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Semi-Analytical approach to Fractional Differential Equations and its Applications

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Background of the study (Motivation)

Fractional Calculus

In a letter to L'Hôpital on 30^{th} September 1695, Leibniz raised the possibility of generalizing the operation of differentiation to non-integer orders, that is finding $\frac{d^n}{dx^n}f(x)$, and L'Hôpital asked what would be the result of half differentiating the function $f(x) = x$; that is $\frac{d^{1/2}}{dx^{1/2}}[x]$. Leibniz replied: "It leads to a paradox, from which one day useful consequences will be drawn". The paradoxical aspects arise because there are several different ways of generalizing the differentiation operator into non-integer orders, leading to inequivalent results. We can say with accuracy that this query was dated the 30^{th} September, 1695 and that it gave birth to "Fractional Calculus"; therefore this subject of fractional calculus with half derivatives and half integrals is as old as conventional Newtonian or Leibniz's calculus. However, this subject of fractional-calculus was dormant until the beginning of the century, and only now has it started finding applications in science and engineering [34].

We already know about $\frac{d}{dx}f(x)$ or $\int_0^x f(y)dy$ i.e. the usual classical derivative and classical integration operation in order to get expressions for n fold differentiation and integration. The concept of the generalisation is like that from natural numbers i.e. $n = 1, 2, 3, ...$ we generalise the number line to the negative side i.e. $n = -1, -2, -3, ...$ and call them integer numbers. Then we have, in between these numbers: $\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{3}{4}, \pm\frac{3}{2}, ...$ or even irrational ones like $\pi, \sqrt{2}, -\sqrt{3}$ etc. and we call them the real line. Similarly, we extend one whole differentiation i.e. $\frac{d}{dx}[f(x)]$ to have twice the differentiation i.e. $\frac{d^2}{dx^2}[f(x)]$, then thrice i.e. $\frac{d^3}{dx^3}[f(x)]$ and generalise to get the expression for n folds, that is $\frac{d^n}{dx^n}[f(x)]$. Similarly we generalise one whole integration i.e. $\int_0^x f(y)dy$ to n folds integration, that is $\int_0^x f(y)(dy)^n$, or $\int_0^x dx_{n-1}\int_0^{x_{n-1}} dx_{n-2}...\int_0^{x_2} dx_1\int_0^{x_1} f(y)dy$. The extension of our classical calculus to fractional calculus is a generalisation of the theory of calculus. This extension in terms of mathematics is an 'analytic-continuation' of the operation i.e. $n-$ fold differentiation and integration from the n integer to entire complex plane z. So we can have the derivatives (or integrals) in between like $\frac{d^{1/2}}{dx^{1/2}}f(x)$. That is, we have operation $\frac{d^\alpha}{dx^\alpha}f(x)$, where α is arbitrary, a 'real number', a 'complex number', even a 'function' as a continuous distribution. A positive α will signify a differentiation process and a negative α will give us integration. This is a generalised theory of calculus, a subject as old as Leibniz's or Newtonian calculus, and is called fractional calculus. Like further generalisation, we can have α as a complex number, a continuous distribution function, and a system. Therefore, there is fun in learning the subject of fractional calculus [10].

The definition of the fractional derivative is in itself a developing concept. Numerous definitions have been suggested for the fractional derivative, with almost each definition possessing some form of deficiency. It is generally believed that the choice of the derivative used is dictated by the situation that is being modelled. Since it was discovered that the fractional derivative can be successfully applied to practical problems, it is the Caputo derivative that has been used the most. The only shortcoming of the Caputo derivative is the singularity issue. Other fractional derivatives that have this singularity problem viz. Riemann–Liouville, Caputo–Hadamard, and Riesz. In a bid to address the singularity concern, the Caputo–Fabrizio derivative was proposed, this derivative eliminated the singularity problem through the use of an exponential kernel. Atangana and Baleanu replaced the exponential kernel by the Mittag-Leffler function to create another non-singular kernel derivative called the Atangana–Baleanu derivative. More detailed discussions encompassing both theory and applications of the non-singular derivatives are found in literature [33, 47].

In the last few decades, Fractional Calculus has become an important tool in many areas of engineering [4], solid-state physics [13], signal and image processing [30], generation and handing of Atomic energy problem [10], chemistry [43], biology [44], ecology [49], stochastic-based finance [6], economics [27], control theory [35], viscoelasticity, and fiber optics [45] by converting these problems into mathematical models through fractional orders and several models using fractional differential operators still need to be solved.

Fractional Differential equations

A fractional differential equation (FDE) is an equation that involves derivatives of a function of a non-integer order. The concept of fractional calculus, which deals with fractional order derivatives and integrals, has been gaining attention due to its applications in various fields such as physics [6], engineering [27], finance [35], and biology [45].

Fractional differential equations can be expressed in various forms, such as Caputo, Riemann-Liouville, or Grunwald-Letnikov forms. A Caputo FDE of order α is given by:

$$D_t^{\alpha} y(t) = f(t, y(t)) \tag{1}$$

where $y(t)$ is the unknown function, $f(t, y(t))$ is a given function, α is a real number and Caputo derivative [9] is defined as the fractional integral of the derivative of order α of $y(t)$ with respect to t.

$$D_t^{\alpha} y(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{\partial^n y(\tau)}{\partial \tau^n} d\tau, & n-1 < \alpha < n \\ \frac{\partial^n y(\tau)}{\partial \tau^n}, & \alpha = n \in N. \end{cases} \tag{2}$$

The Riemann-Liouville and Grunwald-Letnikov forms are other well-known alternative definitions of fractional derivatives.

Solving FDEs can be challenging due to its non-locality and non-integer order of the derivatives. However, there are various analytical, semi-analytical, and numerical methods available for the exact or approximate solutions of FDEs. FDEs have applications in various fields, such as the modelling of viscoelastic materials [24], the analysis of anomalous diffusion processes [15], the control of fractional-order systems [25], and the modelling of financial markets [26, 57]. Therefore, the study of fractional calculus and FDEs is an active research area with many open problems and challenges.

Analytical and Numerical methods

Fractional Differential equations form the backbone of various physical systems occurring in a wide range of science and engineering disciplines viz. physics [38], chemistry [7], biology [16], economics [21], structural mechanics [24], control theory [6], circuit analysis [33], biomechanics [37], etc. Generally, these physical systems are modeled either using fractional ordinary or partial differential equations (FODEs or FPDEs). In order to know the behavior of the system, we need to investigate the solutions of the governing FDEs. The exact solution of differential equations may be obtained using well-known classical methods. Generally, the physical systems occurring in nature comprise of complex phenomena for which computation of exact results may be quite challenging. In such cases, numerical or semi-analytical methods may be preferred [8, 53].

Integral transform

To fully harness the capability of fractional differential equations in modelling problems that arise in the real world, it is imperative that we have methods of solutions that are computationally inexpensive, easily accessible, and highly accurate. Integral transforms are some of the techniques that have proven their worth, as they are regarded to be easy to implement and demand reasonable labour in terms of computations. Integral transforms offer an alternative to integration in the solution of differential equations. The integral transform maps the domain of the original problem into a different domain consisting of an algebraic equation that is normally easy to manipulate. Taking the inverse of the new domain results in the solution of the original problem. There are different types of integral transforms that are used in the solution of differential equations, but it is the Laplace transform that is mostly applied. Most of the integral transforms that have been suggested are extensions of the Laplace transform. Some of the integral transforms that are closely related to the Laplace transform are the Elzaki transform [12], Sawi transform [31], Kamal transform [28], Sumudu transform [55], Natural transform [42], Shehu transform [32], General transform [22], etc.

Semi-Analytical methods

It is more difficult to obtain solutions of nonlinear fractional ordinary or partial differential equations. Therefore, it may not always be possible to obtain analytical solutions of those nonlinear FODEs or FPDEs. In this case, we use semi-analytical methods giving series solutions. In these kinds of methods, the solutions are sought in the form of series. Semi-analytical methods are based on finding the other terms of the series from given initial conditions for the problem being considered. At this point, we encounter the concept of convergence of the series. So, it is necessary to perform convergence analysis of these methods. As this convergence analysis can be carried out theoretically, one can gain information about the convergence of the series solution by looking at the absolute error between the numerical solution and the analytical solution. In some semi-analytic methods, a very good convergence can be achieved with only a few terms of the series, but more terms can be needed in some problems. That is, if the terms of the series increase, this provides better convergence to the analytical solution [54, 8].

To find an approximate or analytical solutions of nonlinear fractional differential equations various methods are available in literature like Adomian decomposition methods [2], Laplace decomposition method [58], homotopy perturbation method [18], homotopy analysis method [19], homotopy analysis transform method [29], Differential transform method [5], spectral collocation method [60], the tanh-coth method [56], exp-function method [20], Mittag-Leffler function method [17] and many more.

In the second chapter, we used a semi-analytic method namely Residual power series method [1, 11] to solve Time-fractional Korteweg-de Vries (KdV) and Kawahara equations. This method is based on constructing power series expansion solution for different nonlinear equations without linearization, perturbation, or discretization. With the help of residual error concepts, this method computes the coefficient of the power series by a chain of algebraic equations of one or more variables and finally we get a series solution, in practice a truncated series solution. The main advantage of this method over the other method is it can be applied directly to the given problem by choosing an appropriate initial guess approximation.

In the past few years, Various hybrid techniques have been developed by combining the integral transform methods with other analytical methods for fast convergent series solution of FDEs. By motivating this statergy we proposed a novel semi-analytic method entitled as Homotopy Perturbation Sawi Transform Method (HPSTM) by merging Sawi tranform with homotopy perturbation method in Third chapter. And we presented an another technique by uniting Jafari's powerful General transform with homotopy perturbation method namely Homotopy Perturbation General Transform Method (HPGTM) in fourth chapter.

Mathematical Modeling

Many research scholars have reported that modelling with fractional calculus concept is very suitable and reliable to give an accurate description of memory and some physical properties of various materials and processes, which are completely missing in classical or integer-order equations, and a fractional mathematical model can give more reliable information about real life phenomena [51]. In addition, many physical systems encountered in various disciplines have been described by fractional differential equations, which include the hydrology and ground water flow [14], diffusion-like waves [39], pattern formation in chemical and biological processes [41], non-linear movement of earthquakes [61], viscoelastic materials [40] and muscular blood vessel model [62], among several other applications. Also, fractional-order problems are naturally connected to models with memory, which arise in some biological scenarios.

Epidemic Models

Epidemiology is the investigation of how illnesses disseminate in a live entity in relation to its surroundings. The epidemiology of an illness can be studied using numerical simulations. Several studies have attempted to predict and simulate the transmission of contagious ailments in the past, including measles, hepatities b virus [50], rubella [36], HIV [23], dengue fever [48], tuberculosis [59], and more recently, Covid19 [58], ebola [52], and the Zika virus [3]. Mathematical simulation is being employed to investigate not only the transmission of contagious ailments, but also increasingly non-communicable diseases, as research advances. Medications and other environmental ailments can often be modeled. This is possible owing to the characteristics of how it spreads, namely via intimate communication as the media spreads.

In the sixth chapter, we discussed a fractional-order COVID-19 model using homotopy perturbation Laplace transform method by involving Atangana-Baleanu fractional derivative in Caputo sense. In parallel we examined a Diabetes model by considering the fractional ordered differential equations with ABC derivative utilizing Adomian decomposition Laplace transform method in the last chapter. In the both model this whole framework is worked out using Banach fixed point theorem, the existence and stability analysis of the system.

Literature Survey

Table 1: Following are the recent developments in various applications by many authors due to the involvement of fractional calculus.

Authors	Year	Method	Applications	Ref. No.
Ali et al	2022	Direct differentiation approach and Predictor-corrector method	Fractional Zika virus model	[3]
Arqub et al	2022	Residual power series algorithm	Time-fractional derivative arising in shallow water waves	[4]
Ayaz	2004	Differential transform method	System of partial differential equations	[5]
Bhanotar and Kaabar	2021	Conformable triple Laplace transform decomposition method	Nonlinear partial differential equation in the sense of conformable derivative	[7]
Gul et al	2022	Sequential hybrid system method	Typhoid treatment model	[15]
Hamza et al	2021	Sumudu transform method	Space-time fractional telegraph equation	[16]
Haubold et al	2011	Mittag-Leffler function method	Kinetic equations and diffusion equations	[17]
He and Wu	2006	Exp-function method	Non-linear wave equations	[20]
Ismail et al	2020	Residual power series method	Approximate studies of fractional physical phenomina	[21]
Jafari	2021	General integral transform method	To solve interal equations	[22]
Jafari and Kheiri	2022	Adapted Forward-Backward Sweep method	HIV/AIDS virus model	[23]
Khan et al	2023	Lagrange's interpolation polynomial method	Fractional hepatitis C model	[24]
Liu	2022	Homotopy perturbation method	Biot elastic model of porous medial	[30]
Odibat and Momani	2009	Variation iteration method	Fluid mechanics	[37]
Saad and Said	2023	Novel fault diagnosis approach	Nuclear reactor medium voltage power cables	[41]
Sakulrang et al	2017	Adams-Moulton predictor-corrector method	Glucose monitoring data model	[44]
Salahshour et al	2015	Fuzzy Laplace transform method	Basset problem	[45]
Shah et al	2020	Laplace Adomian decomposition method	Dengue fever disease model	[48]
Shalbafian and Ganjefar	2023	Homotopy perturbation method	Variable speed wind turbine control	[49]
Simelane and Dlamini	2021	Adams-Bashforth-Moulton technique	Hepatities b virus model	[50]
Wazwaz	2007	Tanh-coth method	Solitons and Kink solutions	[56]
Yasmin et al	2023	Adomian decomposition transform method and Homotopy perturbation transform method	Modeling of nonlinear structures in fluid mediums	[57]
Yunus et al	2023	Laplace Adomian decomposition method	Fractional-order coronavirus disease model	[58]
Zafar et al	2022	q-Homotopy analysis method	Fractional-order tuberculosis model	[59]
Zayernouri and Karniadakis	2014	Spectral collocation method	Advection-diffusion problem	[60]
Zhang et al	2022	Homotopy perturbation method	Fuzzy fractional Black-Scholes European option pricing equations	[61]

Brief about our work

The journey of our research is starts with learning of Fractional Calculus. we focused to understand the new techniques to solve fractional differential equations in the form of physical problems. In the study of FDEs, we understand the way to develop the new semi-analytic approach. We successfully proposed three different hybrid techniques to deal with FDEs. In the same flow we extended our work to know fractional mathematical models, which can give more reliable information about real life phenomena in many fields. In this regards, we examined fractional-order Covid-19 model and Diabetes model in detail.

Formation of the thesis

We plan to write our thesis in seven chapters. First chapter represents the introduction and preliminaries of our proposed work. Second chapter consists the application of recently developed semi-analytic method (Residual power series method). For that we choose nonlinear time-fractional Korteweg-de Vries equation as an application of shallow water wave and Kawahara equations to study Magneto-Acoustic wave in Plasma theory. In next chapter, we introduce a novel technique as a combination of the Homotopy perturbation method and Sawi transform, entitled as Homotopy perturbation Sawi transform method (HPSTM). Fourth chapter is the showcase of a new hybrid technique, the Homotopy perturbation General transform method (HPGTM) for obtaining a semi-analytic solution for a wide class of FDEs. Several numerical examples including well-known equations viz. radioactive decay model, Riccati equation, backward Kolmogorov equation, Klein–Gordon equation, and Rosenau–Hyman equation are considered. Fifth unit presents the solution of fractional-order Swada-Kotera-Ito equation by using Caputo and Atangana-Baleanu fractional derivative. We utilized the Adomian decomposition Shehu transform method to achieve the results. Chapter sixth and seventh are the study of fractional-order Covid-19 model by HPLTM and Diabetes model by ADLTM, respectively. The existence, uniqueness, and stability analysis is established with the help of the Banach's fixed point theorem. Many comparative numerical and graphical studies discussed to prove the significance of suggested methods.

Chapter 1.

Introduction and Preliminaries

We divided this chapter in three parts. First part of this chapter deals with the origin of Fractional Calculus and contribution of many well-known mathematicians in this field. In the second part, we covered recent developments in many areas of science and engineering, viz. solid-state physics, signal and image processing, chemistry, biology, ecology, stochastic-based finance, economics, control theory, viscoelasticity, and fiber-optics by converting these problems into mathematical models having fractional-order derivatives and integrals. At last, some well-known definitions, lemmas, and theorems are stated, which we have used in our work.

Chapter 2.

Fractional Korteweg-de Vries and Kawahara equations

In this chapter, we discussed Korteweg-de Vries (KdV) and Kawahara equations in the form of a fractional partial differential equations (FPDEs) and used the Fractional Residual Power Series Method (FRPSM) to obtain their semi-analytic solutions. Nonlinear Korteweg-de Vries (KdV) equation is an application of shallow water wave and Kawahara equations are arising in the Magneto-Acoustic waves of Plasma theory. The significance of studying these celebrated equations in the fractional form lies in the fact that using the fractional approach we get more realistic results in real-time situations, as compared to conventional derivatives of integer order.

Chapter 3.

Homotopy perturbation Sawi transform method

In continuation of the study, the present work introduces a novel powerful method as a combination of the Homotopy perturbation method (HPM) [18] and the Sawi transform [31], which we call as, "the Homotopy Perturbation Sawi Transform Method (HPSTM)". Time-fractional logistic equation and Fornberg-Whitham equation have been solved to prove the significance of newly introduced method. The proposed method is capable of dealing with general FDEs in an efficient manner, and can be applied not only on various nonlinear wave equations, but applied with equal efficiency to solve oscillatory equations with discontinuities and boundary value problems.

Chapter 4.

Homotopy perturbation General transform method

We proposed a new hybrid technique, the Homotopy Perturbation General Transform Method (HPGTM) for obtaining a semi-analytic solution for a wide class of time-fractional differential equations in the Caputo sense. The efficiency of HPGTM is analyzed using a comparative study with Adomian Decomposition Method (ADM) [2], Residual Power Series Method (RPSM) [1], and exact solution. Numerical examples including well-known equations viz. radioactive decay model, Riccati equation, backward Kolmogorov equation, Klein–Gordon equation, and Rosenau–Hyman equation are considered in arbitrary fractional-order. The outcomes of numerical simulations clearly reveal the effectiveness of the present method.

Chapter 5.

Fractional Sawada-Kotera-Ito equation through Caputo and Atangana-Baleanu derivatives

In the present chapter, the fractional-order Sawada–Kotera–Ito problem is solved by considering Caputo and Atangana–Baleanu (ABC) derivatives. Time-fractional Sawada-Kotera-Ito equation is a study of sub-diffusion dynamics of nano-precipitate growth and destruction. The methodology consists of the blending of the Shehu transform and the Adomian decomposition method. The obtained solution is more accurate when using the ABC type derivative as compared to the Caputo sense, while using the proposed ADShTM method (Adomian decomposition Shehu transform method). The results so obtained by the ADShTM using Caputo and ABC operators are compared with q-HAM [59], RPSM [1], and ADTM [57] methods establishing the superiority of the proposed method. The numerical results demonstrate that the application of the ABC derivative is not only relatively more effective and reliable but also straightforward to achieve high precision solution.

Chapter 6.

Fractional-order COVID-19 model

Present work showcases the study of SEIR model of COVID-19 using Atangana–Baleanu fractional derivative. An attempt is made to obtain the semi-analytical solution using Homotopy Perturbation Laplace Transform Method. The existence, uniqueness, and stability analysis is established with the help of the Banach's fixed point theorem. Many comparative graphical studies discussed to proving the significance of fractional-order derivative in the model. The current study demonstrates the effectiveness of the mathematical model formed by considering the fractional derivatives, which eventually fits the actual data using the method of least squares. The model performs effectively when tested on COVID-19 real data.

Chapter 7.

Investigation of Fractional Diabetes model

The ultimate aim of this study is to develop and analyze a comprehensive regulatory framework for managing glucose and insulin in blood in the presence of Diabetes mellitus. This integer ordered mathematical model of Diabetes [46] is demonstrated and examined in fractional order by involving ABC fractional derivative. This whole framework is worked out using a semi-analytical technique, namely the Adomian Decomposition Laplace Transform Method. To prove the efficiency of this ADLTM technique, the results are compared with other classical methods viz Homotopy Perturbation Transform Method (HPTM) [57] and Modified Homotopy Analysis Transform Method (MHATM) [61]. Using the Banach fixed point theorem, the existence and stability analysis of the solution has been proved. Certain figures and tables are illustrated for this fractional diabetes model. We used the Maple software to generate all the numerics and graphical plots. This detailed investigation also explores how well the level of glucose and insulin affects the dynamics of disease infection.

Overall Conclusion

In the present work, we investigate certain semi-analytic techniques like Fractinal Residual Power Series Method (FRPSM), Homotopy Perturbation Sawi Transform Method (HPSTM), Adomian Decomposition Shehu Transform Method (ADShTM), and Homotopy Perturbation General Transform Method (HPGTM). Demonstration to solve linear and nonlinear ordinary and partial fractional differential equations have successfully been achieved by employing some of the very sophisticated methods showing their applicability, reliability and effectiveness, and thus uncovering the aesthetic beautiful nature of mathematics. In order to achieve the approximate solution in series form, we need to examine the convergence and error approximation related to technique which is under consideration; and we did this exercise in all the techniques, we explored. Semi-analytical methods can be viewed as a good refinement of the existing analytical methods and are become more popular due to its ease of computation and obtaining the solution in series form, with the successful observation of having obtained enough close approximate solution as compared to exact/analytic solution.

Future scope

Still there are several models representing physical phenomena using fractional differential operators need to be solved. Fractional differential equations have a broad and promising future due to their ability to model memory effects and complex behaviors more effectively than classical differential equations. We believe that our work will pave the way for many real world problems in the nature of fractional-ordered linear and nonlinear ordinary as well as partial differential equations. In this context, we claim that these described methods can be used extensively in many science and technology areas to solve complicated non-linear types of differential equations.

List of Publications

Published papers in International journals

1. Khirsariya, S. R., Rao, S. B., & Chauhan J. P. (2022). Semi-analytic solution of time-fractional korteweg-de vries equation using fractional residual power series method. *Results in Nonlinear Analysis*, 5(3), 222–234. doi: https://doi.org/10.53006/rna.1024308

2. Khirsariya, S. R., Rao, S. B., & Chauhan J. P. (2023). A novel hybrid technique to obtain the solution of generalized fractional-order differential equations. *Mathematics and Computers in Simulation*, 205, 272–290. doi: https://doi.org/10.1016/j.matcom.2022.10.013

3. Khirsariya, S. R., & Rao, S. B. (2023). On the semi-analytic technique to deal with nonlinear fractional differential equations. *Journal of Applied Mathematics and Computational Mechanics*, 22(1), 13–26. doi: https://doi.org/10.17512/jamcm.2023.1.02

4. Khirsariya, S. R., & Rao, S. B. (2023). Solution of Fractional Sawada-Kotera-Ito equation using Caputo and Atangana-Baleanu derivatives. *Mathematical Methods in the Applied Sciences,* 1–20. doi: https://doi.org/10.1002/mma.9438

5. Khirsariya, S. R., & Rao, S. B. (2023). Investigation of Fractional Diabetes Model involving glucose-insulin alliance scheme. *International Journal of Dynamics and Control*, Springer. doi: https://doi.org/10.1007/s40435-023-01293-4

Communicated papers in International journals

1. Khirsariya, S. R., Rao, S. B., & Chauhan J. P. (2022). Solution of fractional modified Kawahara equation: A semi-analytic approach. *Mathematics in Applied Sciences and Engineering*

2. Khirsariya, S. R., & Rao, S. B. (2023). A fractional order SEIR model of COVID-19 using homotopy perturbation Laplace transform method through Atangana-Baleanu fractional derivative. *International Journal of Applied and Computational Mathematics*, Springer.

Presented papers in National/ International Conferences

1. Khirsariya, S. R., Rao, S. B., & Chauhan J. P. (2021). New Hybrid Technique For Solving Fractional Differential Equations, International Conference of *Special Function and their Application* (ICSFA-2021), University of Kerala, India, during December 22–24, 2021.

2. Khirsariya, S. R., & Rao, S. B. (2022). Approximate solution of time-fractional differential equation via Homotopy Perturbation Shehu Transform Method using Atangana-Baleanu derivative operator, 2^{nd} International Conference on *Recent Advances in Computational Mathematics & Engineering* (RACME–22), B. K. Birla Institute of Engineering & Technology, Pilani, Rajasthan, India, during May 30–31, 2022.

3. Khirsariya, S. R., & Rao, S. B. (2022). A fractinal order SEIR model of COVID-19 using homotopy perturbation transform method through Atangana-Baleanu fractional derivative, International Conference on *Dynamical Systems, Control and their Applications* (ICDSCA–2022), IIT Roorkee, India, during July 01–03, 2022.

4. Khirsariya, S. R., & Rao, S. B. (2023). Investigation of fractional diabetes model involving glucose insulin alliance scheme, International Conference on *Fractional Calculus: Theory, Applications and Numerics* (ICFTAN–2023), NIT Puducherry, Karaikal, India, during January 27–29, 2023.

5. Khirsariya, S. R., & Rao, S. B. (2023). A robust computational analysis of Residual Power Series involving General Transform to a wide class of Fractional Differential Equations, National Conference on *Special Functions and Allied Areas*, Sardar Patel University, Vallabh Vidyanagar, Gujarat, India, during March 17–18, 2023.

6. Khirsariya, S. R., & Rao, S. B. (2023). Solution of Fractional Klein-Gordon equation using the modern Laplace Residual Power Series Method, International Conference on *Recent Advances in Applied Science & Engineering* (RAISE–2023), The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat, India, during April 12–13, 2023.

7. Khirsariya, S. R., & Rao, S. B. (2023). Fractional Childhood Disease SIR Model using a prosperous numerical technique, International Conference on *Modelling, Simulation and Optimization of Energy Systems* (MSOES–2023), Canadian University of Dubai, UAE, during June 17–18, 2023.

References

[1] Abu Arqub, O., El-Ajou, A., Bataineh, A. S., and Hashim, I. (2013). A representation of the exact solution of generalized lane-emden equations using a new analytical method. In *Abstract and applied analysis*, volume 2013. Hindawi. https://doi.org/10.1155/2013/378593.

[2] Adomian, G. (1990). A review of the decomposition method and some recent results for nonlinear equations. *Mathematical and Computer Modelling*, 13(7):17–43. https://doi.org/10.1016/0895-7177(90)90125-7.

[3] Ali, A., Islam, S., Khan, M. R., Rasheed, S., Allehiany, F., Baili, J., Khan, M. A., and Ahmad, H. (2022). Dynamics of a fractional order zika virus model with mutant. *Alexandria Engineering Journal*, 61(6):4821–4836. https://doi.org/10.1016/j.aej.2021.10.031.

[4] Arqub, O. A., Al-Smadi, M., Almusawa, H., Baleanu, D., Hayat, T., Alhodaly, M., and Osman, M. (2022). A novel analytical algorithm for generalized fifth-order time-fractional nonlinear evolution equations with conformable time derivative arising in shallow water waves. *Alexandria Engineering Journal*, 61(7):5753–5769. https://doi.org/10.1016/j.aej.2021.12.044.

[5] Ayaz, F. (2004). Solutions of the system of differential equations by differential transform method. *Applied Mathematics and Computation*, 147(2):547–567. https://doi.org/10.1016/S0096-3003(02)00794-4.

[6] Baleanu, D., Balas, V. E., and Agarwal, P. (2022). *Fractional Order Systems and Applications in Engineering*. Elsevier.

[7] Bhanotar, S. A. and Kaabar, M. K. (2021). Analytical solutions for the nonlinear partial differential equations using the conformable triple laplace transform decomposition method. *International Journal of Differential Equations*, 2021:1–18. https://doi.org/10.1155/2021/9988160.

[8] Chakraverty, S., Mahato, N., Karunakar, P., and Rao, T. D. (2019). Advanced numerical and semi-analytical methods for differential equations. *Geophysical Journal International*, John Wiley & Sons.

[9] Caputo, M. (1967). *Linear models of dissipation whose Q is almost frequency independent—II*, 13(5):529–539. https://doi.org/10.1111/j.1365-246X.1967.tb02303.x.

[10] Das, S. (2020). *Kindergarten of fractional calculus*. Cambridge Scholars Publishing.

[11] El-Ajou, A., Arqub, O. A., Zhour, Z. A., and Momani, S. (2013). New results on fractional power series: theories and applications. *Entropy*, 15(12):5305–5323. https://doi.org/10.3390/e15125305.

[12] Elzaki, T. M. (2011). The new integral transform elzaki transform. *Global Journal of pure and applied mathematics*, 7(1):57–64.

[13] Gambo, J. J. and Gambo, D. (2021). On the effect of heat generation/absorption on magnetohydrodynamic free convective flow in a vertical annulus: An adomian decomposition method. *Heat transfer*, 50(3):2288–2302. https://doi.org/10.1002/htj.21978.

[14] González-Gaxiola, O., Biswas, A., Zhou, Q., and Alshehri, H. M. (2022). Numerical study of highly dispersive optical solitons with differential group delay having quadratic-cubic law of refractive index by laplace–adomian decomposition. *Journal of Nonlinear Optical Physics & Materials*, 31(03):2250009. https://doi.org/10.1142/S0218863522500096.

[15] Gul, S., Khan, R. A., Khan, H., George, R., Etemad, S., and Rezapour, S. (2022). Analysis on a coupled system of two sequential hybrid bvps with numerical simulations to a model of typhoid treatment. *Alexandria Engineering Journal*, 61(12):10085–10098. https://doi.org/10.1016/j.aej.2022.03.020.

[16] Hamza, A. E., Mohamed, M. Z., Abd Elmohmoud, E. M., and Magzoub, M. (2021). Conformable sumudu transform of space-time fractional telegraph equation. In *Abstract and Applied Analysis*, volume 2021, pages 1–6. Hindawi Limited. https://doi.org/10.1155/2021/6682994.

[17] Haubold, H. J., Mathai, A. M., Saxena, R. K., et al. (2011). Mittag-leffler functions and their applications. *Journal of applied mathematics*, 2011. https://doi.org/10.1155/2011/298628.

[18] He, J.-H. (1999). Homotopy perturbation technique. *Computer methods in applied mechanics and engineering*, 178(3-4):257–262. https://doi.org/10.1016/S0045-7825(99)00018-3.

[19] He, J.-H. (2000). A coupling method of a homotopy technique and a perturbation technique for non-linear problems. *International journal of non-linear mechanics*, 35(1):37–43. https://doi.org/10.1016/S0020-7462(98)00085-7.

[20] He, J.-H. and Wu, X.-H. (2006). Exp-function method for nonlinear wave equations. *Chaos, Solitons & Fractals*, 30(3):700–708. https://doi.org/10.1016/j.chaos.2006.03.020.

[21] Ismail, G. M., Abdl-Rahim, H. R., Ahmad, H., and Chu, Y.-M. (2020). Fractional residual power series method for the analytical and approximate studies of fractional physical phenomena. *Open Physics*, 18(1):799–805. https://doi.org/10.1515/phys-2020-0190.

[22] Jafari, H. (2021). A new general integral transform for solving integral equations. *Journal of Advanced Research*, 32:133–138. https://doi.org/10.1016/j.jare.2020.08.016.

[23] Jafari, M. and Kheiri, H. (2022). Free terminal time optimal control of a fractional-order model for the hiv/aids epidemic. *International Journal of Biomathematics*, 15(05):2250022. https://doi.org/10.1142/S179352452250022X.

[24] Khan, H., Alzabut, J., Baleanu, D., Alobaidi, G., and Rehman, M.-U. (2023). Existence of solutions and a numerical scheme for a generalized hybrid class of n-coupled modified abc-

fractional differential equations with an application. *AIMS Mathematics*, 8(3):6609–6625. https://doi.org/10.3934/math.2023334.

[25] Khan, H., Alzabut, J., Shah, A., Etemad, S., Rezapour, S., and Park, C. (2022). A study on the fractal-fractional tobacco smoking model. *Aims Math*, 7:13887–13909. https://doi.org/10.3934/math.2022767.

[26] Khan, H., Begum, R., Abdeljawad, T., and Khashan, M. M. (2021a). A numerical and analytical study of se (is)(ih) ar epidemic fractional order covid-19 model. *Advances in Difference Equations*, 2021(1):1–31. https://doi.org/10.1186/s13662-021-03447-0.

[27] Khan, H., Ibrahim, M., Abdel-Aty, A.-H., Khashan, M. M., Khan, F. A., and Khan, A. (2021b). A fractional order covid-19 epidemic model with mittag-leffler kernel. *Chaos, Solitons & Fractals*, 148:111030. https://doi.org/10.1016/j.chaos.2021.111030.

[28] Khan, Z. H. and Khan, W. A. (2008). N-transform properties and applications. *NUST journal of engineering sciences*, 1(1):127–133.

[29] Liao, S. (2004). On the homotopy analysis method for nonlinear problems. *Applied mathematics and computation*, 147(2):499–513. https://doi.org/10.1016/S0096-3003(02)00790-7.

[30] Liu, T. (2022). Porosity reconstruction based on biot elastic model of porous media by homotopy perturbation method. *Chaos, Solitons & Fractals*, 158:112007.

[31] Mahgoub, M. M. A. and Mohand, M. (2019). The new integral transform"sawi transform". *Advances in Theoretical and Applied Mathematics*, 14(1):81–87.

[32] Maitama, S. and Zhao, W. (2019). New integral transform: Shehu transform a generalization of sumudu and laplace transform for solving differential equations. *arXiv preprint arXiv:1904.11370*. https://doi.org/10.48550/arXiv.1904.11370.

[33] Meddahi, M., Jafari, H., and Ncube, M. (2021). New general integral transform via atangana–baleanu derivatives. *Advances in Difference Equations*, 2021:1–14. https://doi.org/10.1186/s13662-021-03540-4.

[34] Miller, K. S. and Ross, B. (1993). *An introduction to the fractional calculus and fractional differential equations*. Wiley.

[35] Momani, S., Chen, Y., Agarwal, P., Machado, J. A. T., and Baleanu, D. (2019). *Fractional Calculus: ICFDA 2018, Amman, Jordan, July 16-18*. Springer.

[36] Mukdasai, K., Sabir, Z., Raja, M. A. Z., Sadat, R., Ali, M. R., and Singkibud, P. (2022). A numerical simulation of the fractional order leptospirosis model using the supervise neural network. *Alexandria Engineering Journal*, 61(12):12431–12441. https://doi.org/10.1016/j.aej.2022.06.013.

[37] Odibat, Z. and Momani, S. (2009). The variational iteration method: an efficient scheme for handling fractional partial differential equations in fluid mechanics. *Computers & Mathematics with Applications*, 58(11-12):2199–2208. https://doi.org/10.1016/j.camwa.2009.03.009.

[38] Qureshi, S. (2021). Fox h-functions as exact solutions for caputo type mass spring damper system under sumudu transform. *Journal of Applied Mathematics and Computational Mechanics*, 20(1). https://doi.org/10.17512/jamcm.2021.1.08.

[39] Rashid, S., Kubra, K. T., Jafari, H., and Lehre, S. U. (2022). A semi-analytical approach for fractional order boussinesq equation in a gradient unconfined aquifers. *Mathematical Methods in the Applied Sciences*, 45(2):1033–1062. https://doi.org/10.1002/mma.7833.

[40] Raza, A., Khan, S. U., Al-Khaled, K., Khan, M. I., Haq, A. U., Alotaibi, F., Abd Allah, A. M., and Qayyum, S. (2022). A fractional model for the kerosene oil and water-based casson nanofluid with inclined magnetic force. *Chemical Physics Letters*, 787:139277. https://doi.org/10.1016/j.cplett.2021.139277.

[41] Saad, M. H. and Said, A. (2023). Machine learning-based fault diagnosis for research nuclear reactor medium voltage power cables in fraction fourier domain. *Electrical Engineering*, 105(1):25–42. https://doi.org/10.1007/s00202-022-01649-7.

[42] Saadeh, R. Z. and Ghazal, B. f. (2021). A new approach on transforms: Formable integral transform and its applications. *Axioms*, 10(4):332. https://doi.org/10.3390/axioms10040332.

[43] Sajjadi, N. A. and Asad, J. H. (2022). Fractional treatment: an accelerated mass-spring system. *Romanian Reports in Physics*, 74:122. https://rrp.nipne.ro/2022/AN74122.pdf.

[44] Sakulrang, S., Moore, E. J., Sungnul, S., and de Gaetano, A. (2017). A fractional differential equation model for continuous glucose monitoring data. *Advances in Difference Equations*, 2017(1):1–11. https://doi.org/10.1186/s13662-017-1207-1.

[45] Salahshour, S., Ahmadian, A., Senu, N., Baleanu, D., and Agarwal, P. (2015). On analytical solutions of the fractional differential equation with uncertainty: application to the basset problem. *Entropy*, 17(2):885–902.

[46] Saleem, M. U., Farman, M., Aqeel H., Haque, E., and Ahmad, M. (2020). A Caputo Fabrizio fractional order model for control of glucose in insulin therapies for diabetes. *Ain Shams Engineering Journal*, 11(4):1309–1316. https://doi.org/10.1016/j.asej.2020.03.006

[47] Samko, S. G., Kilbas, A. A., Marichev, O. I., et al. (1993). *Fractional integrals and derivatives*, volume 1. Gordon and breach science publishers, Yverdon Yverdon-les-Bains, Switzerland.

[48] Shah, K., Jarad, F., and Abdeljawad, T. (2020). On a nonlinear fractional order model of dengue fever disease under caputo-fabrizio derivative. *Alexandria Engineering Journal*, 59(4):2305–2313. https://doi.org/10.1016/j.aej.2020.02.022.

[49] Shalbafian, A. and Ganjefar, S. (2023). Variable speed wind turbine control using the homotopy perturbation method. *International Journal of Precision Engineering and Manufacturing-Green Technology*, 10(1):141–150.

[50] Simelane, S. and Dlamini, P. (2021). A fractional order differential equation model for hepatitis b virus with saturated incidence. *Results in Physics*, 24:104114. https://doi.org/10.1016/j.rinp.2021.104114.

[51] Singh, H., Kumar, D., and Baleanu, D. (2019). *Methods of mathematical modelling: fractional differential equations*. CRC Press.

[52] Srivastava, H., Saad, K. M., and Khader, M. (2020). An efficient spectral collocation method for the dynamic simulation of the fractional epidemiological model of the ebola virus. *Chaos, Solitons & Fractals*, 140:110174. https://doi.org/10.1016/j.chaos.2020.110174.

[53] Sun, Z.-Z. and Gao, G.-h. (2020). Fractional differential equations. In *fractional Differential equations*. De Gruyter.

[54] Ugurlu, Y., Kaya, D., and Inan, I. E. (2011). Comparison of three semi-analytical methods for solving (1+ 1)-dimensional dispersive long wave equations. *Computers & Mathematics with Applications*, 61(5):1278–1290. https://doi.org/10.1016/j.camwa.2010.12.026.

[55] Watugala, G. (1993). Sumudu transform: a new integral transform to solve differential equations and control engineering problems. *Integrated Education*, 24(1):35–43. https://doi.org/10.1080/0020739930240105.

[56] Wazwaz, A.-M. (2007). The tanh–coth method for solitons and kink solutions for nonlinear parabolic equations. *Applied Mathematics and Computation*, 188(2):1467–1475. https://doi.org/10.1016/j.amc.2006.11.013.

[57] Yasmin, H., Abu Hammad, M., Shah, R., Alotaibi, B. M., Ismaeel, S. M., and El-Tantawy, S. A. (2023). On the solutions of the fractional-order sawada–kotera–ito equation and modeling nonlinear structures in fluid mediums. *Symmetry*, 15(3):605. https://doi.org/10.3390/sym15030605.

[58] Yunus, A. O., Olayiwola, M. O., Adedokun, K. A., Adedeji, J. A., and Alaje, I. A. (2022). Mathematical analysis of fractional-order caputo's derivative of coronavirus disease model

via laplace adomian decomposition method. *Beni-Suef University Journal of Basic and Applied Sciences*, 11(1):144. https://doi.org/10.1186/s43088-022-00326-9.

[59] Zafar, Z. U. A., Zaib, S., Hussain, M. T., Tunç, C., and Javeed, S. (2022). Analysis and numerical simulation of tuberculosis model using different fractional derivatives. *Chaos, Solitons & Fractals*, 160:112202. https://doi.org/10.1016/j.chaos.2022.112202.

[60] Zayernouri, M. and Karniadakis, G. E. (2014). Fractional spectral collocation method. *SIAM Journal on Scientific Computing*, 36(1):A40–A62. https://doi.org/10.1137/130933216.

[61] Zhang, J., Wang, Y., and Zhang, S. (2022a). A new homotopy transformation method for solving the fuzzy fractional black–scholes european option pricing equations under the concept of granular differentiability. *Fractal and Fractional*, 6(6):286. https://doi.org/10.3390/fractalfract6060286.

[62] Zhang, Y., Zhang, X., Xiong, F.-R., Liu, L., and Zhang, S. (2022b). Active vibration control of typical piping system of a nuclear power plant based on fractional pi controller. *International Journal of Dynamics and Control*, pages 1–13. https://doi.org/10.1007/s40435-022-00926-4.