Chapter 5

Future Scope

5.1 Introduction

The study of Bose Stars—self-gravitating configurations of bosonic fields—has gained considerable attention due to their relevance in astrophysical and cosmological contexts, particularly as dark matter candidates. One of the critical aspects of their dynamics involves the **transfer of angular momentum** between the Bose Star and its surrounding field. However, the methodology to rigorously quantify this exchange is still in its nascent stages. This chapter outlines the current limitations, the strategies employed in our study, and future directions to advance this area of research.

5.2 Current Status

At present, there is **no well-established or standardized method** to quantify the angular momentum transfer in Bose Stars from the surrounding field. The non-linear nature of the governing equations and the complex interactions between the star and its environment render this task particularly challenging. In our study, we adopted the **vorticity magnitude** as a preliminary tool to identify and understand the signatures of angular momentum exchange.

5.3 Role of Vorticity in Angular Momentum Studies

Vorticity, defined as the curl of the velocity field, serves as a measure of local rotation in a fluid or field configuration. In the context of Bose Stars:

- Regions of **high vorticity magnitude** may indicate areas where angular momentum is being transferred or dissipated.
- By tracking the evolution of vorticity within and around the Bose Star, it is possible to obtain indirect but insightful clues about the **momentum dynamics**.

While this method is not a direct quantification, it provides a qualitative understanding that can guide further development.

5.4 Key Challenges

Several key challenges must be addressed to enable precise modeling and measurement of angular momentum transfer:

1. Random Brownian Motion

The Bose Star exhibits **stochastic movement** within the simulation box due to interactions with surrounding field fluctuations. This Brownian-like motion makes it difficult to isolate coherent angular momentum signals.

2. Intrinsic Rotation

Many Bose Star configurations possess **inherent angular momentum**, adding another layer of complexity when distinguishing internal dynamics from externally induced momentum exchanges.

3. Interplay Between Motion and Rotation

The **coupling between random translational motion and intrinsic spin** makes the angular momentum dynamics highly non-trivial. Modeling this interplay requires sophisticated numerical techniques and a deeper theoretical framework.

5.5 Efforts and Preliminary Approaches

Our current methodology involves:

- **Tracking vorticity** fields to detect emerging patterns.
- **Analyzing temporal evolution** of the vorticity magnitude within different regions of the simulation.
- Comparing the Bose Star's **center-of-mass motion** with localized rotational effects to disentangle different sources of angular momentum.

These initial efforts lay the groundwork for a more rigorous formulation in future studies.

5.6 Future Directions

To enhance the understanding and measurement of angular momentum transfer, future research should focus on:

1. Developing Quantitative Frameworks

Formulate methods that can directly compute the **angular momentum flux** through defined boundaries using conserved currents derived from the field theory.

2. Improving Modeling of Brownian Dynamics

Utilize techniques from statistical physics to model the **random walk behavior** of the Bose Star and isolate its effect from rotational motion.

3. Incorporating Angular Momentum Decomposition

Apply **mode decomposition techniques** (e.g., spherical harmonics or azimuthal mode analysis) to break down the angular momentum contributions in different regions.

4. Higher Resolution Simulations

Employ **high-resolution and long-duration simulations** to minimize numerical artifacts and obtain cleaner signals of angular momentum evolution.

5. Machine Learning for Pattern Recognition

Explore **machine learning tools** to detect subtle, non-linear patterns in the vorticity and field configurations that may be associated with angular momentum transfer.

5.7 Conclusion

The quantification of angular momentum transfer in Bose Stars remains an open and intriguing problem. While the use of vorticity magnitude offers a promising starting point, it is clear that a **more comprehensive framework** is needed. Overcoming the challenges posed by random motion and intrinsic spin is essential to deepen our understanding of these exotic objects. Continued advancements in simulation techniques, analytical tools, and theoretical modeling will pave the way for significant progress in this field.