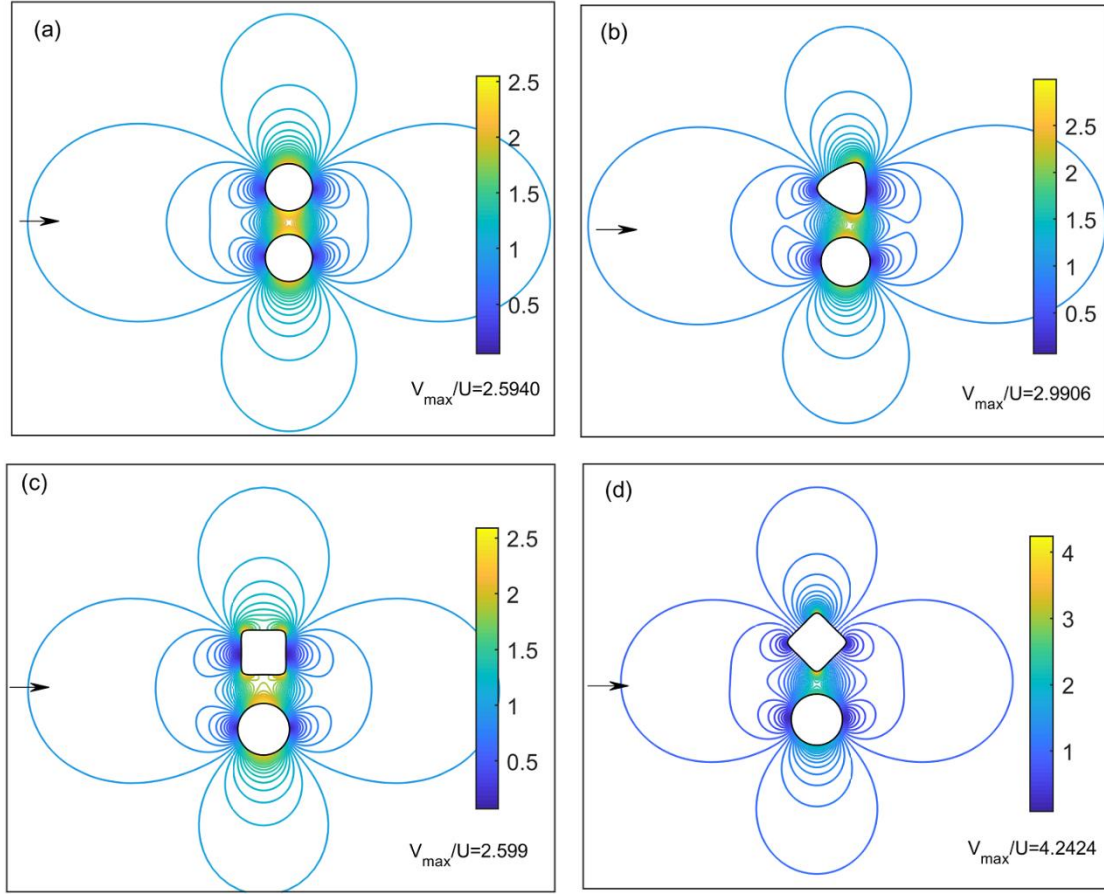


Figure 4.47 Velocity contours around two-cylinders in side-by-side arrangement with center distance $H = 1.5d$ in uniform potential flow along positive x -direction using complex potential function for the cases of (a) circular-circular cylinder pair ($N = \infty$ or $h = 0$) (b) triangular -circular cylinder pair ($N = 3, \gamma = 180^\circ$, and $h = 0.1$) (c) square-circular cylinder pair ($N = 4, \gamma = 45^\circ$, and $h = 0.1$) (d) 45° rotated square -circular cylinder pair ($N = 4, \gamma = 0^\circ$, and $h = 0.1$)

For various other configurations, the effect of center distance on the maximum velocities and minimum pressure co-efficients are shown in Figure 4.50. For tandem configuration of cylinders (flow direction along the line connecting centers of cylinders), the extremum values of velocity decreases and the extremum values of pressure co-efficient increases with decrease of center distance due to the obstruction of flow between the cylinders. As the distance between cylinders increases the obstruction of flow between polygonal and circular cylinder diminish and the extremum values of velocities and pressure co-efficient converges to the values those corresponding to single cylinder. For side by side configuration (flow direction normal to the line connecting the centers of cylinders), the extremum value of velocity increases and pressure co-

efficient decreases with decrease of center distance due to Venturi effect that fluid experiences while passing between polygonal and circular cylinders. For large center distance ($H > 10D$), the Venturi effect dies out and flow parameters around the cylinders behaves similar to that around single cylinder in uniform flow. The intensity of interaction also depends on the shape, size, corner radius and orientation of polygonal cylinder.

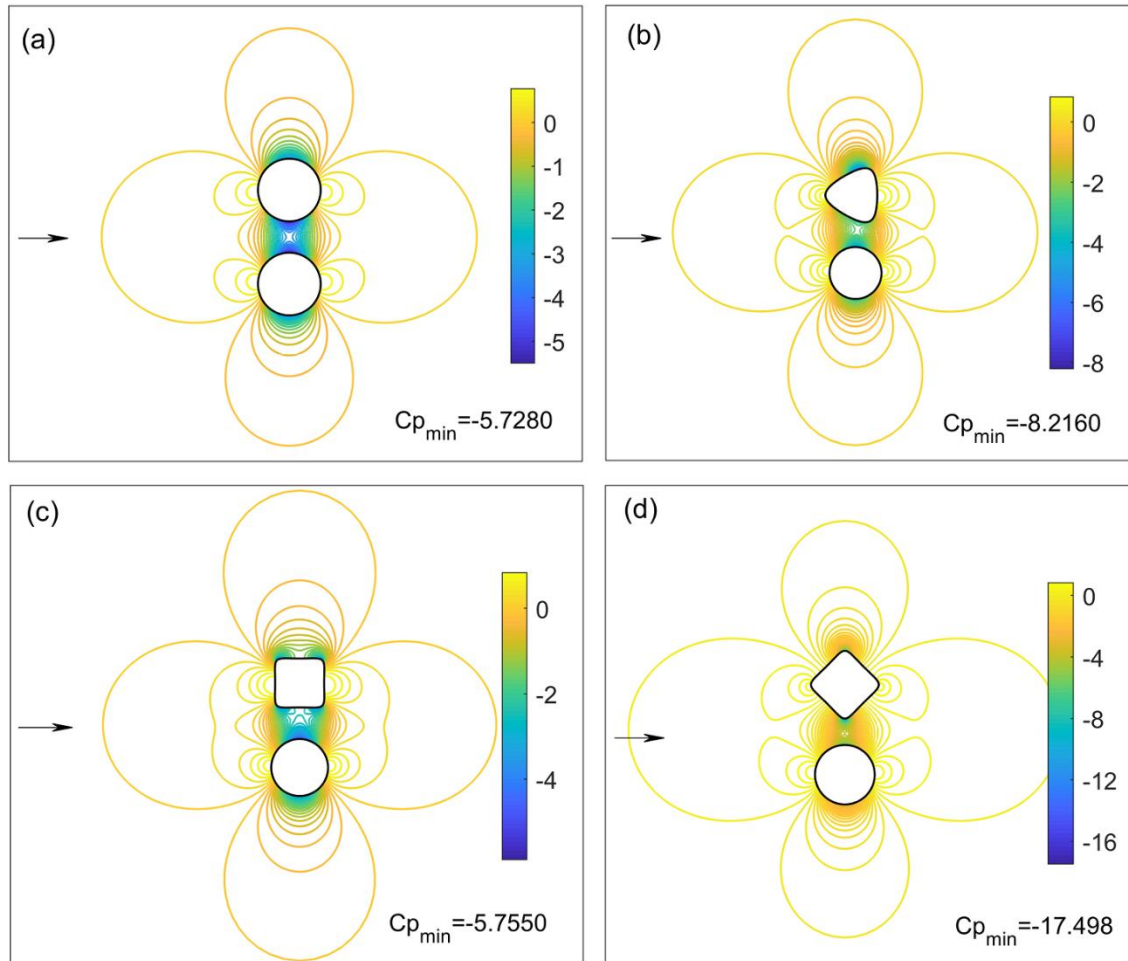


Figure 4.48 Pressure contours around two-cylinders in side-by-side arrangement with center distance $H = 1.5d$ in uniform potential flow along positive x -direction using complex potential function for the cases of (a) circular-circular cylinder pair ($N = \infty$ or $h = 0$) (b) triangular-circular cylinder pair ($N = 3, \gamma = 180^\circ$, and $h = 0.1$) (c) square-circular cylinder pair ($N = 4, \gamma = 45^\circ$, and $h = 0.1$) (d) 45° rotated square-circular cylinder pair ($N = 4, \gamma = 0^\circ$, and $h = 0.1$)

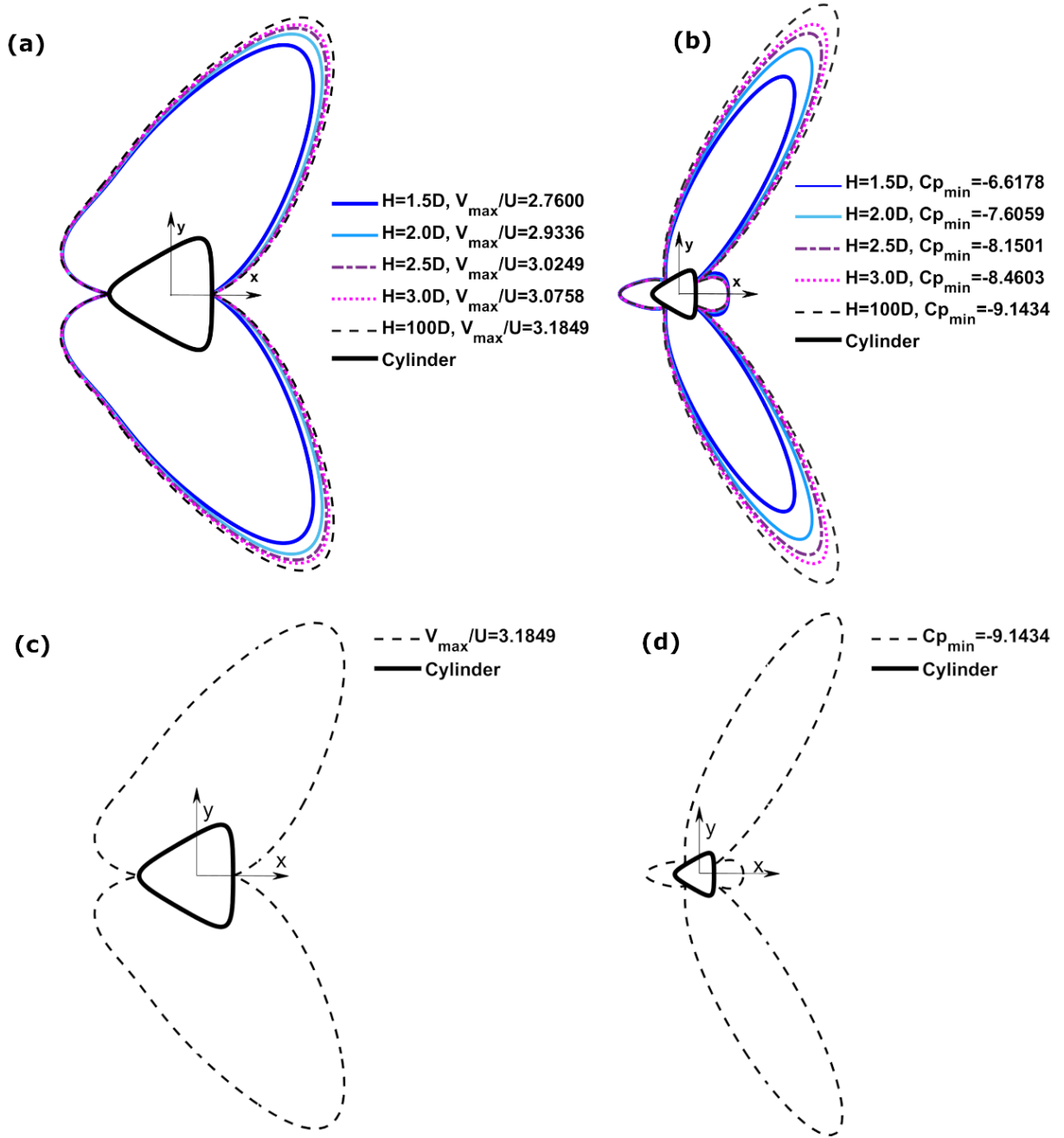


Figure 4.49 Flow parameters on the surface of triangular cylinder ($N = 3, \gamma = 180^\circ, h = 0.15$), (a) velocity profile and (b) pressure co-efficient profile obtained using present method while interacting with circular cylinder at the center distances $H = 1.5D, 2D, 2.5D, 3D$ and $100D$, (c) velocity profile and (d) pressure profile on single triangular cylinder place in uniform potential flow.

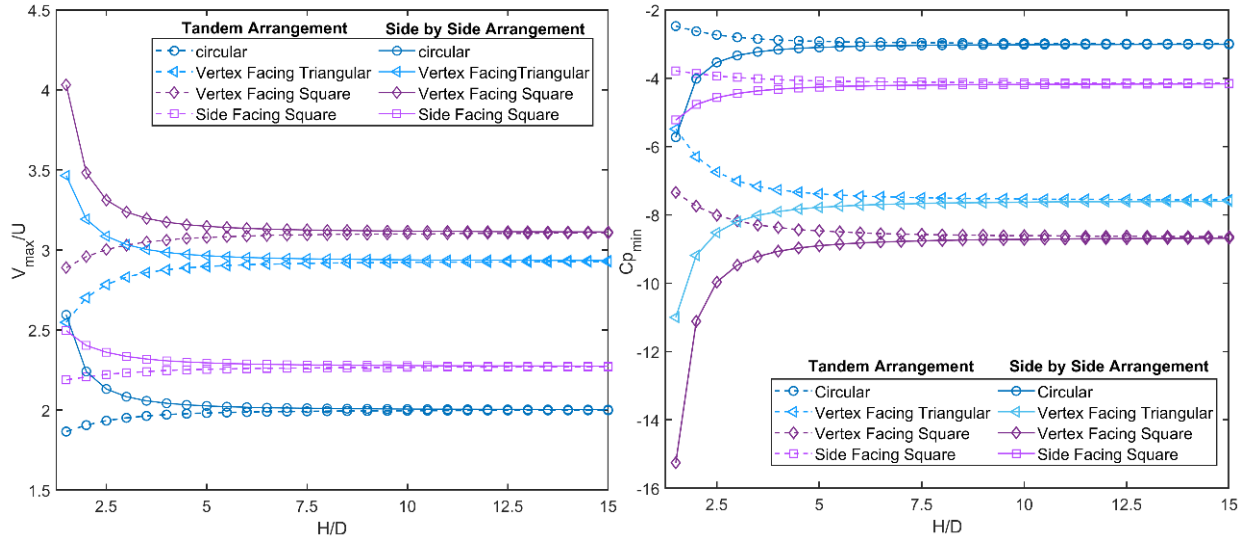


Figure 4.50 Maximum velocity (left) and minimum pressure co-efficient (right) around polygonal-circular cylinder pair in tandem and side-by-side configuration for varying center distance H/D between the cylinders in uniform potential flow. Continues line shows the results for side by side arrangement and dashed line shows the results for tandem arrangement of cylinders

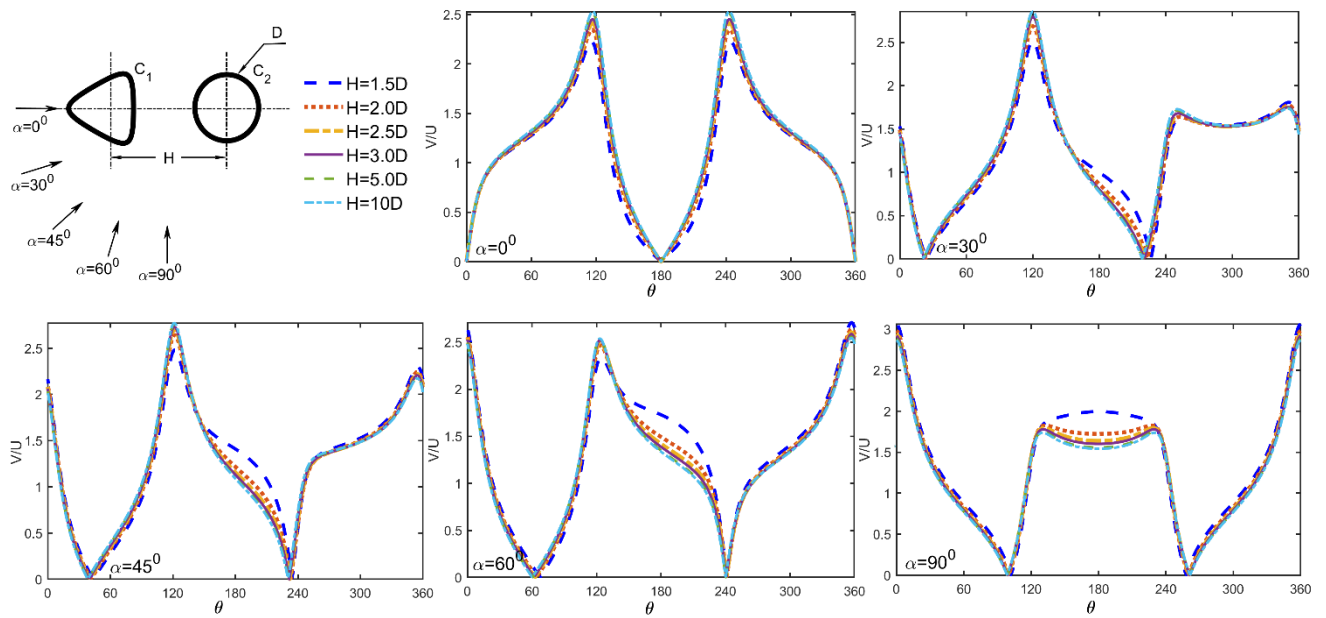


Figure 4.51 Velocity distribution on the surface of triangular cylinder in triangular-circular cylinder pair ($N = 3$, $\gamma = 180^\circ$, and $h = 0.1$) for center $H/D = 1.5, 2.0, 2.5, 3.0, 5.0, 10$ and for angle of attack $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$. Angle of attack is measured from positive x -axis.

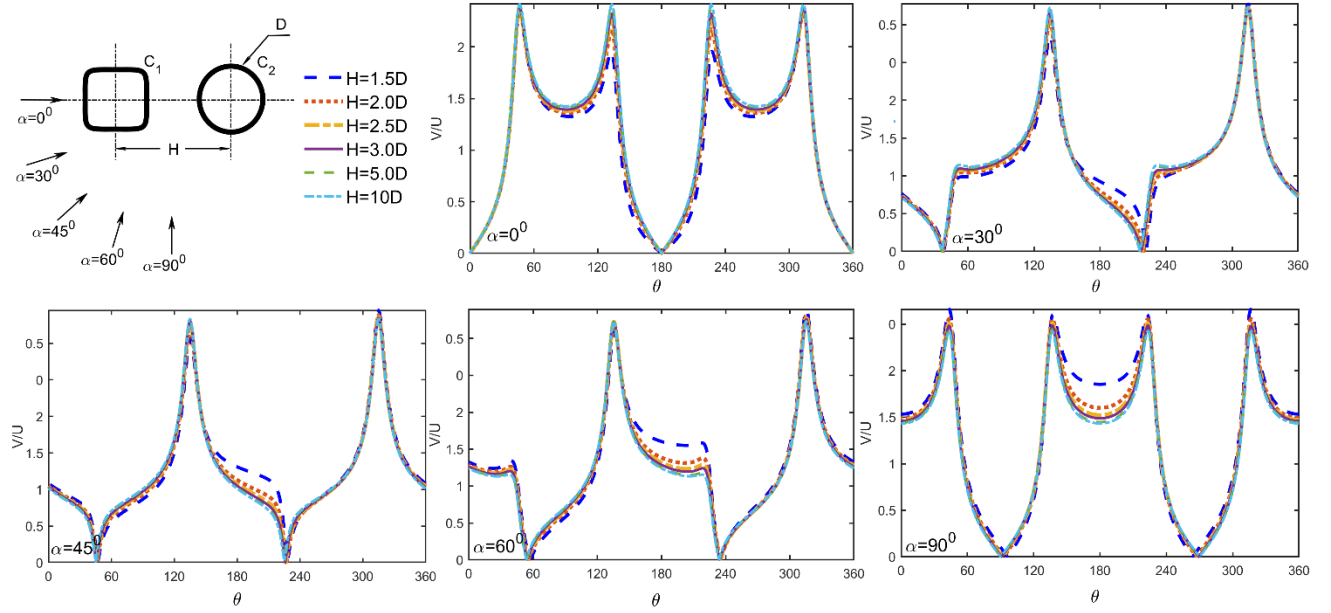


Figure 4.52 Velocity distribution on the surface of square cylinder in square-circular cylinder pair ($N = 4$, $\gamma = 45^\circ$, and $h = 0.1$) for center $H/D = 1.5, 2.0, 2.5, 3.0, 5.0, 10$ and for angle of attack $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$. Angel of attack is measured from positive x -axis

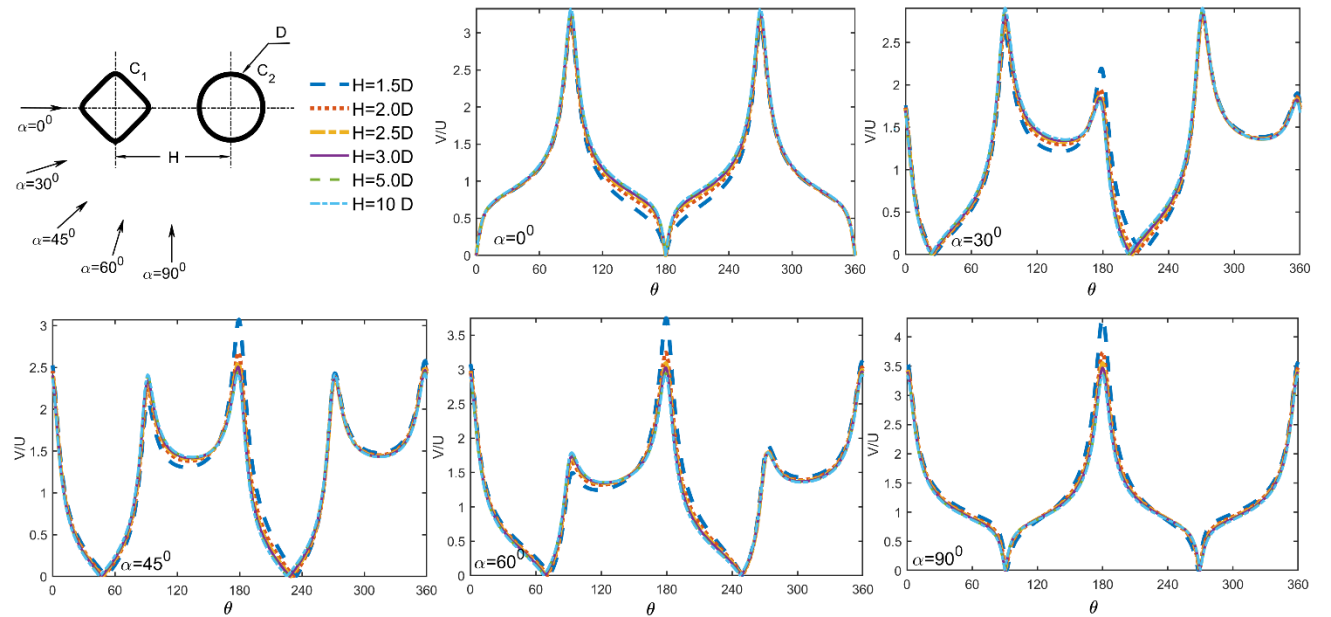


Figure 4.53 Velocity distribution on the surface of square cylinder in 45° rotated square-circular cylinder pair ($N = 4$, $\gamma = 0^\circ$, and $h = 0.1$) for center $H/D = 1.5, 2.0, 2.5, 3.0, 5.0, 10$ and for angle of attack $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$. Angel of attack is measured from positive x -axis.

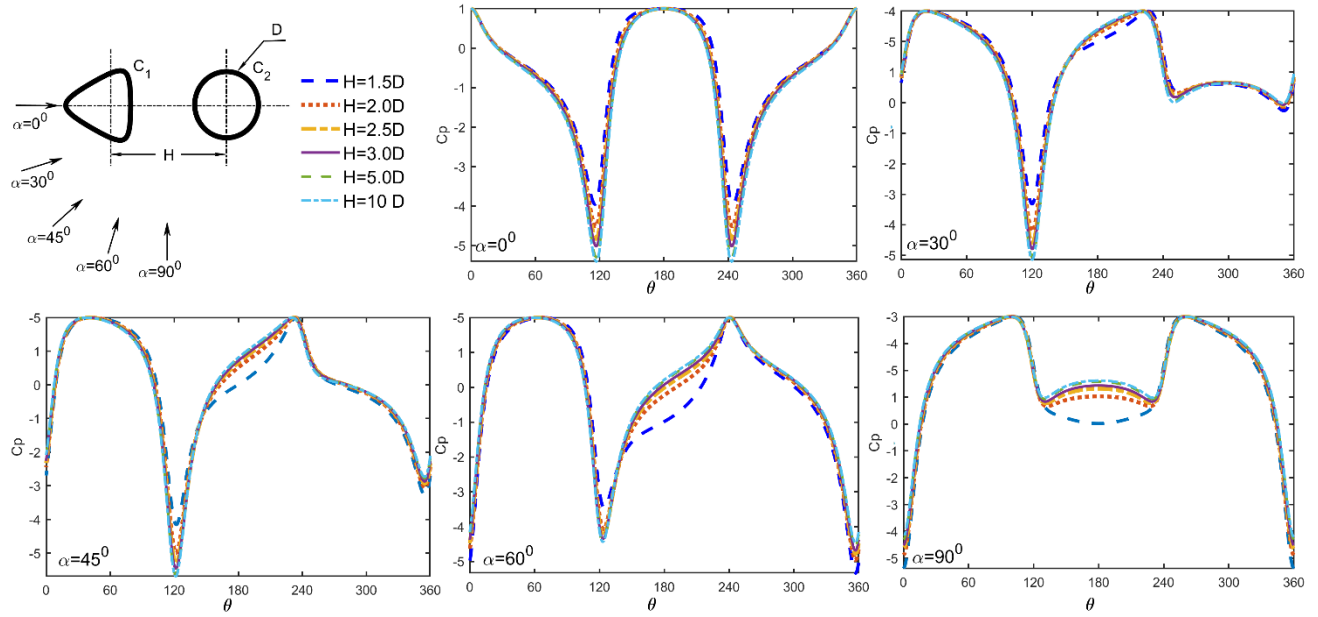


Figure 4.54 Pressure distribution on the surface of triangular cylinder in triangular-circular cylinder pair ($N = 3$, $\gamma = 180^\circ$, and $h = 0.1$) for center $H/D = 1.5, 2.0, 2.5, 3.0, 5.0, 10$ and for angle of attack $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$. Angle of attack is measured from positive x -axis.

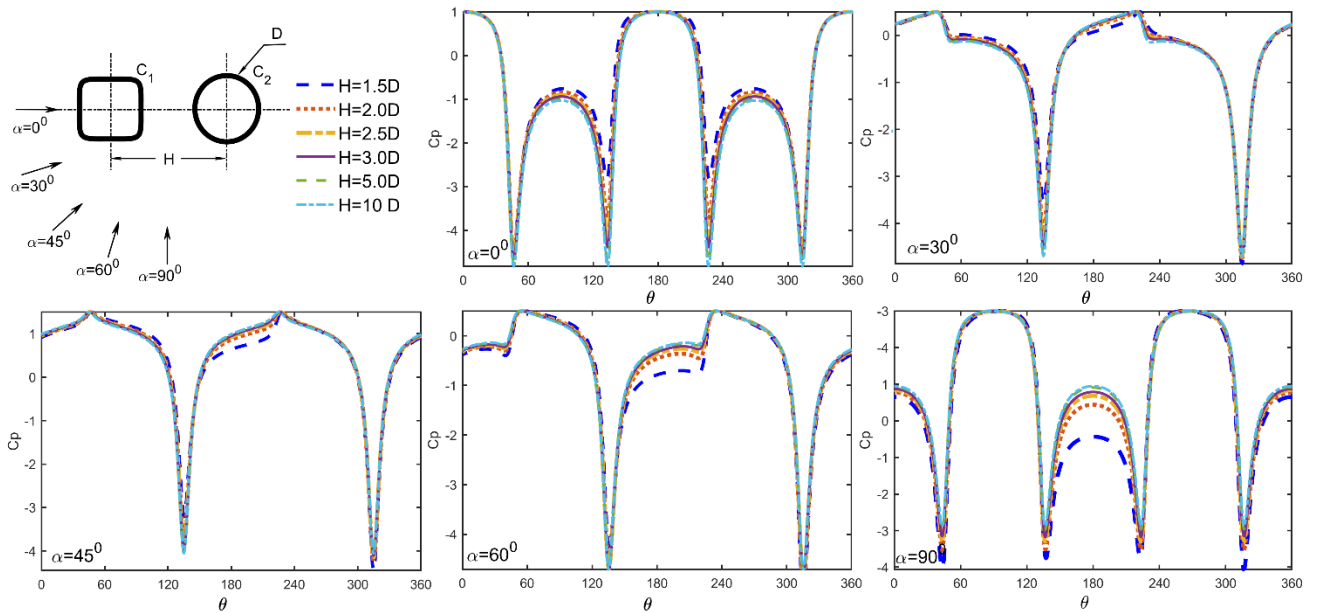


Figure 4.55 Pressure distribution on the surface of square cylinder in 45° rotated square-circular cylinder pair ($N = 4$, $\gamma = 45^\circ$, and $h = 0.1$) for center $H/D = 1.5, 2.0, 2.5, 3.0, 5.0, 10$ and for angle of attack $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$. Angle of attack is measured from positive x -axis.

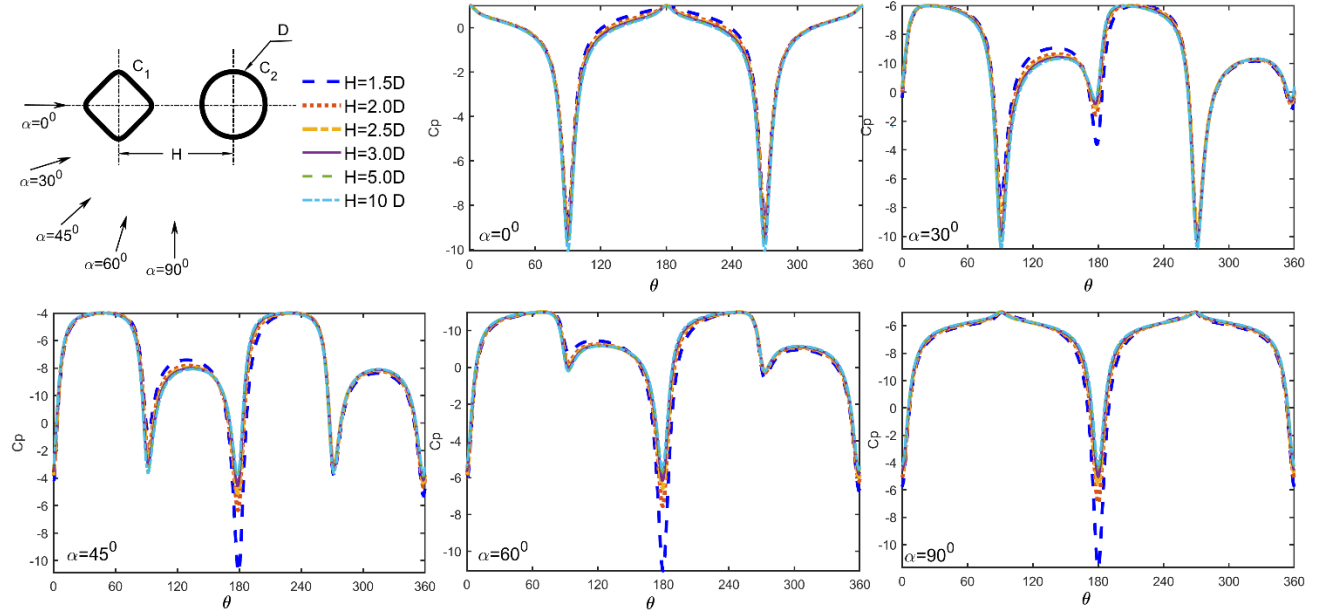


Figure 4.56 Pressure distribution on the surface of square cylinder in 45° rotated square-circular cylinder pair ($N = 4$, $\gamma = 0^\circ$, and $h = 0.1$) for center $H/D = 1.5, 2.0, 2.5, 3.0, 5.0, 10$ and for angle of attack $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$. Angle of attack is measured from positive x -axis.

4.5.1.2 Effect of Flow Angle

For polygonal cylinder located at infinite distance away from the circular cylinder in uniform potential flow, the maximum velocity and minimum pressure are observed at the vertex (vertices) on the line joining it (them) to the center of polygonal cylinder, if the line is perpendicular to the flow. The fluid accelerates at this vertex (vertices) and the velocity and pressure attain their extremum values.

Figure 4.51 to Figure 4.56 shows the velocity and pressure co-efficient on the surface of triangular ($N = 3, h = 0.1$), square ($N = 4, h = 0.1$) and 45° rotated square ($N = 4, h = 0.1$) cylinders interacting with circular cylinder for different center distances $H = 1.5D, 2.0D, 2.5D, 3.0D, 5.0D$ and $10D$ for different flow angles $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° . As the center distance between the cylinders reduces, the hydrodynamic interaction becomes effective and lead to significant changes in the flow parameters around the cylinders. The maximum values of the velocities corresponding to different angle of attack ($0^\circ \leq \alpha \leq 180^\circ$) and center distances ($H = 1.5D, 2.0D, 2.5D, 3.0D, 5D$ and $10D$) for various configurations are shown in Figure 4.57. These extremum values occur at some point (points) on the surface of polygonal cylinders and the

locations of point (points) on the cylinders also get affected by the center distance and flow angles along with orientation of the polygonal cylinder. It can be seen from the Figure 4.58 that for $H = \infty$, the flow parameters around the triangular cylinder ($N = 3, h = 0.15$) attains the extremum value $V_{max}/U = 3.6364$ and $Cp_{min} = -12.2231$ for flow angles $\alpha = 30^\circ$ and $\alpha = 90^\circ$ at the locations $\theta = -60^\circ$ and $\theta = 180^\circ$ respectively. The angles $\theta = -60^\circ$ and $\theta = 180^\circ$ represent orientation of the radial vector perpendicular to the flow. For the center distance $H = 1.5D$, the flow parameter attains extremum values $V_{max}/U = 3.8987$ and $Cp_{min} = -14.2003$ for $\alpha = 90^\circ$ at location $\theta = 180^\circ$, but for $\alpha = 30^\circ$ the values no longer remain extremum. The comparison of the magnitudes and locations of the extremum values of flow parameters on the surface of triangular cylinder interacting with circular cylinder in uniform flow for center distances $H = 1.5D$ and ∞ for various flow angles is shown in Table 4.9.

Table 4.9 The extremum values of velocities and pressure co-efficient and their locations on the surface of triangular cylinder interacting with circular cylinder at center distance $H = 1.5D$ and ∞ .

Flow angle ' α '	$H = 1.5D$			$H = \infty$		
	V_{max}/U	Cp_{min}	θ	V_{max}/U	Cp_{min}	θ
0°	2.7600	-6.6178	$\pm 63.2276^\circ$	3.1849	-9.1434	$\pm 62.0^\circ$
30°	3.1982	-9.2285	-53.9307°	3.6364	-12.2231	-60.0°
45°	3.1399	-8.8592	-58.6039°	3.5212	-11.3987	-59.0°
60°	3.4085	-10.6182	177.9877°	3.1949	-9.1434	178.0° -58.0°
90°	3.8987	-14.2003	180°	3.6364	-12.2231	180°

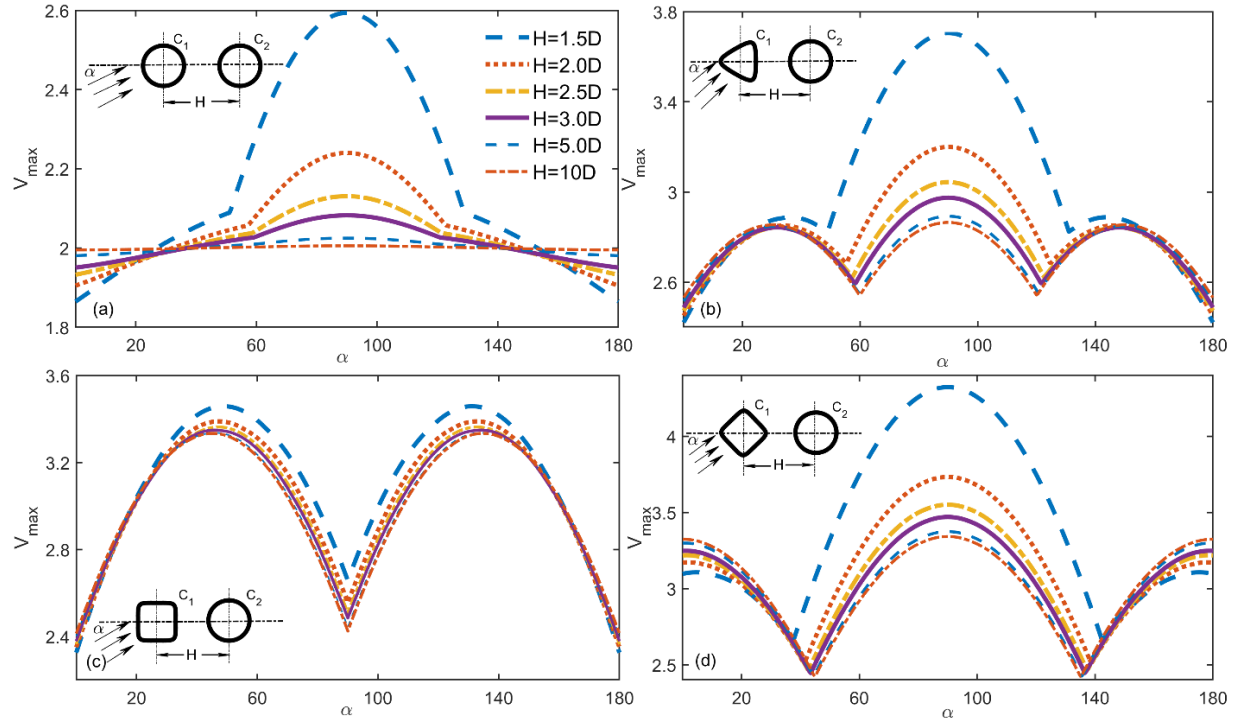


Figure 4.57 Maximum velocity around polygonal-circular cylinder pair in uniform potential flow for angle of attack $\alpha = 0^\circ$ to 180° measured from positive x -axis. The distance between two-cylinders H/D is varied from 1.5 to 10.

4.5.2 Hydrodynamic Forces on Cylinders

In previous section, it is observed that the proximity of cylinders alters the velocity and pressure profile around the cylinders significantly. The change in velocity and pressure profile leads to the change in hydrodynamic forces on the cylinders. The hydrodynamic forces around the cylinders are obtained by integrating the pressure field using Blasius theorem as discussed in Section 0. The effect of various geometrical and flow parameters on the hydrodynamic forces are discussed in this section.

The impact of center distance on the force co-efficient (C_{fx}) on polygonal shaped cylinders ($N = 3$ and $N = 4$) for corner radius $\rho = 1, 0.1$, and 0.01 is shown in Figure 4.59. For the given center distance and orientation, as the corner radius (ρ) decreases the hydrodynamic forces acting on polygonal cylinders increases. For the corner radius $\rho = 0.01$ and center distance $H = 1.5D$, the values of force co-efficient are found to be $C_{fx} = -0.8287, -0.4189, -0.4570$, and -0.5867 for

triangular-circular , 180° rotated triangular-circular, 45° rotated square-circular cylinder, and square-circular cylinder pairs respectively. For the corner radius $0.01 \leq \rho \leq 0.5$, the hydrodynamic forces acting on polygonal and circular cylinder in tandem and side by side arrangements are given in Table 4.10.

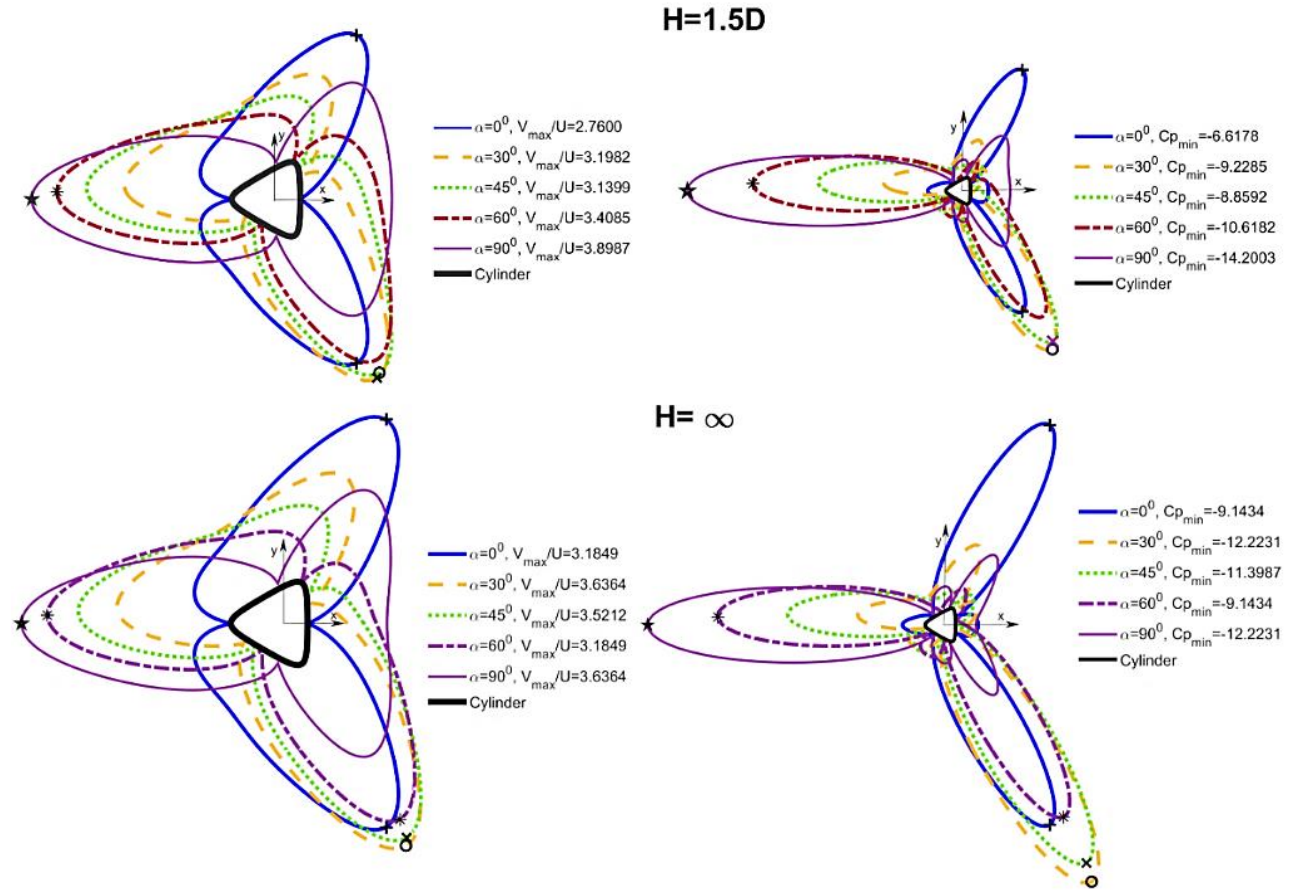


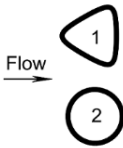
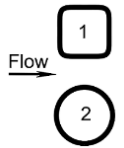
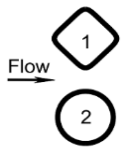
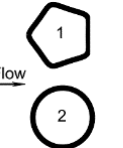
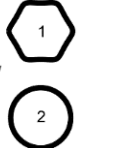
Figure 4.58 Velocity (left) and Pressure (right) profile on the surface of triangular cylinder ($N = 3, h = 0.15$) interacting with circular cylinder for the center distance $H = 1.5D$ and ∞ in uniform potential flow are shown for different flow angles $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° . Flow angles are measured from positive x -axis. Markers shows the locations of extremum values of flow parameters.

The force co-efficient C_{fx} acting on polygonal cylinder is plotted against the angle of attack α varying from 0° to 180° as shown in Figure 4.60. For the given finite center distance (H/D), the hydrodynamic forces acting on Cylinder 1 along the center line increases with angle of attack (α), and attain the maxima at $\alpha = 90^\circ$ for the cases under consideration. The directions of the forces

acting on Cylinder 1 changes from negative to positive and so the forces between the cylinders changes repulsive to attractive at certain angle of flow (α). These angles, where sign of force coefficients changes are shown in Table 4.11 for center distance $H/D = 1.5, 2.0, 2.5, 3.0, 5.0$ and 10, and corner radius $\rho = 0.2$. Also, the forces in the direction normal to the line joining the centers of cylinders are found to be non-zero for the angle of attack $0^\circ < \alpha < 90^\circ$.

Table 4.10 Force co-efficient acted upon polygonal and circular cylinders in tandem ($\alpha = 0^\circ$) and side-by-side arrangement ($\alpha = 90^\circ$) for different corner radius $0.01 \leq \rho \leq 0.5$ and $H = 1.5D$ in uniform potential flow. The potential flow around polygonal-circular cylinder is obtained using complex potential function along with conformal mapping.

Tandem $\alpha = 0^\circ$										
	C_{fx}		C_{fx}		C_{fx}		C_{fx}		C_{fx}	
ρ	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2
0.5	-0.3788	0.3699	-0.3588	0.3542	-0.3429	0.3385	-0.3516	0.3487	-0.3477	0.3457
0.4	-0.3934	0.3774	-0.3656	0.3569	-0.3437	0.3357	-0.3549	0.3495	-0.3492	0.3454
0.3	-0.4152	0.3868	-0.3756	0.3603	-0.3436	0.3323	-0.3606	0.3507	-0.3522	0.3451
0.2	-0.4515	0.3992	-0.3935	0.3651	-0.3529	0.3275	-0.3712	0.3522	-0.3585	0.3447
0.1	-0.5269	0.4177	-0.4324	0.3727	-0.3711	0.3198	-0.3957	0.3549	-0.3747	0.3439
0.01	-0.8287	0.4548	-0.5867	0.3892	-0.4570	0.3031	-0.4999	0.3609	-0.4508	0.3419

Side-by-side $\alpha = 90^\circ$										
	C_{fy}		C_{fy}		C_{fy}		C_{fy}		C_{fy}	
ρ	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2	Cylinder 1	Cylinder 2
0.5	-0.4570	0.4570	-0.4347	0.4291	-0.4852	0.4789	-0.4591	0.4591	-0.4527	0.4501
0.4	-0.4622	0.4622	-0.4307	0.4208	-0.4990	0.4875	-0.4629	0.4629	-0.4537	0.4487
0.3	-0.4719	0.4719	-0.4273	0.4101	-0.5197	0.4986	-0.4694	0.4694	-0.4560	0.4468
0.2	-0.4908	0.4908	-0.4259	0.3952	-0.5541	0.5141	-0.4820	0.4820	-0.4620	0.4441
0.1	-0.5352	0.5352	-0.4316	0.3720	-0.6251	0.5388	-0.5117	0.5117	-0.4788	0.4395
0.01	-0.7297	0.7297	-0.4854	0.3220	-0.8952	0.5938	-0.6400	0.6400	-0.5646	0.4282

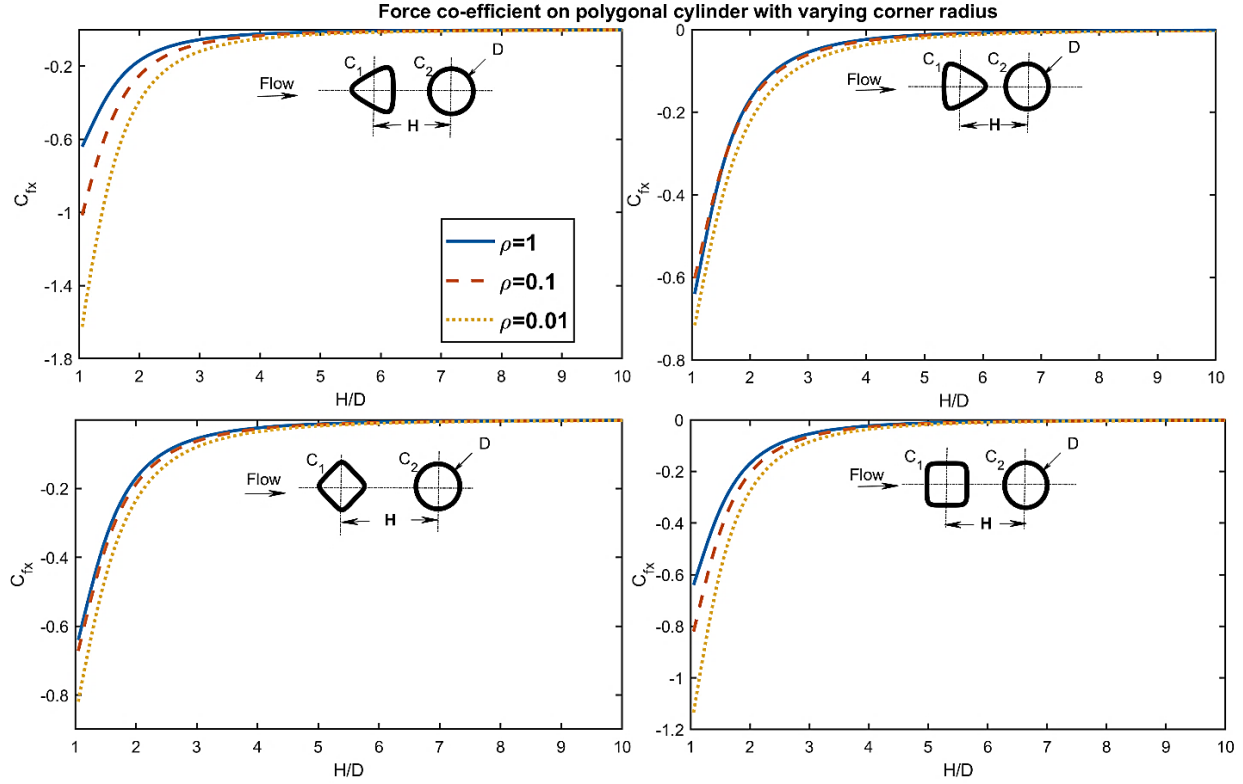


Figure 4.59 Hydrodynamic force co-efficient on polygonal cylinders of corner radius $\rho = 1, 0.1$ and 0.01 interacting with circular cylinder for varying center distance H/D in uniform potential flow along the line connecting the centers of cylinders.

4.6 Hydrodynamic Interaction between two Polygonal Cylinders

The hydrodynamic interaction between polygonal and circular cylinder is discussed in previous section. In this section, the hydrodynamic interaction between two polygonal cylinders is presented and discussed. The effects of flow angle (α), the distance between the centers of cylinders (H), corner radii (ρ_1 and ρ_2), orientation angles (γ_1 and γ_2), and number of sides (N_1 and N_2) of polygonal cylinders on the velocity field, pressure profile and hydrodynamic forces around interacting polygonal cylinders in uniform potential flow are considered in discussion. To vary the center-to-center distance (H) between the cylinders, the upstream cylinder is fixed at the origin and the location of downstream cylinder is changed on the positive real axis.

The results obtained using present method are validated by comparing some of them with results obtained from ANSYS. The pair of polygonal geometries are surrounded by two-dimensional rectangular flow domain generated using ANSYS Design-Modeler. The boundary conditions

similar to those shown Figure 4.42 are applied. The inviscid flow model is selected and the SIMPLE algorithm alongwith Second Order Upwind scheme is used for the solution of the problem.

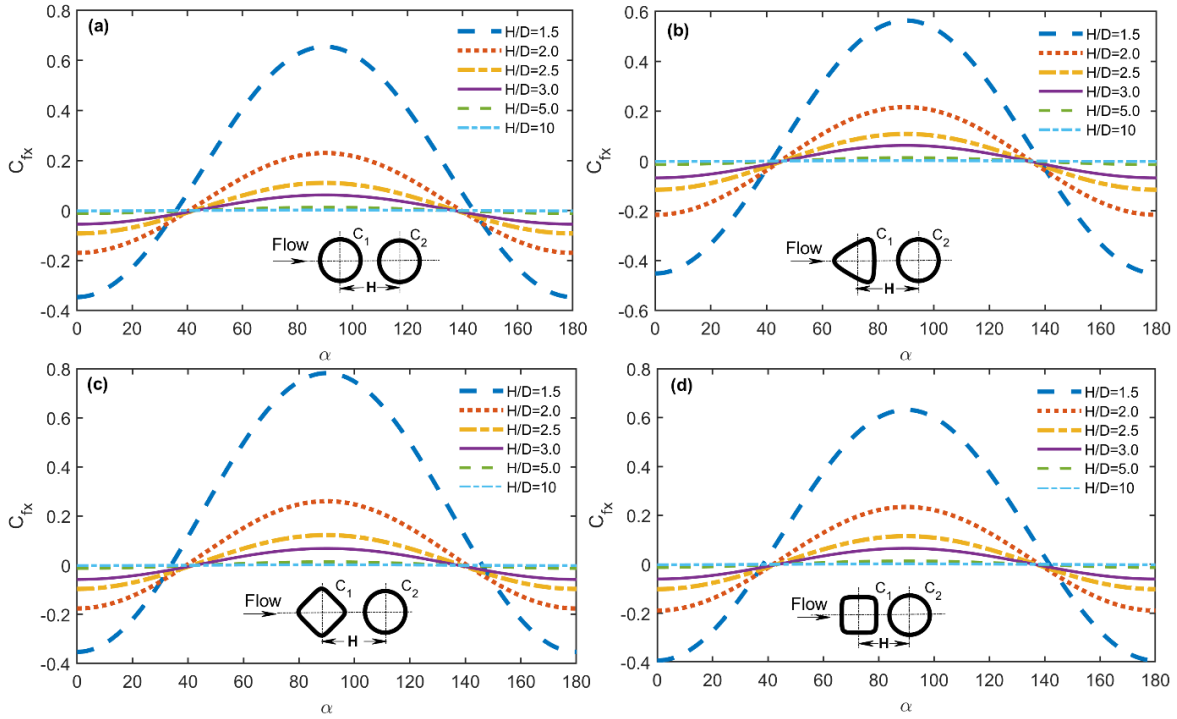


Figure 4.60 Force co-efficient C_{fx} on polygonal cylinder interacting with circular cylinder for different center distances $H/D = 1.5, 2, 2.5, 3, 5$, and 10 in uniform potential flow of varying angle of attack $\alpha = 0^\circ$ to 180° , the angle of attack is measured from positive x -axis. The corner radius of the polygonal shaped cylinders is considered $\rho = 0.2$.

Table 4.11 The angle of attack α at which the hydrodynamic force acting on polygonal Cylinder 1 and circular Cylinder 2 along the line joining the centers of cylinders attain zero for center distance $H/D = 1.5, 2.0, 2.5, 3.0, 5.0$ and 10 , the corner radius of polygonal cylinders is considered $\rho = 0.2$.

Geometry																				
H/D	1.5	2.5	5.0	10	1.5	2.5	5.0	10	1.5	2.5	5.0	10	1.5	2.5	5.0	10	1.5	2.5	5.0	10
α_1	36	42	44	45	31	36	43	44	42	46	46	46	34	42	44	45	38	43	45	45

4.6.1 Velocity and Pressure Distribution

The velocity and pressure distribution on the surface of two interacting polygonal cylinders in uniform potential flow are presented for various geometrical and flow parameters. In Figure 4.61, the velocity and pressure coefficient on the surface of triangular cylinder interacting with the square cylinder are computed using present method, and compared with the results obtained by performing inviscid flow simulation in ANSYS for same geometrical parameters ($N_1 = 3, N_2 = 4, \gamma_1 = \pi, \gamma_2 = \pi/4, \rho_1 = \rho_2 = 0.1$, and $H = 4$) and flow conditions ($\alpha = 0^\circ$ and $U = 1$). The velocities and pressure coefficients computed using present method are in close confirmation with the results obtained using ANSYS.

The velocity and pressure coefficient results are further obtained for flow around triangular-triangular, triangular-diamond, triangular-square, diamond-triangular, diamond-diamond, diamond-square, square-triangular, square-diamond, and square-square cylinder pairs and discussed for different values of center distances (H), flow angles (α), orientation angles (γ_1 and γ_2) and number of sides (N_1 and N_2).

4.6.1.1 Effect of center distance

The pressure coefficient (C_p) on the surface of two polygonal cylinders interacting in uniform potential flow along the line connecting the centers of cylinders is shown in Figure 4.62 (for the cases of triangular-triangular, triangular-diamond, and triangular-square cylinder pairs), Figure 4.63 (for cases of diamond-triangular, diamond-diamond, and diamond-square cylinder pairs), and Figure 4.64 (for the cases of square-triangular, square-diamond, and square-square cylinder pairs). The results are plotted for free stream velocity $U = 1$, corner radii $\rho_1 = \rho_2 = 0.1$, and the center distance $H = 3, H = 5$, and $H = 10$. The minimum pressure coefficient $C_{p_{min}} = -13.2349$ is found at $\theta_1 = 118.6^\circ$ and 241.4° on the surface of triangular cylinder in triangular-triangular, triangular-diamond and triangular-square configurations for center distance $H = 10$. The extremum value of pressure coefficient ($C_{p_{min}}$) for the center distance $H = 3$ is found to be $C_{p_{min}} = -9.8436$ at the same angular location. The pressure coefficients for the downstream cylinders in the said combination for center distance $H = 10$ are found as $C_{p_{min}} = -13.3421$ at $\theta_2 = 61.3^\circ$ and 298.6° , $C_{p_{min}} = -14.6941$ at $\theta_2 = 90^\circ$ and 270° , and $C_{p_{min}} = -7.03$ at $\theta_2 = 133.2^\circ$ and 226.7° for triangular ($N = 3, \gamma_2 = 180^\circ$), diamond ($N = 4, \gamma_2 = 0^\circ$) and square ($N =$

4, $\gamma_2 = 45^\circ$) shaped cylinders respectively. These extremum values of pressure coefficients increase to $Cp_{min} = -12.3483$ ($\theta_2 = 61.2^\circ, 298.7^\circ$), $Cp_{min} = -12.81$ ($\theta_2 = 89.8^\circ, 270.15^\circ$) and $Cp_{min} = -4.2859$ ($\theta_2 = 132.6^\circ, 227.4^\circ$) for the center distance $H = 3$ for downstream triangular, diamond and square cylinders respectively. It is observed that for the flow parallel to the center line of cylinders, the pressure coefficient increases as the distance between the cylinder decreases due to obstruction of flow between the cylinder. The extremum values of pressure coefficients for other configurations are presented in Table 4.12. For the flow direction normal to the line connecting the centers of cylinders, the minimum pressure coefficient on the surface of polygonal cylinders decreases with decrease of distance between the cylinder (refer Table 4.12).

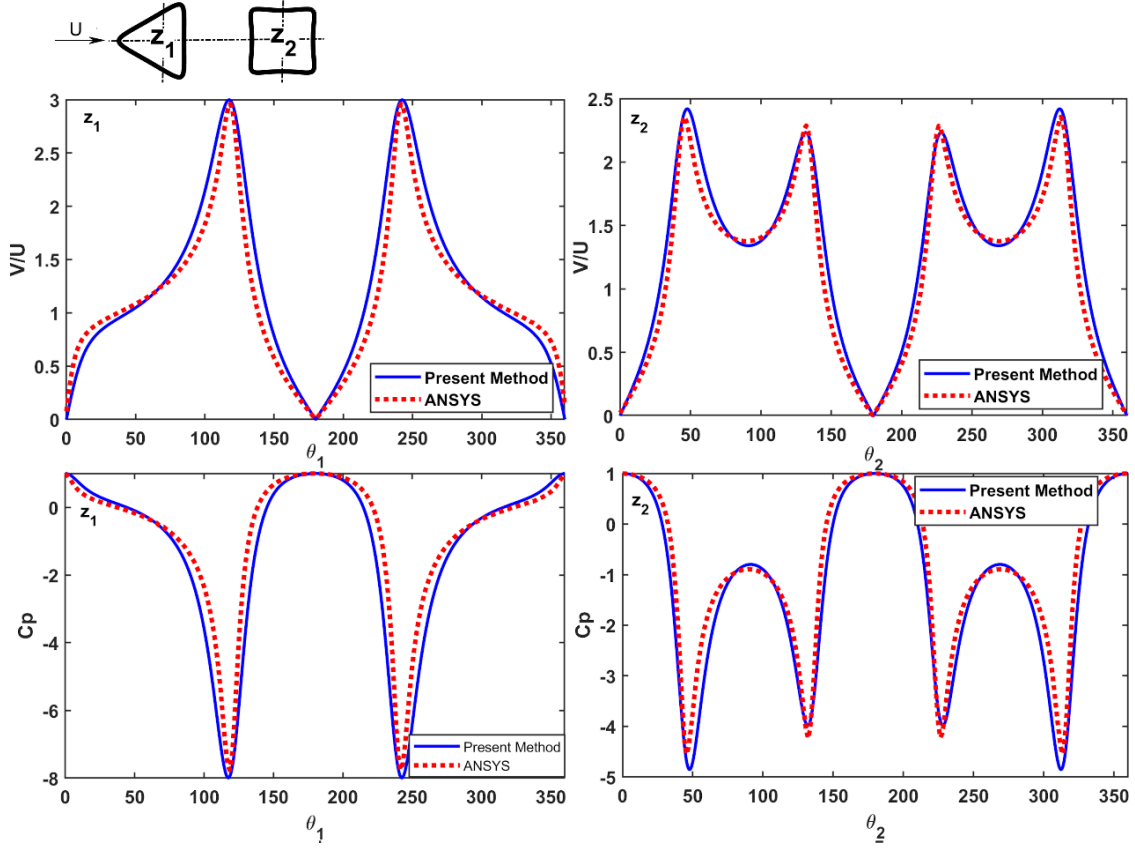


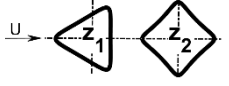
Figure 4.61 Velocity (V/U) and pressure co-efficient (Cp) on the surface interacting triangular and square cylinder is shown for center distances $H = 4$, corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ$. The cylinder pair with same geometrical parameters is obtained in ANSYS using key points exported from MATLAB program of the present method.

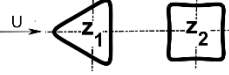
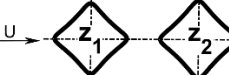
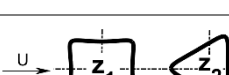
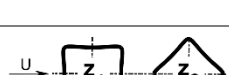
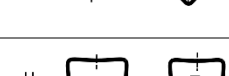
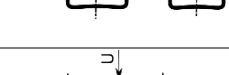
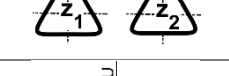
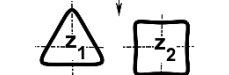
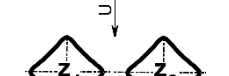
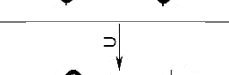
To observe the effect of center distance on velocity distribution, the absolute normalized velocity (V/U) is plotted on the surface of two interacting polygonal cylinders for flow angle $\alpha = 0^\circ$, free stream velocity $U = 1$, corner radii $\rho_1 = \rho_2 = 0.1$ and center distance ranging from $2 < H < 17$ in Figure 4.65 to Figure 4.67. The velocity distribution is plotted only for the upper surface of polygonal cylinders. The velocity distribution on the lower surface of the cylinders will be identical to the one drawn on the upper surface due to the symmetricity of flow and geometries about the center line at $\alpha = 0^\circ$ (refer Figure 4.65 to Figure 4.67).

It can be observed from the figures that the extremum velocities decrease with decrease of center distance (H). After center distance $H \gg 10$, the velocity distribution will remain unaltered as the interaction effect weakens. At larger center distance ($H \gg 10$), the velocity profile is found similar as velocity profile for individual cylinder. In Figure 4.68, the velocity profile around single diamond shaped cylinder ($N = 4, \rho = 0.11, \gamma = 0^\circ$) in uniform potential flow is presented and compared with the velocity profile around diamond shaped cylinder of the same geometry, interacting with pentagonal cylinder ($N = 5, \rho = 0.11, \gamma = 180^\circ$) at center distance $H = 100$. The velocity profiles are found to be identical.

The extremum velocities around two polygonal cylinders for the flow normal to the line joining the centers of cylinders ($\alpha = 90^\circ$) are presented in Table 4.12 for corner radii $\rho_1 = \rho_2 = 0.1$ and center distance $H = 3, 5$, and 10 . It is observed that the extremum values of velocities increase with decrease of center distance (H) between the cylinders.

Table 4.12 Velocity and Pressure co-efficient C_p on the surface of interacting polygonal cylinders for free stream velocity $U = 1$, corner radii $\rho_1 = \rho_2 = 0.1$, center distance $H = 3, 5$, and 10 and flow angles $\alpha = 0^\circ$ and 90° .

Configuration	Center Distance (H)	Cylinder z_1			Cylinder z_2		
		V_{max}/U	$C_{p_{min}}$	θ_1	V_{max}/U	$C_{p_{min}}$	θ_2
	3	3.2929	-9.8436	$118.6^\circ, 241.4^\circ$	3.6535	-12.3483	$61.2^\circ, 298.7^\circ$
	5	3.6226	-12.1234	$118.6^\circ, 241.4^\circ$	3.7250	-12.8757	$61.3^\circ, 298.6^\circ$
	10	3.7729	-13.2349	$118.6^\circ, 241.4^\circ$	3.7870	-13.3421	$61.3^\circ, 298.6^\circ$
	3	3.2929	-9.8437	$118.6^\circ, 241.4^\circ$	3.7162	-12.8100	$89.8^\circ, 270.15^\circ$
	5	3.6226	-12.1234	$118.5^\circ, 241.4^\circ$	3.8639	-13.9296	$89.9^\circ, 270^\circ$
	10	3.7729	-13.2349	$118.6^\circ, 241.4^\circ$	3.9616	-14.6941	$90^\circ, 270^\circ$
	3	3.2929	-9.8436	$118.6^\circ, 241.4^\circ$	2.9910	-4.2859	$132.6^\circ, 227.4^\circ$
	5	3.6226	-12.1234	$118.6^\circ, 241.4^\circ$	7.2421	-6.2421	$133.1^\circ, 226.7^\circ$

	10	3.7729	-13.2349	118.6°, 241.4°	2.8337	-7.0300	133.2°, 226.7°
	3	3.7166	-12.8130	89.8°, 270.1°	3.6535	-12.3483	61.2°, 298.7°
	5	3.8639	-13.9296	89.9°, 270.0°	3.7250	-12.8757	61.3°, 298.6°
	10	3.9612	-14.6914	89.9°, 270.0°	3.7871	-13.3421	61.3°, 298.6°
	3	3.7166	-12.8130	89.8°, 270.1°	3.7166	-12.8130	89.8°, 270.1°
	5	3.8639	-13.9296	89.9°, 270.0°	3.8639	-13.9296	89.9°, 270.0°
	10	3.9612	-14.6914	89.9°, 270.0°	3.9612	-14.6914	89.9°, 270.0°
	3	3.7166	-12.8130	89.8°, 270.1°	2.2991	-4.2859	132.5°, 227.3°
	5	3.8639	-13.9296	89.9°, 270.0°	2.6911	-6.2421	133.1°, 226.8°
	10	3.9612	-14.6914	89.9°, 270.0°	2.8341	-7.0320	133.2°, 226.7°
	3	2.2991	-4.2859	132.61°, 227.3°	3.6535	-12.3483	61.2°, 298.7°
	5	2.6911	-6.2421	133.1°, 226.8°	3.7250	-12.8757	61.3°, 298.6°
	10	2.8341	-7.032	133.2°, 226.7°	3.7871	-13.3421	61.3°, 298.6°
	3	2.2991	-4.2859	132.61°, 227.3°	3.7166	-12.8130	89.8°, 270.1°
	5	2.6911	-6.2421	133.1°, 226.8°	3.8639	-13.9296	89.9°, 270.0°
	10	2.8341	-7.032	133.2°, 226.7°	3.9616	-14.6941	89.9°, 270.0°
	3	2.2991	-4.2859	132.61°, 227.3°	2.2991	-4.2859	132.61°, 227.3°
	5	2.6911	-6.2421	133.1°, 226.8°	2.6911	-6.2421	133.1°, 226.8°
	10	2.8341	-7.032	133.2°, 226.7°	2.8341	-7.032	133.2°, 226.7°
	3	4.4532	-18.8361	207.7°	4.4538	-18.8361	207.7°
	5	4.0144	-15.1155	208.5°	4.0144	-15.1155	208.5°
	10	3.8610	-13.9074	208.6°	3.8610	-13.9074	208.6°
	3	4.4532	-18.8361	207.7°	5.1877	-25.9124	179.9°
	5	4.0144	-15.1155	208.5°	4.2616	-17.1611	179.9°
	10	3.8610	-13.9074	208.6°	4.0499	-15.4016	179.9°
	3	4.4532	-18.8362	207.7°	3.1650	-9.0175	43.4°, 316.5°
	5	4.0144	-15.1155	208.5°	2.9786	-7.8724	43.3°, 316.6°
	10	3.8610	-13.9074	208.6°	2.8989	-7.4036	43.2°, 316.7°
	3	5.1877	-25.9124	179.9°	5.1877	-25.9124	179.9°
	5	4.2557	-17.1116	179.9°	4.2557	-17.1116	179.9°
	10	4.0499	-15.4016	179.9°	4.0499	-15.4016	179.9°
	3	5.1877	-25.9124	179.9°	3.1650	-9.0175	43.4°, 316.5°
	5	4.2557	-17.1116	179.9°	2.9786	-7.8724	43.3°, 316.6°
	10	4.0499	-15.4016	179.9°	2.8989	-7.4036	43.2°, 316.7°
	3	3.1650	-9.0175	43.4°, 316.5°	3.1650	-9.0175	43.4°, 316.5°
	5	2.9786	-7.8724	43.3°, 316.6°	2.9786	-7.8724	43.3°, 316.6°
	10	2.8989	-7.4036	43.2°, 316.7°	2.8989	-7.4036	43.2°, 316.7°

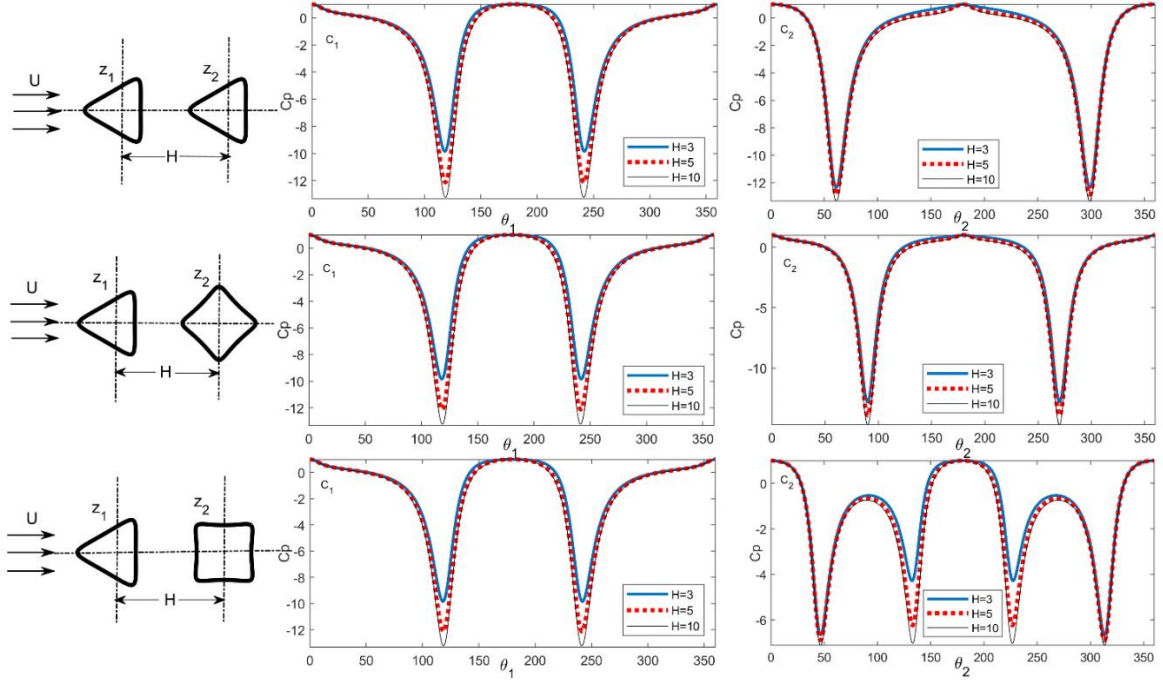


Figure 4.62 Pressure co-efficient (C_p) on the surface of two interacting polygonal cylinders in triangular-triangular, triangular-diamond, and triangular-square cylinder pairs, obtained for center distances $H = 3, 5$, and 10 , corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ$.

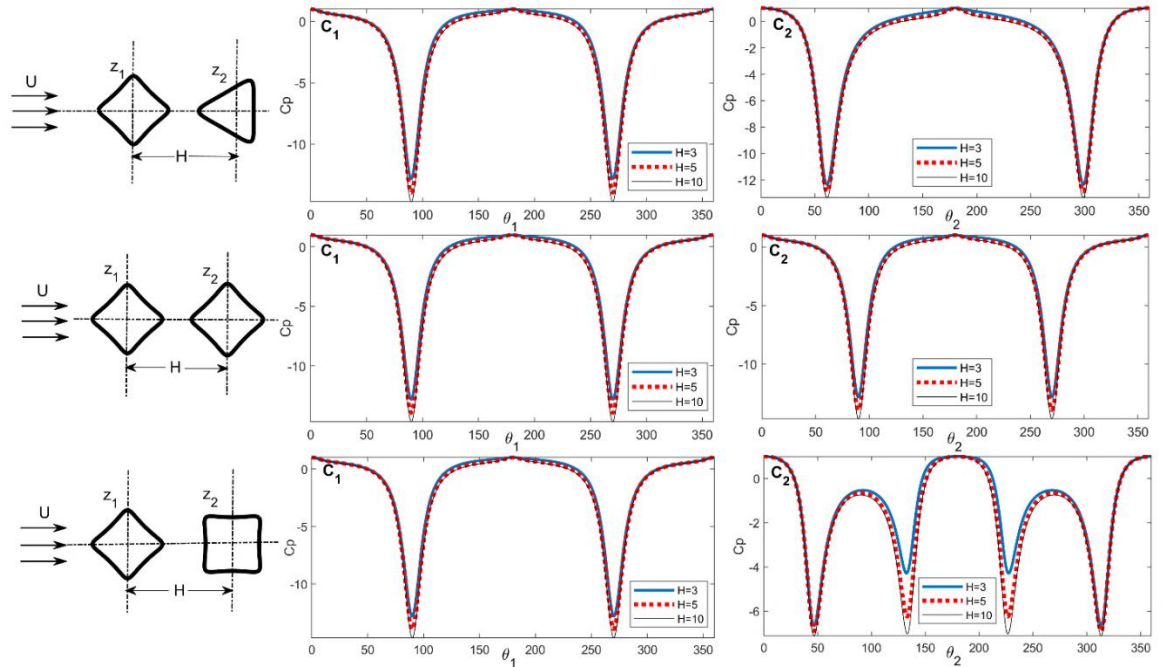


Figure 4.63 Pressure co-efficient (C_p) on the surface of two interacting polygonal cylinders in diamond-triangular, diamond-diamond, and diamond-square cylinder pairs, obtained for center distances $H = 3, 5$, and 10 , corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ$.

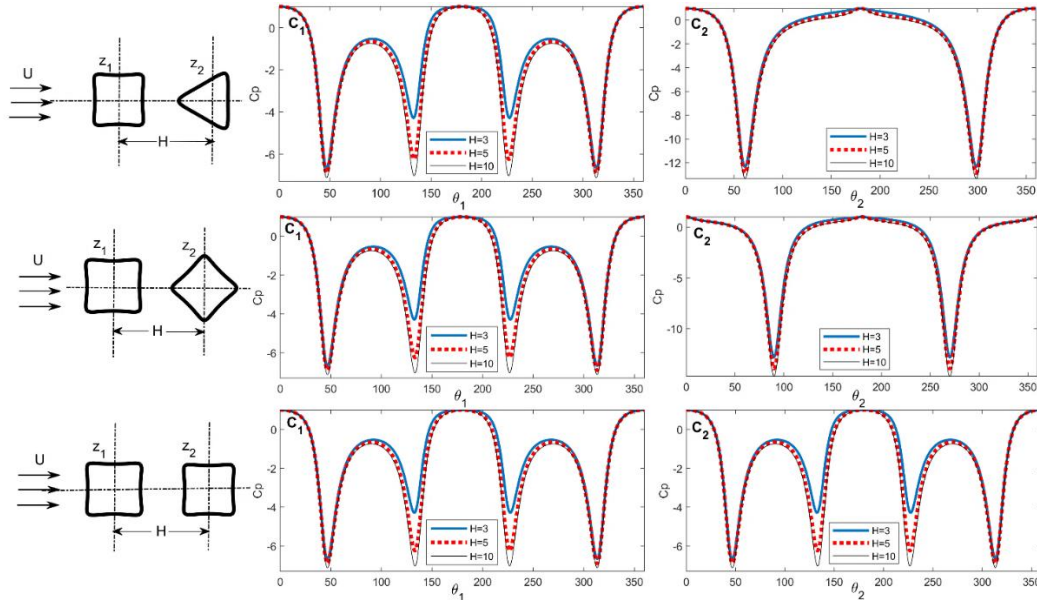


Figure 4.64 Pressure co-efficient (C_p) on the surface of two interacting polygonal cylinders in square-triangular, square -diamond, and square-square pairs, obtained for center distances $H = 3, 5$, and 10 , corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ$.

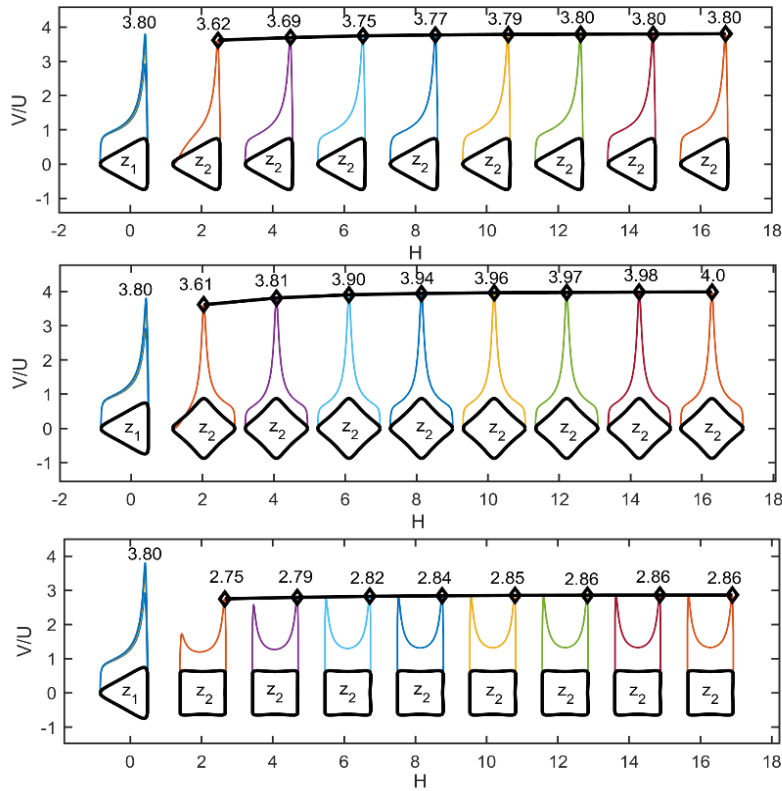


Figure 4.65 Velocity distribution on the surface of interacting polygonal cylinders in triangular-triangular, triangular -diamond, and triangular-square cylinder pairs for different center distances and the corner radii $\rho_1 = \rho_2 = 0.1$

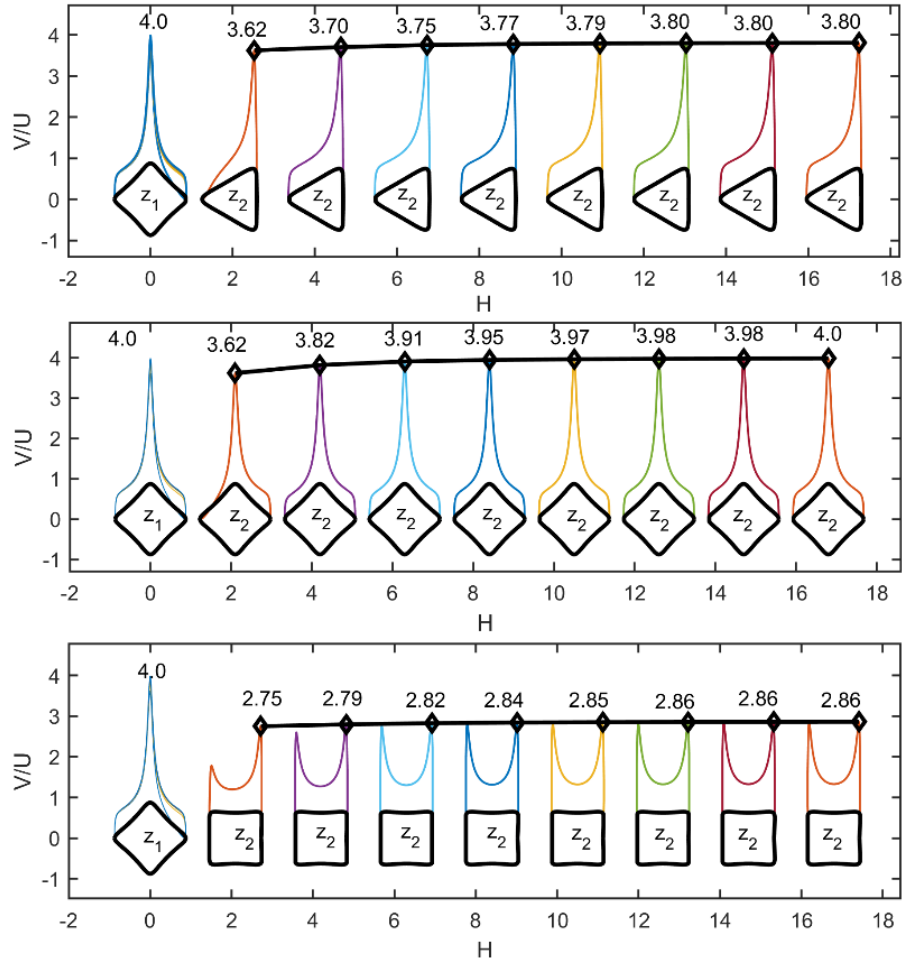


Figure 4.66 Velocity distribution on the surface of interacting polygonal cylinders in diamond-triangular, diamond - diamond, and diamond-square cylinder pairs for different center distances and the corner radii $\rho_1 = \rho_2 = 0.1$

4.6.1.2 Effect of flow angle (α)

To investigate the effect of flow angle on the hydrodynamic parameters, the velocity distributions around the pair of polygonal cylinders are shown in Figure 4.69, Figure 4.70 and Figure 4.71 for the flow angles $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° , center distance $H = 5$, and the corner radii $\rho_1 = \rho_2 = 0.1$.

For the flow angle $\alpha = 90^\circ$, the velocity attains maximum values $V_{max}/U = 4.6524$ on the surface of triangular cylinder (z_2) at $\theta_2=180^\circ$ in triangular-triangular cylinder pair, and $V_{max}/U = 4.5018$ on the surface of triangular cylinder (z_1) at $\theta_1 = 0^\circ$ in triangular-diamond and triangular-square cylinder pairs. In diamond-triangular and square-triangular cylinder pairs, the velocity

attains maximum value $V_{max}/U = 4.631$ at $\theta_2 = 180^\circ$ on the surface of triangular cylinder (z_2), and in diamond-diamond and diamond-square cylinder pairs velocity attains maximum value $V_{max}/U = 4.2306$ on the surface of diamond shaped cylinders (z_1) at $\theta_1 = 180^\circ$ for the same flow angle. In the square-square cylinder pair the velocity attains maximum value $V_{max}/U = 4.0303$ on the surface of both cylinders z_1 and z_2 at $\theta_1 = 45^\circ$ and $\theta_2 = 315^\circ$ respectively for the flow angle $\alpha = 45^\circ$.

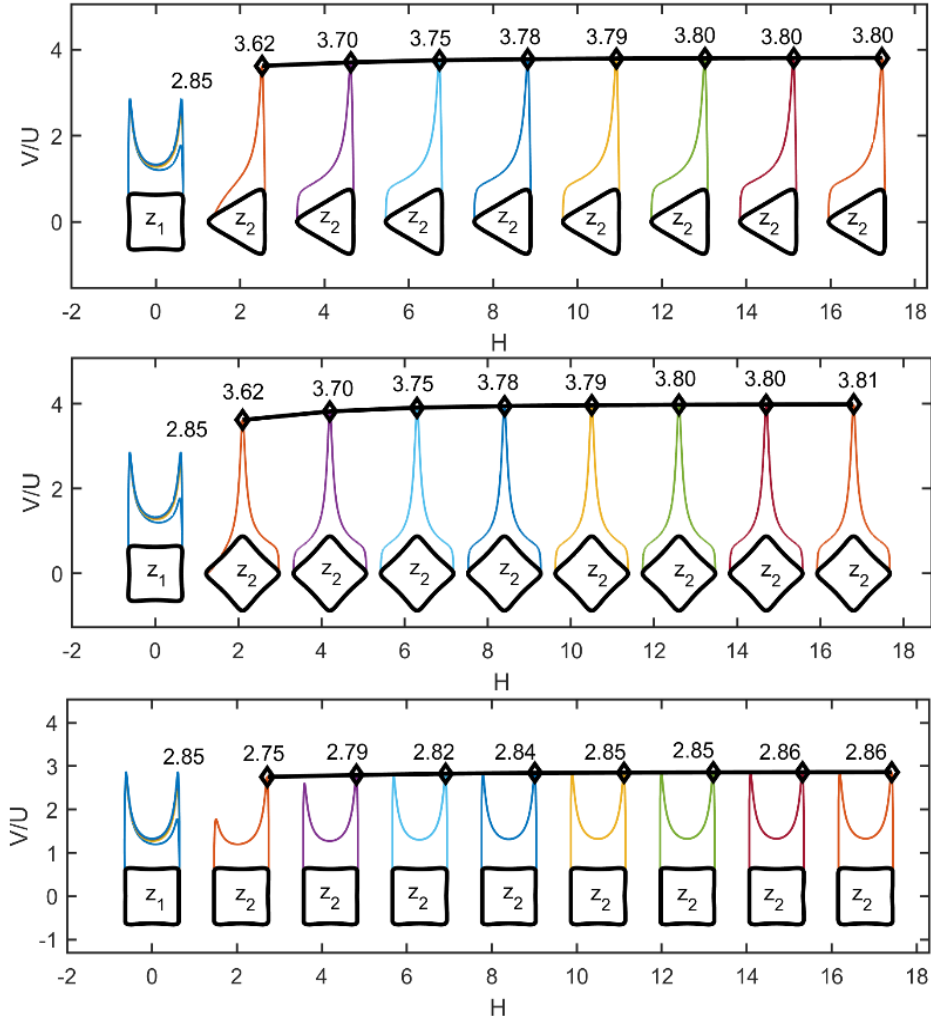


Figure 4.67 Velocity distribution on the surface of interacting polygonal cylinders in square-triangular, square - diamond, and square-square cylinder pairs for different center distances and the corner radii $\rho_1 = \rho_2 = 0.1$

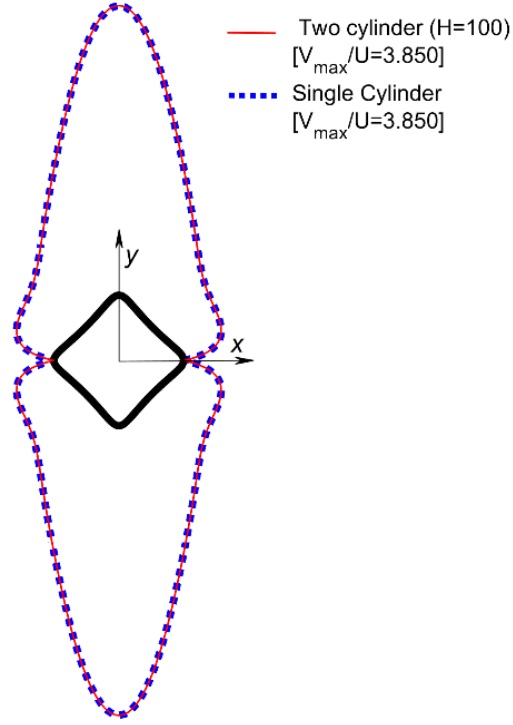


Figure 4.68 Velocity profile on the surface of diamond shaped cylinder ($N_1 = 4, \rho_1 = 0.11, \gamma_1 = 0^\circ$) interacting with pentagonal cylinder ($N_2 = 5, \rho_2 = 0.11, \gamma_2 = 180^\circ$) at center distance $H = 100$ obtained using present method is shown by continues solid line, and compared with the velocity profile on the surface of single diamond shaped cylinder of same geometrical parameters in uniform potential flow is shown by dotted line.

Figure 4.72, Figure 4.73 and Figure 4.74 shows the effect of flow angle (α) on the extremum values of pressure coefficient around polygonal cylinders for the corner radii $\rho_1 = \rho_2 = 0.1$ and center distance $H = 3, 5, 7$ and 10 . The pressure coefficient decreases with increase of flow angle and attains minima at $\alpha = 90^\circ$ on the surface of triangular and diamond shaped cylinders in all configurations under consideration for given finite center distances H . In the case of square cylinder, the pressure coefficient attains the minima at $\alpha = 49^\circ, 47^\circ, 46^\circ$, and 45° for $H = 3, 5, 7$, and 10 respectively. For the decrease of the center distance H , the pressure coefficient (C_p) on the surface of two interacting polygonal cylinders increases for the flow direction along the center line ($\alpha = 0^\circ$) and decreases for the flow direction normal to the center line ($\alpha = 90^\circ$).

Further the effects of the corner radii (ρ_1 and ρ_2), orientation angles (γ_1 and γ_2), and the number of sides (N_1 and N_2) on the flow parameters are investigated and discussed below.

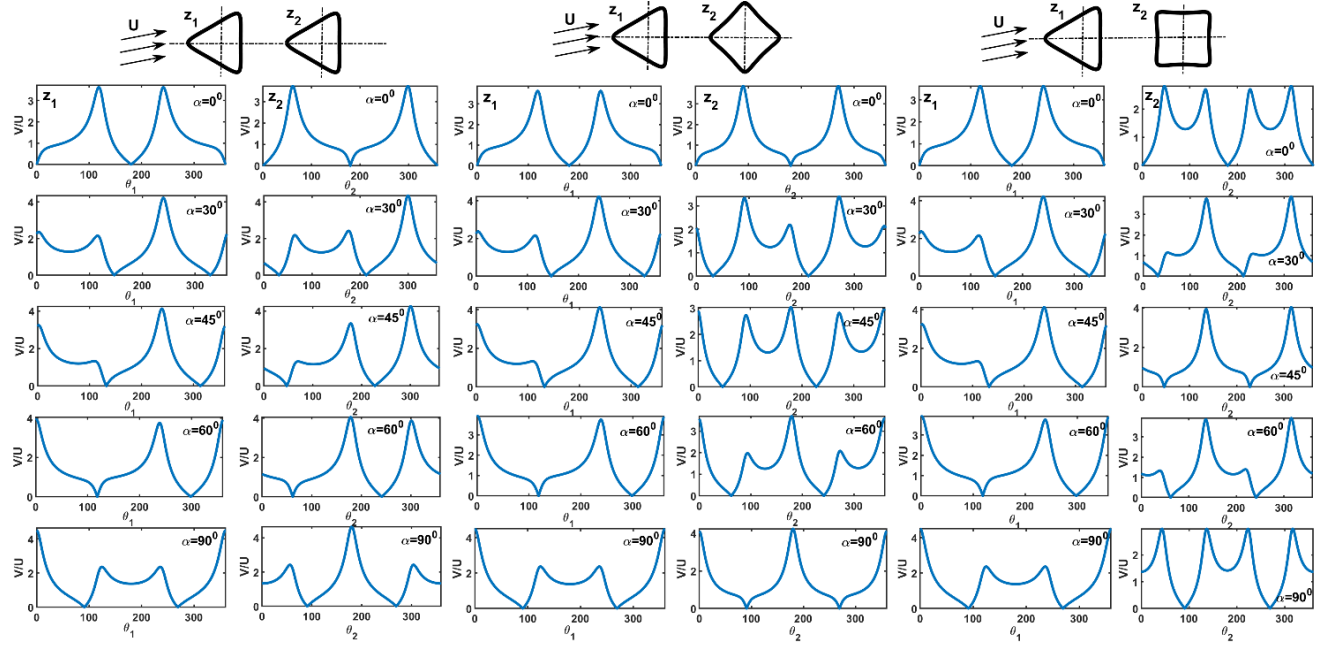


Figure 4.69 Velocity profile on the surface of interacting polygonal cylinders in triangular-triangular, triangular - diamond, and triangular-square cylinder pairs for center distances $H = 5$, corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .

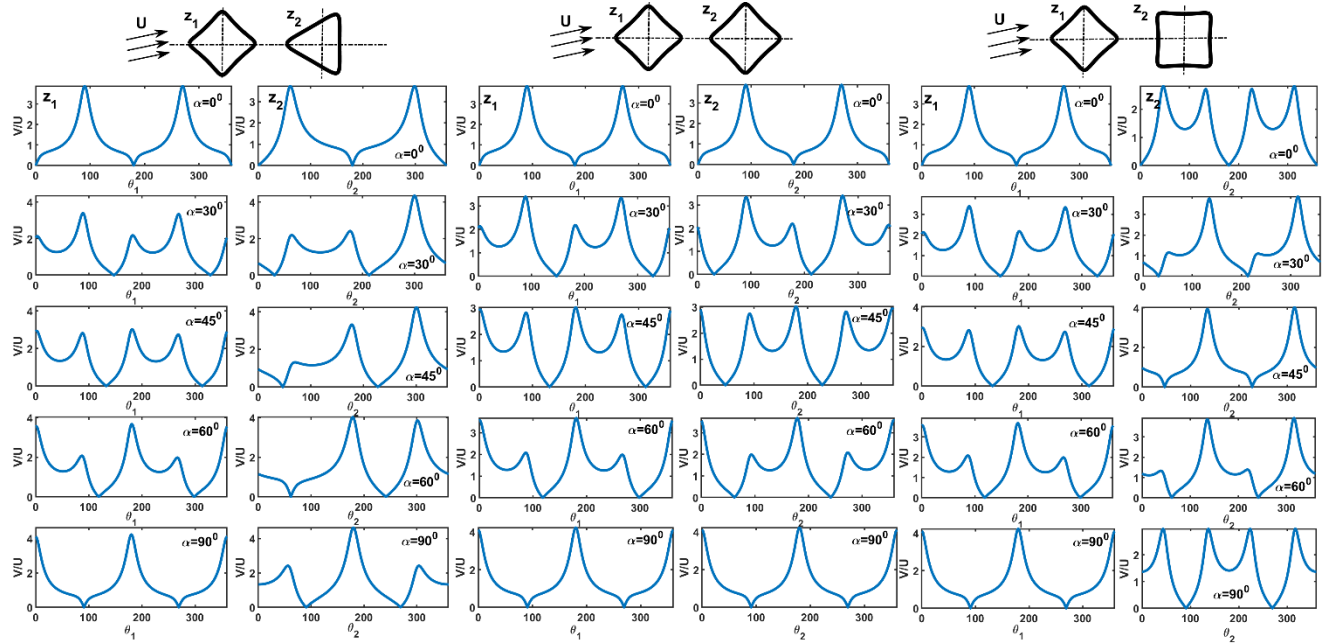


Figure 4.70 Velocity profile on the surface of interacting polygonal cylinders in diamond-diamond, diamond - triangular, and diamond-square cylinder pairs for center distances $H = 5$, corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .

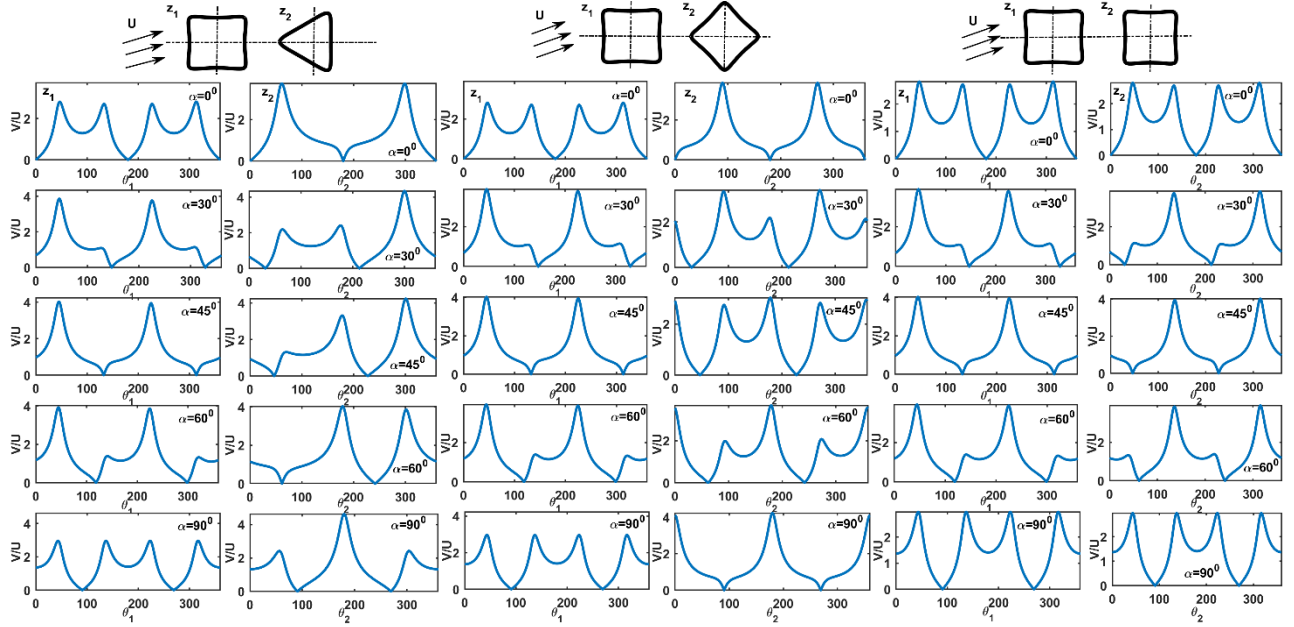


Figure 4.71 Velocity profile on the surface of interacting polygonal cylinders in square-triangular, square-diamond, and square-square cylinder pairs for center distances $H = 5$, corner radii $\rho_1 = \rho_2 = 0.1$, and flow angle $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .

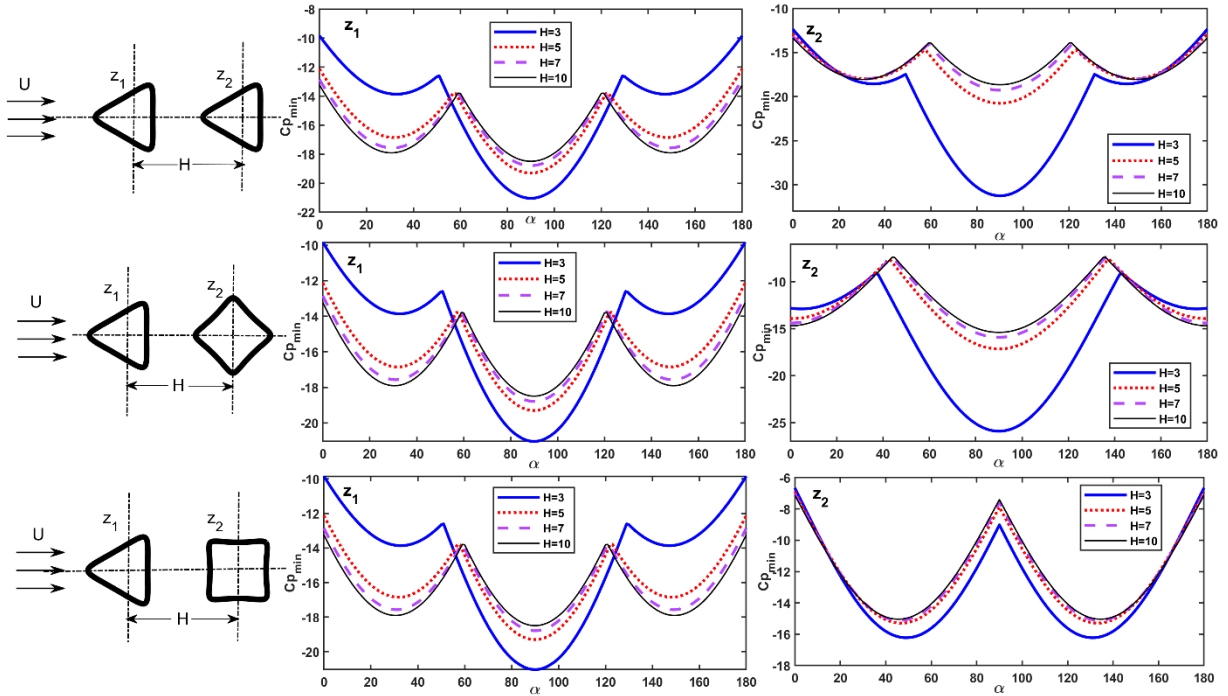


Figure 4.72 Minimum pressure co-efficient (Cp_{min}) on the surface of interacting polygonal cylinders in triangular-triangular, triangular -diamond, triangular -square cylinder pairs for corner radii $\rho_1 = \rho_2 = 0.1$, angle of attack $0^\circ \leq \alpha \leq 180^\circ$, and center distances $H = 3, 5, 7$ and 10 .

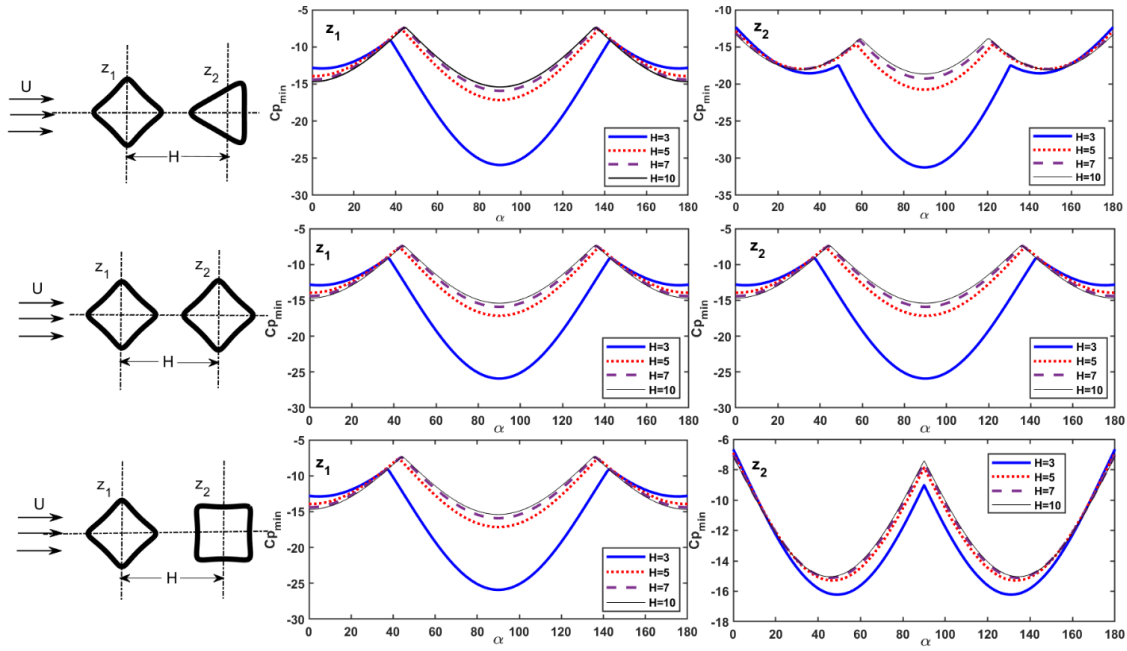


Figure 4.73 Minimum pressure co-efficient ($C_{p_{min}}$) on the surface of interacting polygonal cylinders in diamond-triangular, diamond-diamond, diamond-square cylinder pairs for corner radii $\rho_1 = \rho_2 = 0.1$, angle of attack $0^\circ \leq \alpha \leq 180^\circ$, and center distances $H = 3, 5, 7$ and 10 .

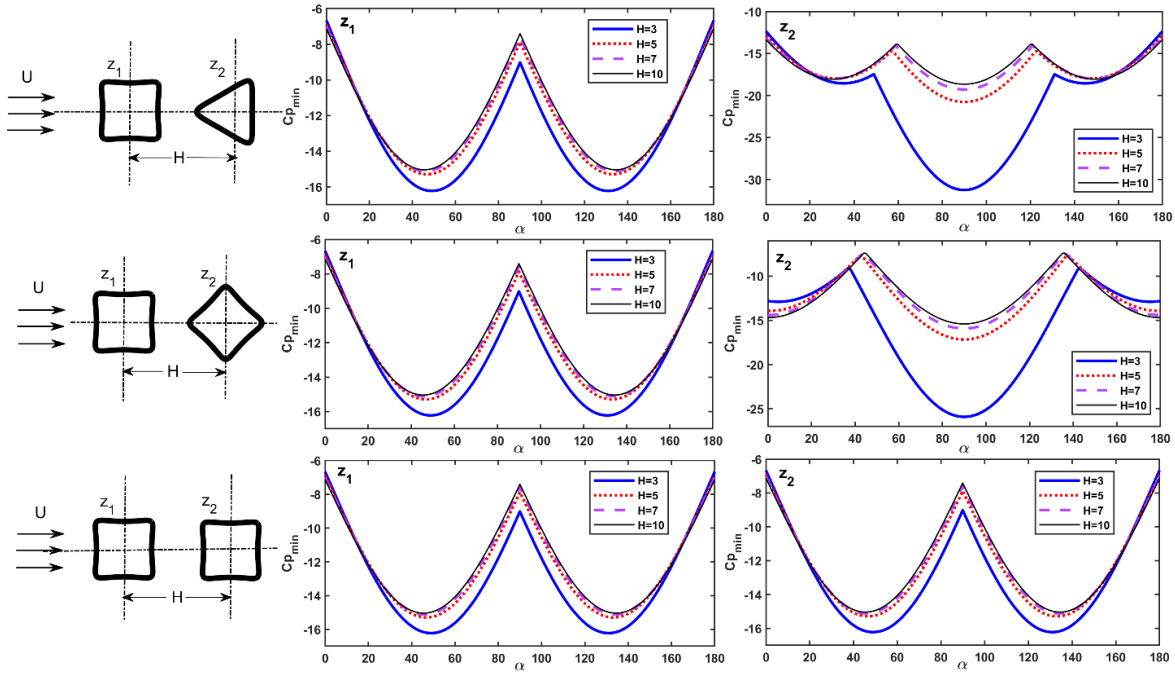


Figure 4.74 Minimum pressure co-efficient ($C_{p_{min}}$) on the surface of interacting polygonal cylinders in square-triangular, square-diamond, square-square cylinder pairs for corner radii $\rho_1 = \rho_2 = 0.1$, angle of attack $0^\circ \leq \alpha \leq 180^\circ$, and center distances $H = 3, 5, 7$ and 10 .

4.6.1.3 Effect of orientation angles

The effect of cylinders orientation angles (γ_1 and γ_2) on the hydrodynamic parameters are investigated for corner radii $\rho_1 = \rho_2 = 0.1$, flow angle $\alpha = 0^\circ$, and center distances $H = 3$. The velocity profiles around triangular-triangular, triangular-diamond, and diamond-diamond cylinder pairs are shown in Figure 4.75 for different orientation angles $\gamma_1 = \gamma_2 = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° . The orientation angles are measured from positive real axis in counter clockwise direction (refer Figure 3.3). The absolute velocities on the upper surface of polygonal cylinders are shown by the solid lines and on the lower surface of cylinders are shown by dotted lines.

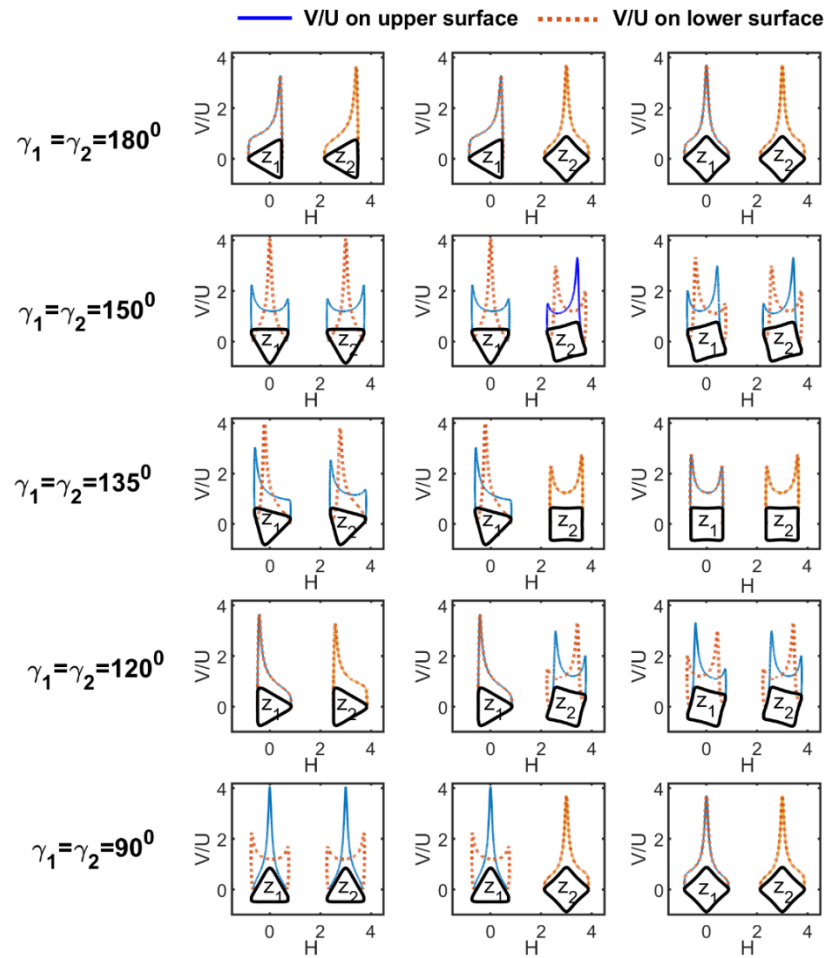


Figure 4.75 Velocities (V/U) on the surface of interacting polygonal cylinders are investigated for corner radii $\rho_1 = \rho_2 = 0.1$, flow angle $\alpha = 0^\circ$, center distance $H = 3$ and orientation angles $\gamma_1 = \gamma_2 = 180^\circ, 150^\circ, 135^\circ, 120^\circ$ and 90° .

The velocity distributions on the upper and lower surface of polygonal cylinders are found identical for $\gamma_1 = \gamma_2 = 0^\circ$ and 180° for flow angle $\alpha = 0^\circ$. The magnitude and locations of the maximum velocities (V_{max}/U) are significantly affected by the orientation angles (γ_1 and γ_2). In triangular-triangular and triangular-diamond cylinder pairs, the extremum values of velocities are observed for the orientation angles $\gamma_1 = \gamma_2 = 90^\circ$ on the surface of triangular cylinder at one of the vertices. In diamond-diamond cylinder pair the velocity attains maxima for the orientation angles $\gamma_1 = \gamma_2 = 90^\circ$ and 180° at two of the vertices.

4.6.1.4 Effects of Number of sides of polygonal cylinders

The effect of number of sides of polygonal cylinder on velocity and pressure coefficient is shown in Figure 4.76 for the corner radii $\rho_1 = \rho_2 = 0.1$, flow angle $\alpha = 0^\circ$, center distances $H = 5$ and cylinder orientation angles $\gamma_1 = \gamma_2 = 180^\circ$. The number of sides of upstream cylinder (z_1) is kept constant $N_1 = 3$ and number of sides of downstream cylinder (z_2) is chosen as $N_2 = 5, 10, 30, 50$ and 500. The velocity profiles and pressure profiles corresponding to the polygonal cylinders are shown by solid lines and that corresponds to the circular cylinder are shown by dotted line.

The extremum values of velocity and pressure coefficient at the corners of downstream polygonal cylinder (z_2) decreases as the number of sides N_2 increases. For $N_2 \rightarrow \infty$, the velocities and pressure coefficients converge to the values that corresponds to the circular cylinder.

The variation of velocities and pressure coefficients at the corners of polygonal cylinder (z_2) from values that correspond to the circular cylinder are different at different angular location θ_2 . The maximum variation is observed at the corners located at or near to the angular location $\theta_2 = 90^\circ$ and $\theta_2 = 270^\circ$ for flow angle $\alpha = 0^\circ$.

4.6.1.5 Effect of Corner radii of cylinders

The hydrodynamic interaction between two polygonal cylinders and associated flow parameters are significantly affected by the corner radii of cylinders. To investigate the effect of corner radii on flow parameters, the velocity (V/U) is plotted on the surface of diamond shaped cylinder interacting with triangular cylinder for different corner radii $\rho_1 = \rho_2 = 0.01, 0.05, 0.1, 0.3, 0.5, 0.8$ and 1, the center distance $H = 3$, and flow angle $\alpha = 0^\circ$ as shown in Figure 4.77. The velocity on the surface of cylinder decreases with increase of the corner radii (ρ_1

and ρ_2) due to deceleration of fluid at the corners of cylinders. At $\rho_1 = \rho_2 = 1$, the velocity attains values equal to that corresponds to two interacting circular cylinders in uniform potential flow for the same flow angle (α) and center distance (H). As the pressure co-efficient is related to the velocity by Bernoulli's relation (Equation (3.66)), the pressure drops on the surface of cylinders are also decrease with increase of corner radii.

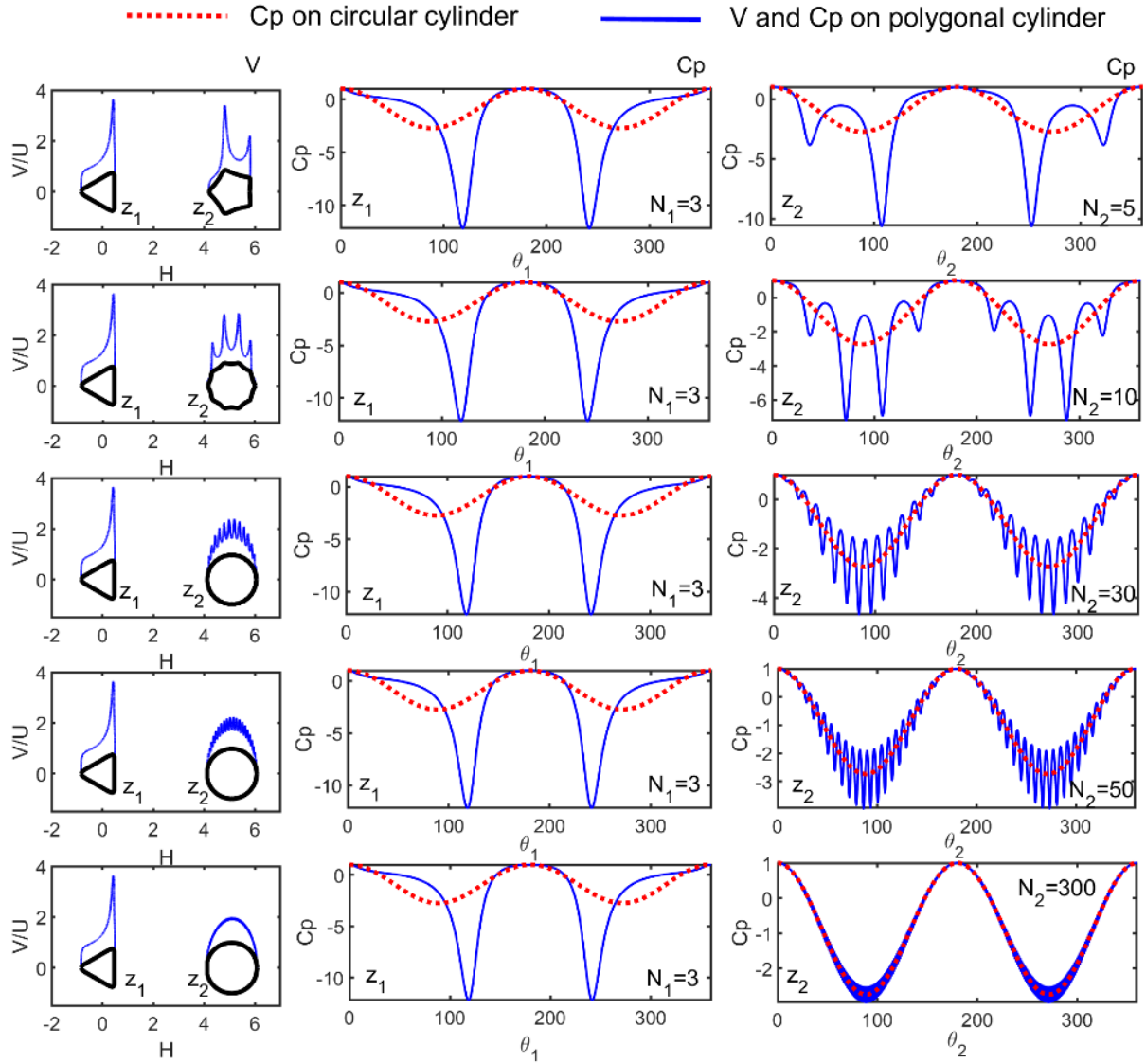


Figure 4.76 The effect of number of sides N_2 of polygonal cylinder (z_2) on the velocity (V/U) and pressure (C_p) distribution for corner radii $\rho_1 = \rho_2 = 0.1$, flow angle $\alpha = 0^\circ$, center distances $H = 5$ and the orientation angles $\gamma_1 = \gamma_2 = 180^\circ$.

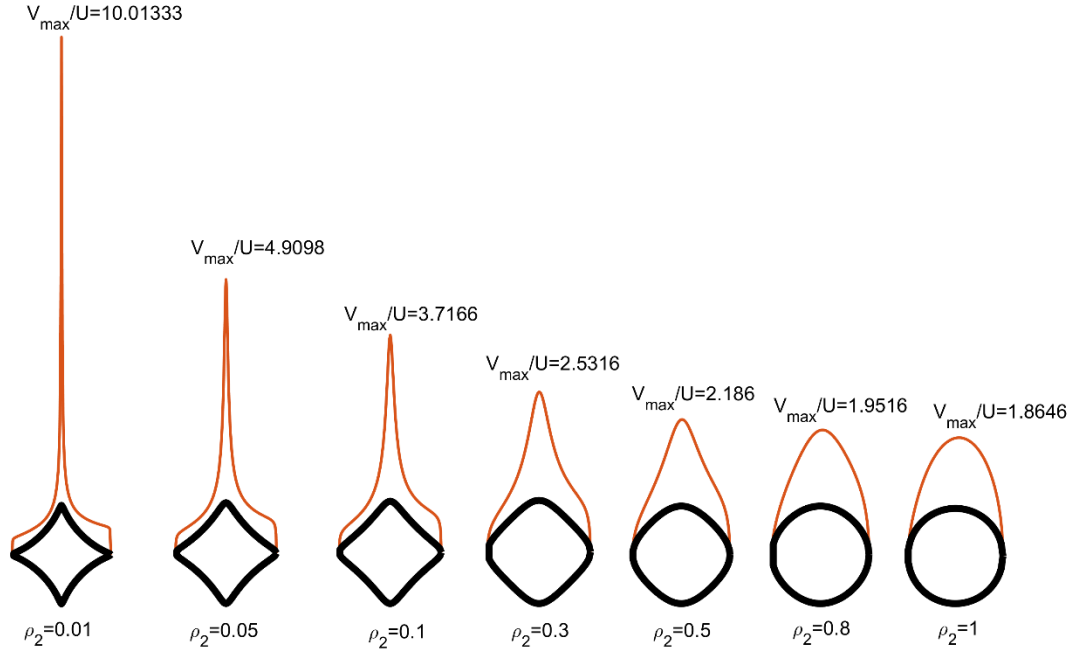


Figure 4.77 Velocity distribution on the surface of diamond shaped cylinder interacting with triangular cylinder for the center distance $H = 3$, flow angle $\alpha = 0^\circ$ and, corner radii $\rho_1 = \rho_2 = 0.01, 0.05, 0.1, 0.3, 0.5, 0.8$ and 1 .

For the corner radii $\rho_1 = \rho_2 = 0$, the cylinders are transformed to the hypocycloidal cylinders with the cusps. For the cusps not aligned to the flow directions, the flow parameters attain infinite values.

4.6.2 Hydrodynamic Forces on Cylinders

The proximity of two cylinders alters the velocity and pressure field around the cylinders significantly as discussed and shown in previous section. The altered non-symmetric pressure distribution around the cylinders induces the hydrodynamic forces on the cylinders. The hydrodynamic forces acting on the cylinders during hydrodynamic interactions in uniform potential flow are computed by integrating the pressure field on the cylinder surface using Blasius theorem.

4.6.2.1 Effect of center distance and corner radius

For the flow direction along the line connecting the centres of cylinders ($\alpha = 0^\circ$), the velocity between two interacting polygonal cylinders decreases, due to which the pressure between the

cylinders increases (refer Table 4.12). The increased pressure between two-cylinders results in repelling force between them along the centreline. For the flow direction normal to the centreline ($\alpha = 90^\circ$), the velocity of flow between two interacting polygonal cylinders increases and the pressure decreases, which results in attracting forces between the cylinders along the centreline. The magnitude and direction of induced hydrodynamic forces acting on the polygonal cylinders are dependent on the center distance (H), corner radii (ρ_1 and ρ_2), orientation angles (γ_1 and γ_2) and flow angle (α).

Figure 4.78 shows the hydrodynamic force acting on the polygonal cylinders for center distance $2.5 \leq H \leq 40$, corner radii $\rho_1 = \rho_2 = 0.01, 0.1$ and 1 , and the flow angle $\alpha = 0^\circ$. The hydrodynamic forces are represented in terms of non-dimensional force coefficient C_{fx} and C_{fy} for the forces acting in the x and y directions, respectively. The hydrodynamic forces acting on cylinders increases with decrease of center distance (H). At a large center distance ($H \geq 10$), the forces acting on cylinders converge to zero and the interaction effect die out.

The effects of corner radii (ρ_1 and ρ_2) on the hydrodynamic forces can also be observed from Figure 4.78. For the given finite center distance ($H < 10$), the hydrodynamic forces acting on upstream and downstream cylinders in different configurations are found maximum for the corner radii $\rho_1 = \rho_2 = 0.01$. For corner radii $\rho_1 = \rho_2 = 1$, the polygonal cylinders are transformed into the circular cylinders and hydrodynamic forces of equal magnitude act on them.

4.6.2.2 Effect of flow incidence angle

The effect of flow angle (α) on the hydrodynamic forces acting on the polygonal cylinders in uniform potential flow is shown in Figure 4.79, Figure 4.80 and Figure 4.81 for the corner radii $\rho_1 = \rho_2 = 0.1$, and center distances $H = 3, 4, 5, 6$, and 10 . The following observations are made from Figure 4.79, Figure 4.80 and Figure 4.81.

1. For the flow angle $\alpha = 0^\circ$ and $\alpha = 180^\circ$, the hydrodynamic forces are found to be acting in $-x$ direction on the cylinder z_1 and $+x$ direction on the cylinder z_2 , i.e. forces make cylinders to repel each other.
2. For the given finite center distance (H) between the cylinders, as the flow angle increases (starting from zero), the magnitude of hydrodynamic forces approaches to zero and at some

threshold value of flow angle (α_1) it alters the directions. This flow angle (α_1) at which the hydrodynamic force changes its direction is the first change point (α_1), and its values depend on the cross-sectional properties of the cylinders. For center distance $H = 3$, the magnitude of hydrodynamic forces achieves zero values at $\alpha_1 = 42^\circ$ on cylinder z_1 and, at $\alpha_1 = 31^\circ, 34^\circ$ and 38° on cylinder z_2 in triangular-triangular, triangular-diamond and triangular-square cylinder pairs, respectively.

3. As the flow angle (α) increases beyond the change point (α_1), the magnitude of hydrodynamic forces increases and attains maximum values at $\alpha = 90^\circ$. After $\alpha = 90^\circ$, as flow angle increases the hydrodynamic forces again shows reduction in magnitude and attains zero values at some flow angle α_2 . This flow angle, α_2 is the second change point. For center distance $H = 3$, the values of $\alpha_2 = 138^\circ$ for cylinder z_1 and, $\alpha_2 = 149^\circ, 146^\circ$ and 142° for cylinder z_2 are observed in triangular-triangular, triangular-diamond and triangular-square cylinder pairs, respectively.
4. At flow angle $\alpha = 90^\circ$, hydrodynamic forces acting on the cylinders takes maximum value due to extremum Venturi effect.
5. For the given geometry and the center distance (H), the hydrodynamic forces are attractive in nature for the flow angles varying from α_1 to α_2 . For all other flow angles, the hydrodynamic forces are repulsive in nature.
6. The maximum attractive forces on the cylinders are observed at $\alpha = 90^\circ$ i.e. flow perpendicular to line joining centers of the cylinders, and maximum repulsive forces are observed at $\alpha = 0^\circ$, i.e. flow parallel to line joining centers of the cylinders.

For various other configurations and center distances (H), the value of flow angles (α) for which the hydrodynamic forces change their nature are presented in Table 4.13.

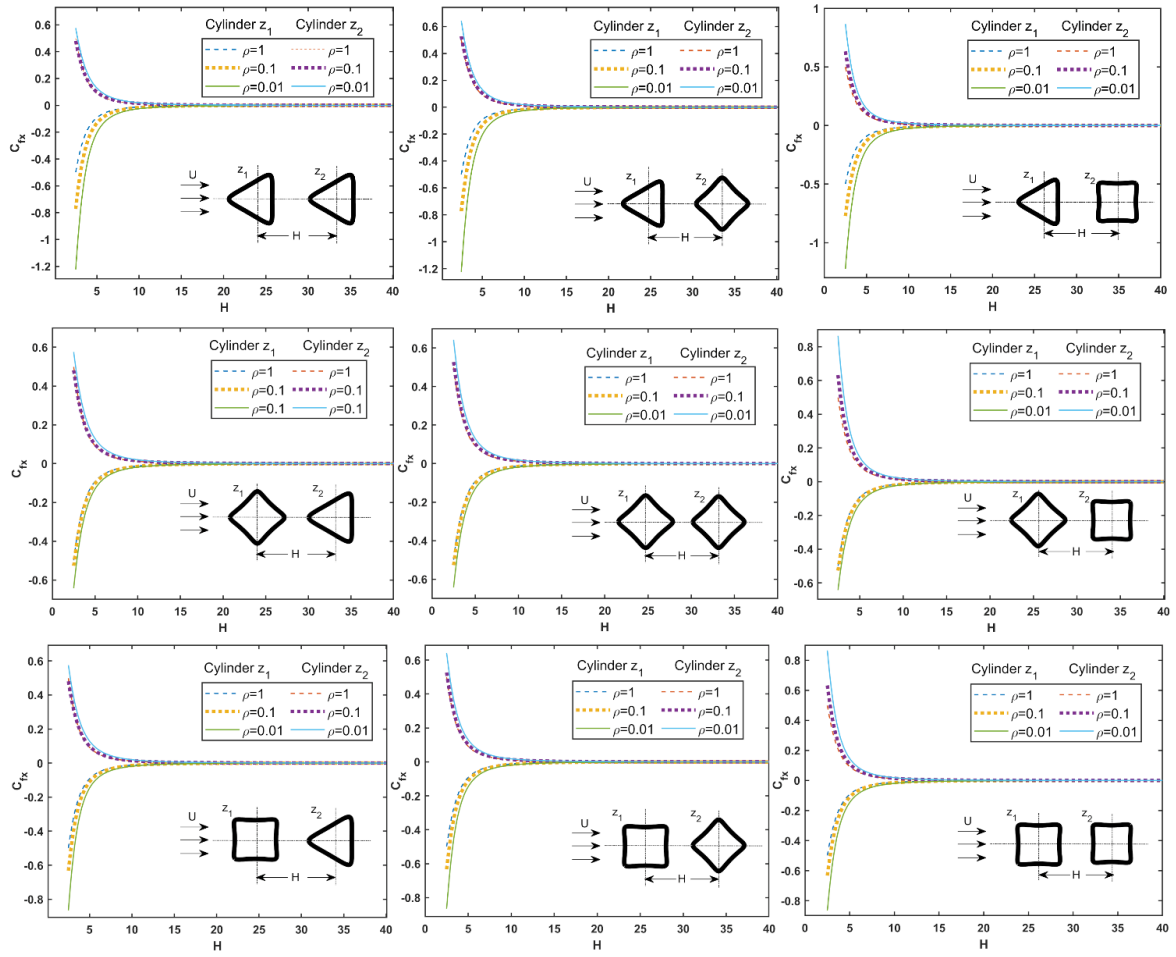


Figure 4.78 Hydrodynamic force (C_{fx}) acting on polygonal cylinders during interaction for center distance $2.5 \leq H \leq 40$, flow angle $\alpha = 0^\circ$, corner radii $\rho_1 = \rho_2 = 0.01, 0.1$ and 1 .

Table 4.13 The flow angles α_1 and α_2 at which the hydrodynamic force acting on polygonal cylinder z_1 and cylinder z_2 along the x -axis attain zero for center distance $H = 3, 4, 5, 6$ and 10 , and corner radii $\rho_1 = \rho_2 = 0.1$ of polygonal cylinders.

Geometry	Change points	Center distances (H)									
		$H = 3$		$H = 4$		$H = 5$		$H = 6$		$H = 10$	
		(z_1)	(z_2)	(z_1)	(z_2)	(z_1)	(z_2)	(z_1)	(z_2)	(z_1)	(z_2)
	α_1	42°	31°	45°	36°	46°	39°	46°	40°	46°	43°
	α_2	138°	149°	135°	144°	134°	141°	134°	140°	134°	137°
	α_1	42°	34°	45°	39°	46°	42°	46°	43°	46°	44°
	α_2	138°	146°	135°	141°	134°	138°	134°	137°	134°	136°

	α_1	42^0	38^0	45^0	42^0	46^0	43^0	46^0	44^0	46^0	45^0
	α_2	138^0	142^0	135^0	138^0	134^0	137^0	134^0	136^0	134^0	135^0
	α_1	34^0	31^0	39^0	36^0	42^0	43^0	43^0	40^0	44^0	43^0
	α_2	146^0	149^0	141^0	144^0	138^0	138^0	137^0	140^0	136^0	137^0
	α_1	34^0	34^0	39^0	39^0	42^0	42^0	43^0	43^0	44^0	44^0
	α_2	146^0	146^0	141^0	141^0	138^0	138^0	137^0	137^0	136^0	136^0
	α_1	34^0	38^0	39^0	42^0	42^0	43^0	43^0	44^0	44^0	45^0
	α_2	146^0	142^0	141^0	138^0	138^0	137^0	137^0	136^0	136^0	135^0
	α_1	38^0	31^0	42^0	36^0	43^0	40^0	44^0	40^0	45^0	43^0
	α_2	142^0	149^0	138^0	144^0	137^0	140^0	136^0	140^0	135^0	137^0
	α_1	38^0	34^0	38^0	39^0	43^0	42^0	44^0	43^0	45^0	44^0
	α_2	142^0	146^0	142^0	141^0	137^0	138^0	136^0	137^0	135^0	136^0
	α_1	38^0	38^0	38^0	38^0	43^0	43^0	44^0	44^0	45^0	45^0
	α_2	142^0	142^0	142^0	142^0	137^0	137^0	136^0	136^0	135^0	135^0

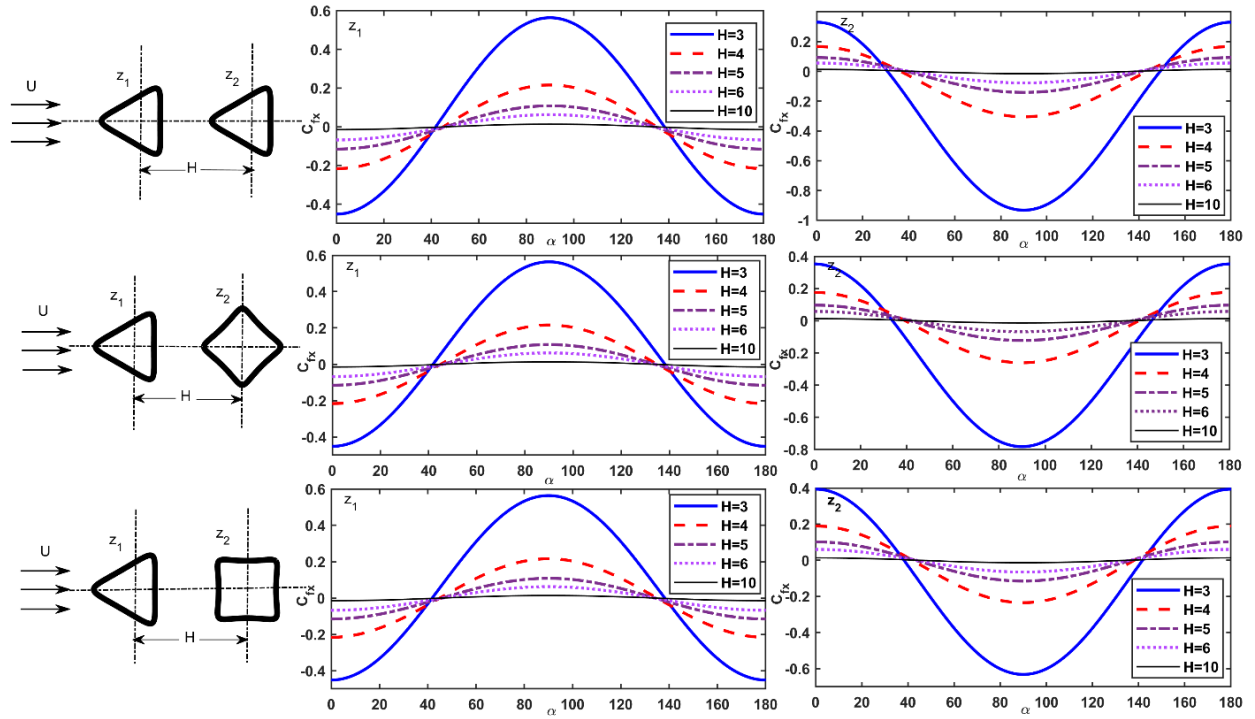


Figure 4.79 Effect of flow angle ($0^0 \leq \alpha \leq 180^0$) on hydrodynamic force (C_{fx}) acting on cylinders in triangular-triangular, triangular-diamond, triangular-square cylinder pairs for center distance $H = 3, 4, 5, 6$, and 10 , and corner radii $\rho_1 = \rho_2 = 0.1$.

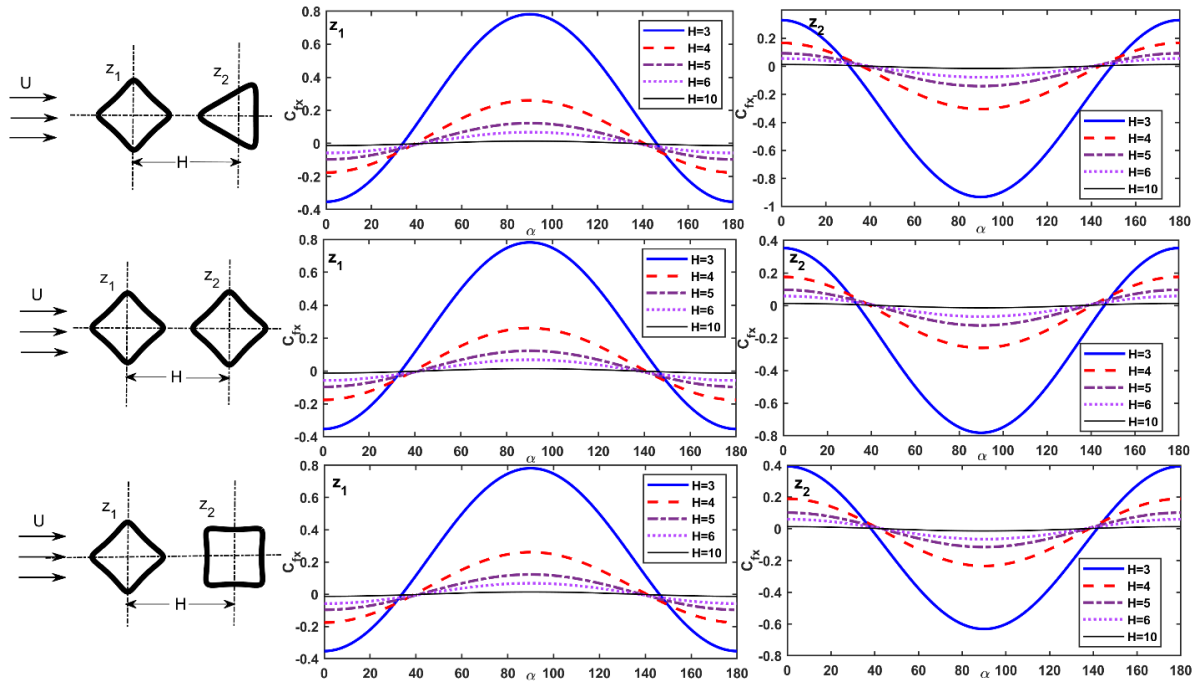


Figure 4.80 Effect of flow angle ($0^\circ \leq \alpha \leq 180^\circ$) on hydrodynamic force (C_{fx}) acting on cylinders in diamond-triangular, diamond-diamond, diamond-square cylinder pairs for center distance $H = 3, 4, 5, 6$, and 10, and corner radii $\rho_1 = \rho_2 = 0.1$.

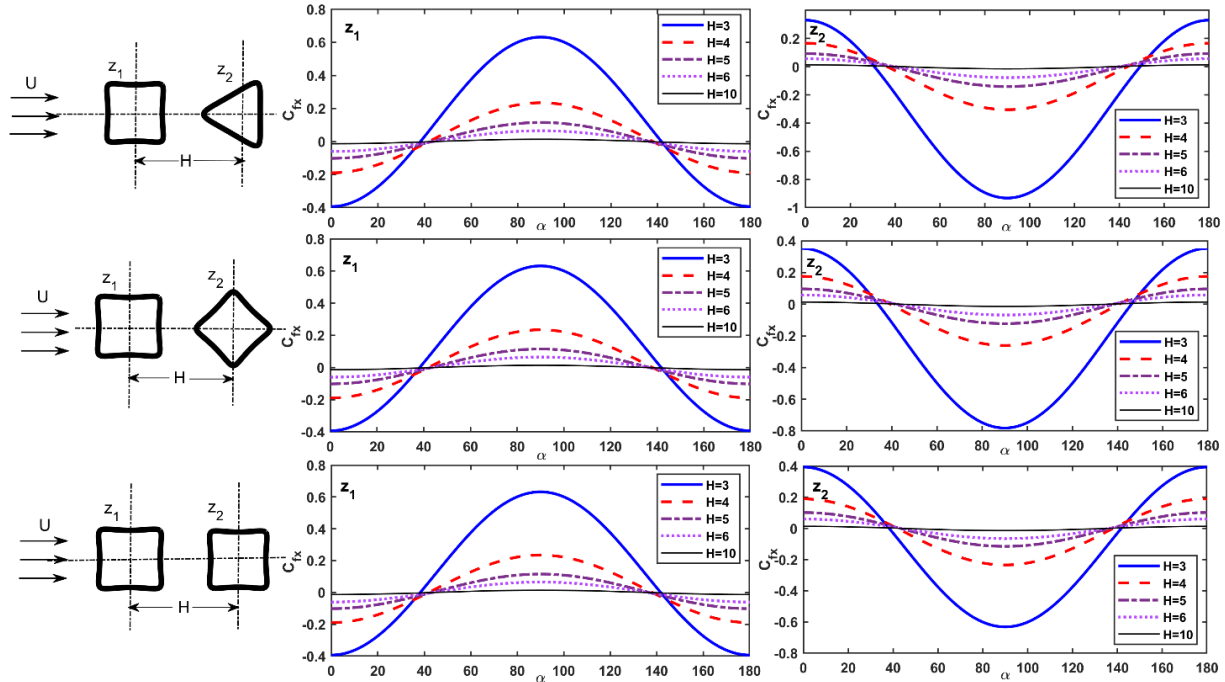


Figure 4.81 Effect of flow angle ($0^\circ \leq \alpha \leq 180^\circ$) on hydrodynamic force (C_{fx}) acting on cylinders in square-triangular, square-diamond, square-square cylinder pairs for center distance $H = 3, 4, 5, 6$, and 10, and corner radii $\rho_1 = \rho_2 = 0.1$.

4.7 Closing Remark

The results of potential flow around polygonal cylinder and the effect of various geometrical parameters of cylinder on the velocity and pressure field around the cylinder is discussed in this chapter. Also, the hydrodynamic interaction between two circular cylinders, polygonal and circular cylinder and between two polygonal cylinders is discussed for various geometrical and flow conditions. The center distance between the cylinders, orientation angles of cylinders, size and shape of cylinders, and the flow angle significantly affects the velocity and pressure around the cylinders, and the hydrodynamic forces acting on the cylinders while interacting in fluid flow.