

Abstract

Fractional calculus is a field of mathematical study, which grows in the same way as that of traditional integral and derivative operators of normal calculus, by replacing fractional exponents in the place of integer values. This idea of fractional calculus involving fractional derivative and integral operators is not a new one, but historically its roots go back to 1695 when L'Hôpital asked Leibniz about the result of half times derivative of function $f(x) = x$. Although, unlike integer-ordered calculus, fractional-ordered integration and differentiation do not have well-defined clear physical and geometrical interpretations, but due to its applicability in various physical phenomena to represent time-continuous processes more efficiently, it is highly popular in research areas of pure and applied mathematical reasoning as far as problems of addressing real-world problems are concerned. Especially, Fractional calculus is used in studies of viscoelastic materials, as well as in many fields of science and engineering including fluid flow, rheology, diffusive transport, electrical networks, electromagnetic theory, and probability.

A fractional differential equation (FDE) is an equation that involves derivatives of a function of a non-integer order. Solving FDEs can be challenging due to its non-locality and non-integer ordered derivatives. Several analytic, semi-analytic, and numerical methods are available in the literature for the exact or approximate solutions of FDEs. Fractional Differential equations form the backbone of various physical systems occurring in a wide range of science and engineering disciplines, and hence, the study of Fractional calculus and FDEs is an active research area with many open problems and challenges.

The present thesis has been divided to total of Seven chapters. First one represents an introduction and preliminaries regarding the proposed work. Second chapter is all about the application of the recently developed semi-analytic method (Residual power series method). For that, we choose nonlinear time-fractional Korteweg-de Vries equation as an application of shallow water wave and Kawahara equations to study Magneto-Acoustic

wave in Plasma theory.

In the Third chapter, there introduced a novel technique combining the Homotopy Perturbation Method with the Sawi transform and named it as a Homotopy Perturbation Sawi Transform Method (HPSTM). Logistic and Fornberg-Whitham equations have been solved using this method demonstrating its effective applications towards efficient solutions.

Fourth chapter showcases a new hybrid technique, the Homotopy Perturbation General Transform Method (HPGTM) for obtaining a semi-analytic solution for a wide class of FDEs. Several numerical examples including well-known equations viz. radioactive decay model, Riccati equation, backward Kolmogorov equation, Klein–Gordon equation, and Rosenau–Hyman equation are considered, establishing the key place of HPGTM in the list of semi-analytical methods to deal with FDEs.

Fifth chapter, in its nature uses Caputo and Atangana-Baleanu fractional derivatives while solving the Swada-Kotera-Ito equation (representing a study of sub-diffusion dynamics of nano-precipitate growth and destruction), using Adomian Decomposition Method blended with Shehu transform and called it here this new method as Adomian Decomposition Shehu Transform Method (ADShTM).

Chapter Six and Seven are about the study of the fractional-ordered COVID-19 model by the Homotopy Perturbation Laplace Transform Method (HPLTM) and fractional-order Diabetes model by Adomian Decomposition Laplace Transform Method (ADLTM). The existence, uniqueness, and stability analysis are established with the help of Banach's fixed point theorem. Many comparative numerical and graphical studies discussed to prove the significance of the above suggested methods.