Chapter 4

Decay of Boson Stars in Galactic Magnetic Field

Although the QCD axions were initially proposed to solve the strong-CP problem [129, 130, 131], they are also regarded as one of the well-motivated candidates for the cold-dark-matter(CDM) in the field of cosmology. The non-relativistic cold dark-matter is required in the standard ΛCDM cosmology model for structure formation [132]. Axion dynamics are described by a hypothetical scalar field whose mass, decay-properties and interaction with other standard model particles are not properly understood and there exist several bounds on their values [133]. A significant production of axions can happen in a primordial Universe under various physical scenarios. One such well-known scenario is the mis-alignment mechanism [134]where coherent oscillations of the axion-fields begin at the time of QCD phase-transition. In the other popular scenario where a relaxation of the string network at Pecci-Quinn phase transition, at temperature different than the QCD phase transition scale, can lead to a significant axion production [**?**]. Besides QCD-axions, "so called" axion-like particles (ALPs) are also considered to be a suitable candidate for the cold-dark matter in the Universe [for a review see [135]]. Dynamics of ALPs are also described by a pseudo-scalar field which naturally arises in Kaluza-Klein and Superstring theories [136]. ALPs are weakly coupled to the standard model particles and their coupling with photons are described by the interaction Lagrangian [please see for example [137]] which has a form similar to that of the axion-photon interaction:

$$\mathcal{L} = g_{a\gamma\gamma}\mathbf{E}\cdot\mathbf{B}a \tag{4.0.1}$$

where $g_{a\gamma\gamma}$ is the axion-photon coupling constant while $\mathbf{E}\,\&\,\mathbf{B}$ respectively denote the three-vectors describing electric and magnetic field strengths and a is either an ALP or axion field. It is quite likely that a self-gravitating cloud of axions/ALPs can thermalise through their own gravitational interaction by forming a Bose-Einstein condensate (BEC) and form a Bose star [138]. Indeed the numerical simulations in Ref.[139, 89] shows a formation of a Bose star for a self-gravitating initial cloud of axions/ALPs with a randomly distributed matter density in the space.

As QCD axions or ALPs can interact with electromagnetic field and decay in to photons, there exists several realistic scenarios in astrophysical situations where it may be possible to detect their existence[[140], see also [141]]. An oscillating axion field in the presence of an external magnetic field can drive an electric field and likewise, an oscillating electric field in the presence of a magnetic field can produce axions or ALPs. This has lead to possibilities of discussing the stability of axionic/ALP dark matter and studying the signature of axions. For example in the case of pulsar the cold-collisionless plasma environment is created around the neutron star according the Goldreich-Julian model [142]. This together with the intrinsic magnetic field of pulsar can produce a detectable signature of axion decay if its mass is comparable to the plasma frequency [143, 144]. In [145] it was shown that the axion created during the QCD-hadron phase transition would survive (remain stable) till the late time. In this study the authors perform linear mode analysis in the homogeneous and isotropic medium consisting electromagnetic plasma in the magnetohydrodynamic(MHD) limit and show that indeed the axion decay rate or the imaginary part of the frequency is quite small to ensure the stability of the axions.

Recently in [146], Authors investigated the problem of a dilute axion star converting into photons in the presence of Milkyway's magnetic field. In this work the author argues that that the transverse waves(photon) emitted in the interstellar plasma shall be having dispersion relation $k^2 = \omega^2 - \omega_p^2$, where, k, &ω respectively denote wave-vector and frequency of the emitted photons and ω_p denote the plasma frequency of the interstellar medium. Using the following formula for the power radiated per unit solid angle [147, 148]

$$\frac{dP}{d\Omega} = \frac{\pi^4 (g_{a\gamma\gamma}\phi_0\omega^2 R^2)^2}{32k\omega} \left(\frac{\tanh\left(\frac{\pi k R}{2}\right)}{\cosh\left(\frac{\pi k R}{2}\right)} \right)^2 |B_0|^2, \tag{4.0.2}$$

where, k, R &B_0 respectively describe wave-vector of the emitted radiation, the radius of the star and magnitude of the Milky-way's magnetic field, the author argues that the power emitted by the axion star converting into photon can be detected with the instruments at NCLE [149]. In the present work, we intend to study modification of the dispersion relation of transverse waves in the medium containing ALP and the interstellar plasma. In particular, based on the the techniques of linear mode analysis we obtain a modified dispersion relation by assuming that unperturbed distribution of the axion cloud and the interstellar plasma is homogeneous. We believe that such an analysis could provide some insight into understanding the process conversion of an axion star into photons. A priori, it is not clear that under such a realistic situation the plasma response can be described either by MHD approximation or the more-detailed description of the plasma is needed here. For this purpose, in this work we regard electrons in the interstellar medium as a fluid which has the effect of collision with ions described by collision frequency. But the ions and neutrals are considered to be stationary [150, 151]. The dispersion relation thus derived in our analysis is more general and one can obtain the MHD limit as a special case.

One can write the total Lagrangian density for the axion+photon system:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - J^\mu A_\mu + g_{a\gamma\gamma}\frac{1}{4}aF^{\mu\nu}\tilde{F}^{\mu\nu}, \tag{4.0.3}$$

where J^μ is the electromagnetic current and A^μ is the gauge potential.

The Euler–Lagrange equation in this case produces a version of Maxwell's equations that includes the effects of an axion field, as discussed in Refs. [138, 152]:

$$\Box a + m_a^2 a = g_{a\gamma}\mathbf{E}\cdot\mathbf{B} \tag{4.0.4}$$

$$\nabla\cdot\mathbf{E} = \rho - g_{a\gamma\gamma}\mathbf{B}\cdot\nabla a \tag{4.0.5}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{4.0.6}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{4.0.7}$$

$$\nabla\times\mathbf{B} = \frac{\partial\mathbf{E}}{\partial t} + \mathbf{J} - g_{a\gamma\gamma}\left(\mathbf{E}\times\nabla a - \frac{\partial a}{\partial t}\mathbf{B}\right) \tag{4.0.8}$$

where, ρ and J respectively denote is the electric charge and current densities and they can be described by the basic equations describing the plasma. For the case when the system is considered to be homogeneous, one can neglect the spatial derivative terms in equations (4-8) to obtain:

$$\begin{aligned}\dot{\vec{E}} &= -g\vec{B}\dot{a} - \vec{J}\,,\\ \ddot{a} + m^2 a &= g\vec{B}\cdot\vec{E}\,.\end{aligned} \tag{4.0.9}$$

These equations are equivalent to the ones considered in [145], if one regards $\mathbf{J} = \sigma\mathbf{E}$ in the MHD approximation. The electrical conductivity $\sigma = \dfrac{\omega_p^2}{\nu_c}$, where the ω_p and ν_c denote the plasma and collision frequency respectively. We shall refer to the above equation later.

To proceed further to consider linear analysis, we write any physical quantity $A \sim A_0 + \delta A$, where A_0 is the background quantity while δA is perturbation. Only δA has space and time dependance. Since we do not consider any electric field in background and the magnetic field B_0 is considered to be constant in space, one can write equations (5-8) as :

$$\nabla \cdot \delta\mathbf{E} = \delta\rho - g_{a\gamma\gamma} B_0 \cdot \nabla \delta\mathbf{a} - g_{a\gamma\gamma} \delta\mathbf{B} \cdot \nabla a_0 \tag{4.0.10}$$

$$\nabla \cdot \delta\mathbf{B} = 0 \tag{4.0.11}$$

$$\nabla \times \delta\mathbf{E} = -\frac{\partial \delta\mathbf{B}}{\partial t} \tag{4.0.12}$$

$$\nabla \times \delta\mathbf{B} = \frac{\partial \delta\mathbf{E}}{\partial t} + \delta\mathbf{J} - g_{a\gamma\gamma} \left(\delta\mathbf{E} \times \nabla a_0 - \frac{\partial \delta a}{\partial t} B_0 \right) \tag{4.0.13}$$

Next, by regarding that all the perturbed quantity $\delta\mathbf{A}$ varies with space-time dependance like $exp[-i\omega t + i\mathbf{k} \cdot \mathbf{x}]$ one rewrites the above equation as:

$$i\mathbf{k} \cdot \delta\mathbf{B} = 0 \tag{4.0.14}$$

$$i\mathbf{k} \times \delta\mathbf{E} = i\omega \delta\mathbf{B} \tag{4.0.15}$$

$$i\mathbf{k} \times \delta\mathbf{B} = -i\omega \delta\mathbf{E} + \delta\mathbf{J} - g_{a\gamma\gamma} \left(\delta\mathbf{E} \times \mathbf{k} a_0 + i\omega \delta a \mathbf{B_0} \right) \tag{4.0.16}$$

We consider transverse Eletromagnetic wave propagating in the medium i.e. $\mathbf{k} \cdot \delta\mathbf{E} = 0$ and $\mathbf{k} \cdot \delta\mathbf{B} = 0$ therefore we are not considering gauss law. From Eqn[15], one can write that,

$$k_y \delta E_z = \omega \delta B_x \tag{4.0.17}$$

And from Eqn[16], one can write that,

$$-ik_y \delta B_x \hat{z} = -i\omega \delta E_z \hat{z} + \delta J_z \hat{z} - g_{a\gamma\gamma} \left(-i\delta E_z k_y a \hat{x} + i\omega \delta a B_0 \hat{z} \right) \tag{4.0.18}$$

The z-component of above equation yields:

$$-ik_y \delta B_x = -i\omega \delta E_z + \delta J_z - i\omega \delta a \beta \tag{4.0.19}$$

The axion field's equation of motion yields,

$$(-\omega^2 + k_y^2 + m^2)\delta a = -g_{a\gamma\gamma}\ \delta E_z B_0 \tag{4.0.20}$$

It is reasonable to assume for the interstellar plasma as electrons are much lighter than the ions, the electric current is given by $J = -4\pi e n_e \vec{v}_e$, where $-e$, n_e &$\vec{v}_e$ denote charge, density and velocity of the electron fluid. Here we have ignored the contribution of ions as they have negligible mobility. We write electron-fluid equation as:

$$m_e n_e \left(\partial_t \vec{v}_e + \vec{v}_e \cdot \vec{\nabla} \vec{v}_e\right) = -e n_e \left(\vec{E} + \vec{v}_e \times \vec{B}\right) - m_e n_e \nu_c \vec{v}_e \tag{4.0.21}$$

Further, if we assume that there is no streaming velocity for the electron-fluid and thus we regard the current to be written as $\vec{J} \approx -e n_{e0} \delta \vec{v}_e$. In this situation the electron-fluid equation can be written as:

$$m_e n_{e0} \partial_t \delta \vec{v}_e = -e n_{e0} \left(\delta \vec{E} + \delta \vec{v}_e \times \vec{B}_0\right) - m_e n_{e0} \nu_c \delta \vec{v}_e \tag{4.0.22}$$

It is to be noted that n_{e0} is the unperturbed electron density, while $\vec{B}_0$ is the external magnetic field which is assumed to be constant in space and time. All the quantities with δ are perturbed quantities and they are assumed to be functions of space and time with variation like $e^{-i\omega t}$, one obtains expression for the current as:

$$J = i \frac{\omega_p^2}{\omega + i\nu_c} \vec{E}, \tag{4.0.23}$$

where, $\omega_p^2 = \frac{4\pi e^2 n_e}{me}$ is the plasma frequency. Therefore,

$$\delta J_z = i \frac{\omega_p^2}{\omega + i\nu_c} \delta E_z \tag{4.0.24}$$

Now, Substituting Eqn(17, 21, & 24) in Eqn(19), one gets:

$$\left[(-\omega^2 + k^2)(\omega^2 - k^2 - m^2) + \beta^2\omega^2\right](\omega + iv_c) = -\omega_p^2\omega(\omega^2 - k^2 - m^2) \qquad (4.0.25)$$

The above is a more general form of the dispersion relation which can also be written as:

$$\left[(-\omega^2 + k^2) + \frac{\omega_p^2\omega}{(\omega + iv_c)}\right](\omega^2 - k^2 - m^2) = -\beta^2\omega^2 \qquad (4.0.26)$$

Before considering the solution of equation(26), first it should be noted that if one sets parameter $\beta = 0$, the above dispersion relations splits into two separate branches describing propagation of transverse waves in a collisional plasma and the propagation axions in the absence of the magnetic field. Parameter β helps in describing axion to photon conversion. Further, the factor $\frac{\omega_p^2}{(\omega + iv_c)}$ can be expressed as $\frac{\omega_p}{i\nu_c}\frac{1}{1 - i\frac{w}{\nu_c}}$.

Parameter	Values (eV)
Plasma Frequency	10^{-12}
Conductivity	10^{42}
Collision Frequency	10^{-66}
β	10^{-47}
Gyrofrequency	10^{-15}

Table 4.1: Interstellar Medium Parameters

Before we analyse the solutions of equation(26), it is important to consider the following two special cases: (a) magnetohydrodynamic limit[150] when $\omega << \nu_c$ and (b) non-MHD limit $\omega >> \nu_c$.

- In the limit $|\omega| << \nu_c$: In this limit the factor can be approximated as,

$$\frac{\omega_p^2}{(\omega + i\nu_c)} \simeq -i\sigma$$

where, the parameter

$$\sigma = \frac{\omega_p^2}{\nu_c}$$

describes the electrical conductivity. In this case the general dispersion relation can be reduced to the following form:

$$\left[(-\omega^2 + k^2) - i\sigma\omega\right](\omega^2 - k^2 - m^2) = -\beta^2\omega^2 \tag{4.0.27}$$

If the spatial homogeneity is considered i.e.

$$k \to 0$$

, one recovers the dispersion relation obtained in [145]:

$$(\omega + i\sigma)(\omega^2 - m^2) = \beta^2\omega \tag{4.0.28}$$

- For the non-MHD limit since the condition $\omega >> \nu_c$ is satisfied, the dissipation due to plasma will be insignificant, since there is no ν_c dependence. In this case the factor $\frac{\omega_p^2}{(\omega+i\nu_c)}$ can be approximated as $\frac{\omega_p^2}{\omega}$.

By analyzing the real and imaginary parts of the solutions ω, we can determine the conditions for stability and the frequency and damping rate of the oscillatory modes. In particular, the imaginary part of the solutions determines the damping rate of the oscillations, which can be used to estimate the lifetime of the axion star.

Now, let us define $\omega/m = z$, $\sigma/m = \epsilon$, and, $\beta^2/m^2 = \delta$.Then, Eqn(29) can be written as,

$$\left(z^2 - 1\right)(z + i\epsilon) = \delta z \tag{4.0.29}$$

Substituting $z = x + \iota y$, and separating the real and imaginary part of the equation (4.0.29), The real part of the (4.0.29) can be written as:

$$x^2 = \delta + 3y^2 + 2y\epsilon + 1 \tag{4.0.30}$$

And the imaginary part of (4.0.29) can be:

$$x^2(3y + \epsilon) = y^2\epsilon + y^3 + \delta y + y + \epsilon \tag{4.0.31}$$

Using (4.0.30) in (4.0.31), we get:

$$8y^3 + 8y^2\epsilon + 2y\left(\delta + \epsilon^2 + 1\right) + \delta\epsilon = 0 \tag{4.0.32}$$

The solution of (4.0.32) will give us information about the damping, (4.0.32) has three solutions, but only one of them is real solution, since by definition y is real, Therefore only the real solution is valid, viz:

$$y = -\frac{\epsilon}{3} - \frac{A}{12 \cdot 2^{2/3}\sqrt[3]{\sqrt{4A^3 + B^2} + B}} + \frac{\sqrt[3]{\sqrt{4A^3 + B^2} + B}}{24\sqrt[3]{2}} \tag{4.0.33}$$

where $A = 48\delta - 16\epsilon^2 + 48$ and $B = -576\delta\epsilon + 128\epsilon^3 + 1152\epsilon$

Now, let us estimate the parameter σ for Intergalactic medium, The conductivity of the medium is defined as:

$$\sigma = \frac{\omega_p^2}{\nu_c} \tag{4.0.34}$$

where the plasma frequency ω_p is $\dfrac{n_e e^2}{\epsilon_0 m_e}$ and the collision frequency (ν_c) is $n_e \sigma_c v_{th}$. The collision cross-section σ_c can be estimated using the Rutherford scattering cross section formula:

$$\sigma_c = \frac{\pi\alpha^2}{2(KE)^2} \tag{4.0.35}$$

where α is the fine structure constant and $KE(\simeq T)$ is the average kinetic energy of the particles.

The thermal velocity v_{th} of the particles is $\sqrt{\dfrac{2T}{m_e}}$. Therefore, the (4.0.34) can be written as:

$$\sigma = \frac{8}{\alpha}\frac{T^{3/2}}{\sqrt{m_e}} \tag{4.0.36}$$

For a typical stable region in Intergalactic medium, the temperature $T < 200K$ and has the electron density $n_e \sim 0.03/cm^3$. Consider the gas temperature to be around $10K$, the conductivity of the medium $\sigma \simeq 10^{-5}eV$

Now estimating the parameter $\beta = g_{\alpha\gamma\gamma}B$ for Intergalactic medium, for a typical axion mass in the range of μeV, the value of axion-photon coupling constant $g_{\alpha\gamma\gamma}$ is $\sim 10^{-15} GeV^{-1}$ and the Intergalactic magnetic field $B \sim \mu G \simeq 10^{-26} GeV^2$. Therefore, $\beta \simeq 10^{-32} eV$ for an intergalactic medium.

It can be seen that in intergalactic medium, $\sigma >> \beta$. Now, the typical axion mass range is from 10^{-3} eV to 10^{-6} eV. Therefore the range of parameter ϵ will vary from 10^{-2} to 10, and the parameter δ will vary from 10^{-58} to 10^{-52}.

Since $\delta << \epsilon$, Eqn(33) can be expanded in δ by retaining only it's first order term, Therefore,

$$y = \delta \left(\frac{-3\epsilon^5 - 13\sqrt{3}\epsilon^4 - 66\epsilon^3 - 54\sqrt{3}\epsilon^2 - 63\epsilon - 9\sqrt{3}}{12A^4\left(\epsilon^2+1\right)} - \frac{3\epsilon^3 + 5\sqrt{3}\epsilon^2 + 3\epsilon - 3\sqrt{3}}{12A^2\left(\epsilon^2+1\right)} \right) \tag{4.0.37}$$

where,

$$A = \sqrt[3]{\epsilon^3 + 3\sqrt{3}\left(\epsilon^2+1\right) + 9\epsilon} \tag{4.0.38}$$

Now, if we plot the the above solution by keeping the value of $\delta = 10^{-52}$,

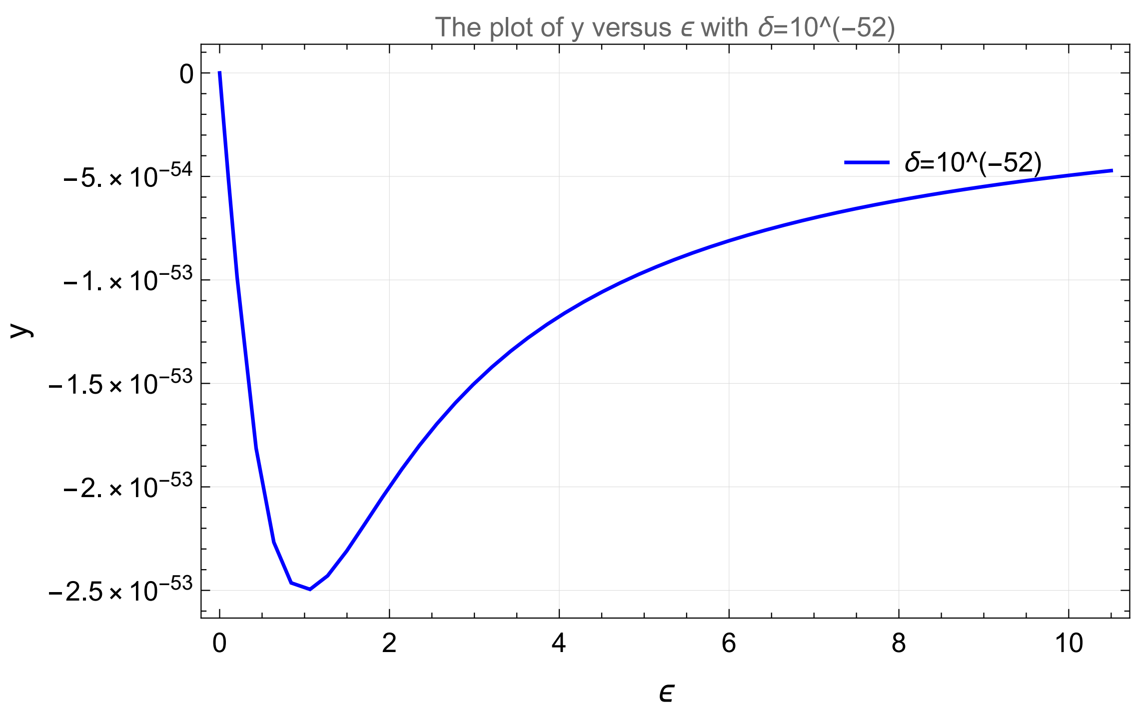

Figure 4.1: y versus ϵ

Now, it can be seen from the plot the for all value of ϵ, y yields negative values, since the values of are of the order of 10^{-54}, then the imaginary part of frequency i.,e, $\omega_I = ym$. Now, if we take $y \simeq 10^{-52}$ and $m \simeq 10^{-6}eV$, Then, $\omega_I \simeq 10^{-58}eV \simeq 10^{-42}s^{-1} \simeq 10^{-34}yr^{-1}$. Therefore the decay time scale,i.e $\omega_I^{-1} \simeq 10^{34}$ years.

If we retains only the highest order terms of ϵ in Eqn[38]

$$y = -\frac{\delta}{2\epsilon} \tag{4.0.39}$$

viz. $-\dfrac{\beta^2}{2\sigma}$, this is exactly what has been derived by Raffelt. et. al [145].

If we imagine a scenario where an Axion star encounters a Magnetar which has high magnetic field of the range 10^15 Gauss, Therefore, $\beta \simeq 10^{-2}eV$, now, here $\sigma \simeq \beta$. y yields values of the order of 10^{-4}, then the imaginary pary of frequency i.e. $\omega_I = ym$. Now, if we take $y \simeq 10^{-4}$ and $m \simeq 10^{-6}eV$, Then, $\omega_I \simeq 10^{-10}$ eV $\simeq 10^6 s^{-1}$. Therefore the decay time scale,i.e $\omega_I^{-1} \simeq 10^{-6}$ seconds. In [145], the MHD equation for the current i.e. $\vec{J} = \sigma\vec{E}$ was used to derive the above expression along with the spatial homogeneity. From now onward we shall regards the condition $|\omega| << \nu_c$ as the MHD limit.

Next, we consider the case when the condition $|\omega| >> \nu_c$ is satisfied. In this case we write the factor involving the collision frequency as the collision frequency play a subdominant role:

$$\frac{\omega_p^2}{(\omega + iv_c)} \simeq \frac{\omega_p^2}{\omega} \tag{4.0.40}$$

Thus the linear response of the medium can be described by

$$\left[(-\omega^2 + k^2) + \omega_p^2\right](\omega^2 - k^2 - m^2) = -\beta^2\omega^2 \tag{4.0.41}$$

Here we would like to note that for the galactic environments the parameters β & ν_c are very small in comparison with the plasma frequency of the interstellar medium. Therefore it is quite useful to analyse the the solutions of the general dispersion relation by regarding β and ν_c small and employing the perturbative techniques. In the subsequent discussion we analyse the solutions of the general dispersion relation in MHD limit as well as in the non-MHD ($|\omega| >> \nu_c$) limits using the perturbative methods.Taking $\omega = \omega_0 + \omega_1$ in Eq[29], the zeroth order solutions are as follows:

$$\omega_0 = -m,\ \omega_0 = m,\ \omega_0 = -i\sigma \tag{4.0.42}$$

And the first order solution is,

$$\omega_1 = \frac{\beta^2\omega_0}{(-m^2 + 2i\sigma\omega_0 + 3\omega_0^2)} \tag{4.0.43}$$

Substituting, $\omega_0 = m$ in above expression leads to

$$\omega_1 = \frac{m\beta^2}{2m^2 + 2im\sigma} \tag{4.0.44}$$

And for, $\omega_0 = -i\sigma$,

$$\omega_1 = \frac{i\beta^2\sigma}{m^2 + \sigma^2} \tag{4.0.45}$$

In the subsequent discussion, we shall consider perturbative solution of the equation (26) while regarding the parameters β and ν_c to be much smaller than ω_p. Here one proceed as before by substituting for $\omega \simeq \omega_0 + \omega_1$ in equation (26) and retaining terms unto linear order in ω_1. In this case zeroth order solution of equation (26) would not contain parameters $\beta \& \nu_c$ and they can be given by

$$\omega_0 = 0,\ \omega_0 = -\sqrt{k^2 + m^2},\ \omega_0 = \sqrt{k^2 + m^2},\ \omega_0 = -\sqrt{k^2 + \omega_p^2},\ \omega_0 = \sqrt{k^2 + \omega_p^2} \tag{4.0.46}$$

Here solutions $\omega_0 = \pm\sqrt{k^2 + m^2}$ describes the zeroth order axion dynamics I.e. when $\beta = 0$. The other branches $\omega_0 = \pm\sqrt{k^2 + \omega_p^2}$ describes the propagation of a transverse waves in the interstellar plasma. Next, the first order solutions of equation (26) can be written as,

$$\omega_1 = \frac{-\beta^2\omega_0^3 + i\left(-k^4 v_c - k^2 m^2 v_c + 2k^2 v_c \omega_0^2 + m^2 v_c \omega_0^2 - v_c \omega_0^4\right)}{\left(k^4 + k^2 m^2 - 6k^2\omega_0^2 - 3m^2\omega_0^2 + 5\omega_0^4 + k^2\omega_p^2 + m^2\omega_p^2 - 3\omega_0^2\omega_p^2\right)} \tag{4.0.47}$$

Upon substituting for $\omega_0 = \pm\sqrt{k^2+m^2}$ in the above equation one gets, the following expression for ω_1

$$\omega_1 = \pm\frac{\beta^2\sqrt{k^2+m^2}}{2m^2-2\omega_p^2} \tag{4.0.48}$$

Thus for the axion-dynamics both ω_0 and ω_1 are real and there is no decaying solution. However, when the mass of axion m approaches the plasma frequency ω_1 terms increase in magnitude. Our perturbative treatment shall remain valid till $|\omega_0| \leq |\omega_1|$. We shall comment on this more in the subsequent discussion.

Next, if one consider the transverse waves or the photon branch upon substituting for $\omega_0 = \pm\sqrt{k^2+\omega_p^2}$, into the equation for ω_1, one obtains

$$\omega_1 = -\frac{i\nu_c\omega_p^2}{2\left(k^2+\omega_p^2\right)} \pm \frac{\beta^2\sqrt{k^2+a_p^2}}{2\left(m^2-\omega_p^2\right)} \tag{4.0.49}$$

It is to be noted that the term with ν_c in the above expression is imaginary and it has negative sign. It describes the damping of transverse electromagnetic waves in the collisional plasma. But the term with parameter β is real and it has both positive and negative signs, depending on which branch of ω_0 is used. Since the denominator of this term contains factor $\left(m^2-\omega_p^2\right)$, its magnitude can grow When the axion mass equals the plasma frequency. Here it should be noted that amplitude of oscillations is not growing as the term with β^2 is real. Thus the linear mode analysis presented in here does not contain any resonance effect when the condition $m = \omega_p$ is satisfied.

Further it is important note here that for the physical situation that we have considered in this work we have condition $\beta \ll \nu_c \ll \omega_p$ is satisfied. Typical values of parameter β depends on ALP-photon coupling constant and the magnitude of the interstellar magnetic field. Following the Ref.[153] we take the ALP-photon coupling constant $g_{a\gamma\gamma} \approx 10^{-12}GeV^{-1}$ and considering the magnitude of the interstellar magnetic field $B_0 \sim 10^{-6}G$. Thus the typical value of $\beta \sim 10^{-29}eV$. For the scenario when the axions are converted into photons one requires to have Eqn(49) having -ve sign, while β^2 term in equation have positive sign. In this scenario if the condition $\omega_1 \leq \omega_0$ is satisfied then there could be a significant contribution of axion-to-photon conversion. This would require that $\frac{\beta^2}{m^2-\omega^2} \sim 1$. Since we know the plasma frequency of the medium only up to the the first few significant digits, it is very hard if not impossible to satisfy the condition $\frac{\beta^2}{m^2-\omega^2} \sim 1$.

In conclusion, we have analysed a potentially useful scenario in which an axion-star could convert or decay into photons in the presence of interstellar magnetic field and the plasma environment. We have provided a systematic linear mode analysis describing this process. Since the relevant parameters describing the collisional frequency ν_c between electron and ions and axion-photon coupling with magnetic field β are small in comparison with the plasma frequency of the interstellar medium, we have provided the perturbative solutions of the dispersion relation. In the magneto-hydrodynamical limit ($\omega \ll \nu_c$), our linear-mode analysis clearly demonstrates that the axion star would remain stable as the decay rate of the star turns out to be larger than the age of the Universe. In the non-magnetohydrodynamic limit ($\omega \gg \nu_c$), it is shown that the ALP-to-photon conversion can be very significant if the condition $\frac{\beta^2}{\left(m^2-\omega_p^2\right)} \sim 1$ is satisfied. Since $\beta^2 \sim 10^{-58}(eV)^2$ and the estimated $\omega_p \sim 10^{-12}eV$ for the considered physical scenario, one needs to have high degree fine tuning between the ALP mass and ω_p. Thus we believe that the requirement of fine tuning between the ALP-mass and ω_p may cause difficulty in observing an axion star converting into photons at NCLE. This work can be extended to scenarios where background magnetic field is very high, such as a Bose star in the vicinity of a magnetar, in such cases there may be possiblity of axion decay into photons as shown in [154, 155]