Appendix - A Mathematical Calculation

A.1 Calculation of solar radiation

In the following discussion, a sample calculation is presented to obtain the data for the solar radiation input conditions for the FPSRS for the date of July 21, 2020. The accurate measurement of extra-terrestrial radiation flux is crucial, considering the variation in Earth-Sun distance, which leads to fluctuations within the range of ±3.3%. To address this, Eq. (3.1) provides a simplified equation with adequate precision for most engineering calculations, offering accuracy up to ±0.01%. However, to determine the solar radiation at Earth's surface for a specific day, it's essential to consider the number of days elapsed since a reference date, as described in Eq. (A.1). Subsequently, to calculate the value of solar radiation (G_{on}) on a given day, such as on the summer solstice (n=173, corresponding to June 21st), Eq. (A.2) can be employed. In this equation, G_{sc} represents the solar constant, with a standard value of 1367 W/m^2.

$$G_{on} = G_{sc}\left(1 + 0.033\cos\frac{360n}{365}\right) \quad \text{(A.1)}$$

$$= 1367 * \left(1 + 0.033 * (\cos\frac{(360*173)}{365})\right)$$

$$G_{on} = 1322 \text{ W/m}^2$$

Similarly, it can also be found using the δ, which can be calculated from Eq. 3.2. The value of δ for n=173 is calculated as follows,

$$\delta = 23.45\sin\left(360\frac{284+173}{365}\right)$$

$$\delta = 23.44°$$

The value of G_{on} can now be calculated using Eq. 3.1, while considering δ as follows,

$$G_{on} = G_{sc}(1.000110 + 0.034221\cos(23.44) + 0.001280\sin(23.44) + 0.000179\cos(2 * 23.44) + 0.000077\sin(2 * 23.44))$$

G_{on}= 1345 W/m^2

However, from the above calculation, it is observed that the G_{on} value can be more accurate and easier to calculate when using n=173 instead of relying on δ. This adjustment simplifies the calculation process while maintaining a satisfactory level of accuracy in determining solar radiation input conditions for the given date. By substituting the day of the year (n=173 for July 21) directly into the calculation, the need for calculating the δ is eliminated, streamlining the process and reducing the potential for computational errors.

Now in the following discussion the value of θ is calculated with the use of Eq. (3.3) for a surface which is horizontal, vertical and the inclined located at Vadodara (22.3072° N, 73.1812° E) at 10:30 (solar time) on February 13th, 2020. The inclined surface is 60° from the horizontal plane and pointed 15° on south side (azimuthal aligned). Now for this condition n=44, the value of δ from Eq. (3.2) is -14°. The standard time is calculated from the Eq. (A.2) where, the L_{st} is the standard meridian for the local time zone, L_{loc} is the longitude of the location ($0°<L_{loc}<360°$). The parameter E is the equation of time (in minutes) from Fig. (A.1.1) and with use of Eq. (A.2) (Roelandts, 2007).

$$\text{Solar time} - \text{standard time} = 4(L_{st} - L_{loc}) + E \qquad \text{(A.2)}$$

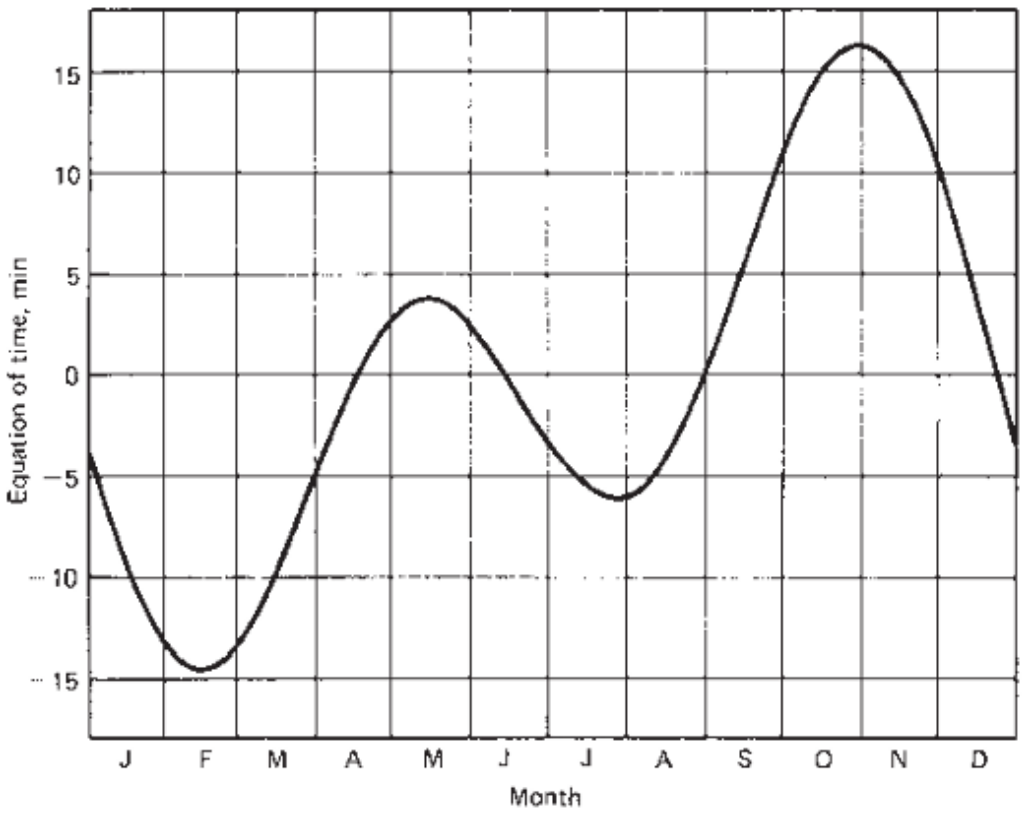

Fig. A.1.1 The equation of time E in minutes as a function of the year (Roelandts, 2007).

$$\text{Standard time} = 4(L_{st} - L_{loc}) + E + \text{solar time}$$

= 4 (90°-22.3072°) + (-15 min) + 10:30

= 4.72-00.15+10.30 = 14.87~15h27m~15h30m.

The value of ω = -22.5° (calculated as 15° per hour times 1.5 hours before noon), and the value of γ_z is obtained from Eq. (3.6). Similarly, the value of θ_z can be obtained from Eq. (3.5), as demonstrated in the following calculation. Additionally, the value of β = 60° is provided from the given data.

$$\cos\theta_z = \cos(22.31)\cos(-14)\cos(-22.5) + \sin(22.31)\sin(-14)$$

θ_z= 43°

$$\gamma_s = -22.5° \left|\cos^{-1}\left(\frac{\cos(43)\sin(22.3)-\sin(-14)}{\sin(43)\cos(22.3)}\right)\right|$$

γ_s= -18.52°

Now the value of θ is calculated from the Eq. (3.3) as follows,

$$\cos\theta = \sin(-22.31)\begin{pmatrix}\sin(-14)\cdot\cos(60)\\ +\cos(-14)\cdot\cos(-18.52)\cdot\cos(-22.5)\cdot\sin(60)\end{pmatrix} +$$
$$\cos(-22.31)\begin{pmatrix}\cos(-14)\cdot\cos(-22.5)\cdot\cos(60)-\\ \sin(-14)\cdot\cos(-18.52)\cdot\sin(60)\end{pmatrix} + (\cos(-14)\cdot\sin(-18.52)\cdot\sin(-22.5)\cdot$$
$$\sin(60))$$

θ = 124°

A.2 Calculation of intersection pints and then angle made by the ray to the reflector

In the following discussion, the three-dimensional geometric equations have been solved to enhance comprehension, specifically focusing on the STC based FPSRS geometry. The TIS and BRS are considered to be square shaped and the dimension of the single side of TIS and BRS is 4m and 2m respectively. The vertical height of FPSRS is 2 m considered. The coordinates for TIS are denoted as E (-1, 2, 3), F (3, 2, 3), G (3, 2, -1), and H (-1, 2, -1), while those for BRS are labelled A (2, 0, 2), B (2, 0, 0), C (0, 0, 0), and D (0, 0, 2).

The first intersection point on the TIS, denoted as P1, is located at coordinates (1, 2, 1), with an incident angle θ of 45°. However, deriving the line equation for P1 proves challenging. To address this, the need for determining the value of P2 arises. P2 is obtained by extending the line originating from P1 at an angle θ until it intersects the bottom horizontal plane, yielding coordinates (3.86, 0, 1).

Subsequently, the line equation is derived using input values of P1 and P2, as outlined in Eq. (3.12) through Eq. (3.14). Upon substituting the coordinates of P1 and P2 into the line equation, constants ‘t’ for x, y, and z are obtained, as depicted in Eq. (A.3) to Eq. (A.5). The parameter ‘t’ varies from 0 to 1, reflecting different positions along the line segment.

$$x = 1 + (1 - 3.86)t \quad \text{(A.3)}$$

$$y = 2 + (1 - 0)t \quad \text{(A.4)}$$

$$z = 1 + (1 - 1)t \quad \text{(A.5)}$$

The plane equation and the corresponding normal vector are derived from Eq. (3.10) and Eq. (3.11), respectively. In these equations, 'a', 'b', and 'c' represent the coefficients of the unit vector. The normal vector can be calculated by considering the coordinates of the plane, as outlined in Table A.1.1. This calculation involves determining the spatial orientation of the plane with respect to its coordinates.

$$a(x - 1) + b(y - 2) + c(z - 1) = 0 \quad \text{(A.6)}$$

Table A.1.1 Coordinates of the plane for calculating the unit vector

Coordinate	X	Y	Z
F	3	2	3
G	3	2	-1
B	2	0	0
A	2	0	2

The vectors $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ are represented in coordinates as (0, 0, -4) and (-1, -2, -3) respectively. Utilizing these numerical values, the final normal to the plane can be determined using Eq. (A.7), which is derived from vector algebra principles.

$$\vec{u} = \begin{bmatrix} i & j & k \\ 0 & 0 & -4 \\ -1 & -2 & -3 \end{bmatrix} \quad \text{(A.7)}$$

The normal vector ($\vec{u}$) of the plane FGBA is represented as -8i + 4j + 0k. By substituting the coordinates from Eq. (A.6) into the plane equation, the final plane equation is obtained as 2x - y = 4. Subsequently, the intersection point on the plane when a line passes through it is calculated. Upon substituting Eq. (A.3) to (A.5) into the plane equation, the value of parameter t is determined to be 0.48. This value is then substituted back into the equation of the line to obtain the coordinate point on the plane, referred to as the intersection point. Furthermore, the angle between the normal vector and

the incident ray on the plane is computed using Eq. (A.7). This angle provides insight into the interaction between the incident ray and the surface, aiding in understanding the behaviour of light or energy propagation. It can be calculated with use of Eq. (A.8).

$$\theta_L = \cos^{-1}\left(\frac{-8*-2.86+4*2+0}{\sqrt{-8^2+4^2+0^2}.\sqrt{-2.86^2+2^2+0^2}}\right) \quad \text{(A.8)}$$

$$\theta_L = 8.42° \quad \text{(A.9)}$$

Thus, from the preceding calculation, the coordinates of the intersection point P3 on reflector R1 are (2.48, 0.96, 1.00), with the incident angle θ_L= 8.42°. Subsequently, this ray undergoes an additional reflection from Reflector R3 before reaching the BRS. Upon reaching the BRS, the final incident angle θ_L is determined to be 71.86°. Similarly, the values of θ and the intersection points on the reflector can be obtained for all the core points on the TIS.