

Journal of Mathematical Extension
Vol. 15, No. 4, (2021) (7)1-12
URL: <https://doi.org/10.30495/JME.2021.1634>
ISSN: 1735-8299
Original Research Paper

A Note on Lyapunov-Type Inequalities for Fractional Boundary Value Problems with Sturm-Liouville Boundary Conditions

A. Chavada

The Maharaja Sayajirao University of Baroda

N. Pathak*

The Maharaja Sayajirao University of Baroda

Abstract. In this note, we study Cauchy-Schwarz-type inequality for fractional Sturm-Liouville boundary value problem containing Caputo derivative of order α , $1 < \alpha \leq 2$. A lower bound for the smallest eigenvalue is determined using this inequality. We give a comparison between the smallest eigenvalue and its lower bound obtained from the Lyapunov-type and Cauchy-Schwarz-type inequalities which indicate the properties of eigenvalues.

AMS Subject Classification:34A08; 34A40; 26D10; 34C10; 33E12

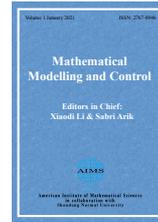
Keywords and Phrases: Lyapunov inequality; Caputo fractional derivative; Cauchy-Schwarz inequality; Mittag-Leffler function.

1 Introduction

The Lyapunov inequality [10] has proved to be very useful in the study of spectral properties and oscillation theory of ordinary differential equations. This inequality can be stated as follows [1]:

Received: April 2020; Accepted: November 2020

*Corresponding Author



Research article

Transmission dynamics of breast cancer through Caputo Fabrizio fractional derivative operator with real data

Anil Chavada and Nimisha Pathak*

Department of Applied Mathematics, Faculty of Technology and Engineering, The Maharaja Sayajirao University of Baroda, Vadodara 390002, Gujarat, India

* **Correspondence:** Email: nimisha.pathak-appmath@msubaroda.ac.in.

Abstract: In this paper, we studied the dynamical behavior of various phases of breast cancer using the Caputo Fabrizio (CF) fractional order derivative operator. The Picard-Lindelof (PL) method was used to investigate the existence and uniqueness of the proposed system. Moreover, we investigated the stability of the system in the sense of Ulam Hyers (UH) criteria. In addition, the two-step Adams-Bashforth (AB) technique was employed to simulate our methodology. The fractional model was then simulated using real data, which includes reported breast cancer incidences among females of Saudi Arabia from 2004 to 2016. The real data was used to determine the values of the parameters that were fitted using the least squares method. Also, residuals were computed for the integer as well as fractional-order models. Based on the results obtained, the CF model's efficacy rates were greater than those of the existing classical model. Graphical representations were used to illustrate numerical results by examining different choices of fractional order parameters, then the dynamical behavior of several phases of breast cancer was quantified to show how fractional order affects breast cancer behavior and how chemotherapy rate affects breast cancer behavior. We provided graphical results for a breast cancer model with effective parameters, resulting in fewer future incidences in the population of phases III and IV as well as the disease-free state. Chemotherapy often raises the risk of cardiotoxicity, and our proposed model output reflected this. The goal of this study was to reduce the incidence of cardiotoxicity in chemotherapy patients while also increasing the pace of patient recovery. This research has the potential to significantly improve outcomes of patients and provide information of treatment strategies for breast cancer patients.

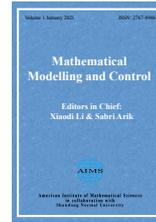
Keywords: cardiotoxicity; epidemiology; PL technique; two-step AB scheme; UH stability

1. Introduction

As per medical experts, breast cancer is characterized by the abnormal development and division of cells inside the breast tissue. This condition poses a significant risk to health and is mostly seen in the female population. According to [1], compared to other types of cancer, the most people get breast cancer. It kills breast tissue and cells, causing the breasts to grow out of control and change shape. Several risk factors have been found that put women at a higher chance of getting breast cancer. Dietary habits, personal experience of malignancy, alcohol consumption, smoking, weight status, dense breast tissue, physical inactivity, reproductive history (including pregnancy and breastfeeding), menstrual history,

genetic predisposition, racial background, life span history, specific breast changes, and getting older are all factors to consider. The main clinical presentations of breast cancer include lymphadenopathy, nipple discharge, nipple retraction, breast/nipple pain, presence of flaky skin on the breast/nipple, skin irritation, appearance of skin dimpling with erythema, changes in breast form or size, localized breast tissue thickening, and either complete or partial breast swelling. Tumor, node, and metastatic stages are used by medical professionals to evaluate a patient's cancer state and calculate their prospects of remission.

The odds of recovery are higher at earlier stages. Bisphosphonates, bone marrow transplants, gene therapy, hormone therapy, immunotherapy, surgery, stem cell therapy, targeted cancer medications, radiation, and



Research article

A fractional mathematical model for assessing cancer risk due to smoking habits

Anil Chavada¹, Nimisha Pathak¹ and Sagar R. Khirsariya^{2,*}

¹ Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, Vadodara 390001, Gujarat, India

² Department of Mathematics, Marwadi University, Rajkot 360003, Gujarat, India

* **Correspondence:** Email: ksagar108@gmail.com.

Abstract: This article presents and analyzes a mathematical model for smoking-related cancer that involves fractional-order derivative with seven different compartments. The model uses the ABC fractional derivative to describe the transmission dynamics of cancer caused by the smoking habit. We employed the Adams-Bashforth-Moulton method to find the numerical and graphical results of the model and we achieved a good level of accuracy. The existence and uniqueness of the model solution were established using Banach's fixed-point theory. For stability, we investigated the steady state points and basic reproduction number of the system. Additionally, the model's stability was discussed using the Hyers-Ulam criterion. The two-dimensional (2D) and three-dimensional (3D) simulations were performed for the different compartments and for the various values of the fractional-order parameters.

Keywords: smoking-related cancer model; Atangana-Baleanu fractional derivative; Adams-Bashforth-Moulton method; stability analysis

1. Introduction

The smoking disease, also known as chronic obstructive pulmonary disease (COPD), is a group of progressive lung diseases that cause breathing difficulties [1]. It is primarily caused by long-term exposure to cigarette smoke, although exposure to second-hand smoke, air pollution, and certain occupational fumes can also contribute to its development. Common symptoms of COPD include chest tightness, wheezing, shortness of breath, and persistent cough. Over time, the disease progressively worsens, resulting in a reduced ability to carry out physical activities and everyday tasks. Individuals with COPD may also experience frequent respiratory infections, fatigue, unintended weight loss, and swelling in the ankles, feet, or legs [2].

This smoking-related disease mainly affects the lungs' alveoli, or air sacs, and airways. The toxic compounds

in tobacco smoke affect the respiratory system and cause damage to the airways, leading to a narrowing of the air passages and mucus production [3]. As a result, the lungs become less efficient at transferring oxygen into the bloodstream, making breathing more difficult. In advanced stages, COPD can also affect the heart, leading to complications such as heart failure [4]. While there is currently no cure for COPD, there are treatments that can help to control the symptoms and delay the illness's progression. In order to effectively manage COPD, changes in lifestyle like giving up smoking, avoiding triggers in the environment, and engaging in regular exercise are essential. Medications like bronchodilators, inhaled corticosteroids, and oxygen therapy are commonly prescribed to improve breathing and reduce inflammation [5].

Smoking is a major cause of various types of cancer, including lung, oral, throat, bladder, kidney, pancreas, and stomach cancer, among others [6]. It is estimated that



Fractional mathematical modeling of breast cancer stages with true data from Saudi Arabia

Anil Chavada^a, Nimisha Pathak^{a,*}, Rutu Raval^b

^a Department of Applied Mathematics, Faculty of Technology and Engineering, The Maharaja Sayajirao University of Baroda, Gujarat, India

^b Heart & Vascular Institute, Division of Cardiology, West Virginia University, VA, USA

ARTICLE INFO

MSC:
92D30
26A33
34A08
34D20
65L05

Keywords:

Breast cancer epidemiology
Fractional order model
Krasnoselskii's fixed point theory
Routh–Hurwitz criterion
Least squares error-minimizing methodology

ABSTRACT

This research focuses on the development, analysis, and simulation of fractional mathematical models to investigate the transmission dynamics of different phases of breast cancer. The suggested breast cancer model incorporates three often-used fractional operators in epidemiology: Caputo, Caputo–Fabrizio–Caputo, and Atangana–Baleanu–Caputo operators. In this study, the determination of the equilibrium point and its stability analysis is conducted using the Routh–Hurwitz criterion. Additionally, we examine the existence and uniqueness of solutions for the fractional system using Krasnoselskii's and Banach fixed-point theory. Moreover, the global stability is discussed via the Ulam–Hyres criterion. Furthermore, the fractional models are being verified using reported occurrences of stage IV breast cancer among females in Saudi Arabia from 2004 to 2016. The real data is used to determine the values of the parameters that are fitted using the least squares error-minimizing methodology. Also, residuals and efficiency rates are computed for the integer as well as fractional-order models. Graphical representations are used to illustrate numerical results by examining different choices of fractional order parameters. Then, the dynamic characteristics of various stages of breast cancer are analyzed to demonstrate the impact of fractional order on breast cancer progression and how the rate of chemotherapy influences its behavior. We provide graphical results for a breast cancer model with effective parameters, resulting in fewer future incidences in the population of stages III and IV. Chemotherapy often raises the risk of cardiotoxicity, and our proposed model output reflects this. The goal of this study is to reduce the incidence of cardiotoxicity in chemotherapy patients while also increasing the pace of patient recovery. This research has the potential to significantly improve outcomes for patients and provide information on treatment strategies for breast cancer patients.

1. Introduction

Breast cancer is a malignancy that originates from the cells of the breast. The initiation of breast cancer often occurs inside the epithelial lining of the milk ducts or the lobules responsible for milk production. A malignant tumor has the ability to metastasize to other regions of the body [1]. An individual diagnosed with breast cancer may have localized cancerous cells in a specific area of the breast, often detectable as a palpable mass. Cancer has the ability to metastasize to either one or both breasts. Occasionally, breast cancer metastasizes to other regions of the body, such as the skeletal system, the liver, or other locations. Following lung cancer,

* Corresponding author.

E-mail addresses: anilc-appmath@msubaroda.ac.in (A. Chavada), nimisha.pathak-appmath@msubaroda.ac.in (N. Pathak), ritu99raval@gmail.com (R. Raval).

<https://doi.org/10.1016/j.rico.2024.100431>

Received 30 November 2023; Received in revised form 21 February 2024; Accepted 7 May 2024

Available online 10 May 2024

2666-7207/© 2024 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

RESEARCH ARTICLE

Fractional-order modeling of Chikungunya virus transmission dynamics

Anil Chavada¹ | Nimisha Pathak¹ | Sagar R. Khirsariya² 

¹Department of Applied Mathematics, Faculty of Technology and Engineering, The Maharaja Sayajirao University of Baroda, Vadodara, India

²Department of Mathematics, Marwadi University, Rajkot, India

Correspondence

Sagar R. Khirsariya, Department of Mathematics, Marwadi University, Rajkot 360003 Gujarat, India.
Email: ksagar108@gmail.com

Communicated by: M. Kirane

Funding information

There are no funders to report for this submission.

This article presents two innovative mathematical models for the dynamics of Chikungunya virus contamination by using Caputo fractional derivative. By applying the recently developed numerical technique to find the approximate solutions for the Chikungunya virus system which allowing us for the valuable insights. Through a rigorous analysis of the obtained numerical and graphical solutions, the impact of fractional orders on the infection dynamics is thoroughly examined. Additionally, Banach's fix point theorem is used to investigate the existence, uniqueness, and stability properties of the solutions, providing a deeper understanding of the key parameters that affect the spread and persistence of the infection.

KEYWORDS

Caputo derivative, Chikungunya virus model, qualitative and stability analysis

MSC CLASSIFICATION

92D30, 26A33, 34A08, 34C60

1 | INTRODUCTION

The Chikungunya virus is an arbovirus that is primarily transmitted to humans through the bite of infected *Aedes* mosquitoes. The virus was first discovered in 1952, during an outbreak in Tanzania. The name “Chikungunya” derives from a word in the Makonde language, meaning “to become contorted,” reflecting the severe joint pain that is a common symptom of the disease [1]. For several years after its discovery, Chikungunya remained mostly confined to regions of Africa, with occasional outbreaks reported in Asia. However, in 2004, the virus started to spread in an unprecedented manner, causing a large-scale outbreak in the Indian Ocean islands of Réunion and Mauritius. This outbreak marked the first instance of Chikungunya virus being transmitted locally in large numbers outside of Africa [2].

From 2005 onwards, Chikungunya continued to spread rapidly, affecting several countries in Asia, Europe, and the Americas. It became a significant global public health concern, with millions of cases reported and a high burden of illness [3]. The virus was able to adapt to new *Aedes* mosquito species present in these regions, allowing for its sustained transmission. The symptoms of Chikungunya virus infection include fever, severe joint pain, muscle pain, headache, fatigue, and rash [4]. While most individuals recover after a few days or weeks, joint pain can persist for months or even years in some cases.

Efforts to control and prevent the spread of Chikungunya virus focus on mosquito control, as there is no specific antiviral treatment available. Mosquito breeding sites need to be eliminated, and personal protective measures like using insect repellents and wearing long sleeves and pants are recommended [5]. The history of the Chikungunya virus serves as a reminder of the ability of mosquito-borne viruses to rapidly spread across continents and cause significant public health challenges [6]. Monitoring and surveillance of mosquito-borne diseases, as well as research and development of effective vaccines and treatments, are crucial in preventing and mitigating future outbreaks [7].

Eigen Value Estimates for Fractional Sturm-Liouville Boundary Value Problem



Anil Chavada and Nimisha Pathak

Abstract In this article, we consider fractional Sturm-Liouville boundary value problem containing Caputo derivative of order α , $1 < \alpha \leq 2$ with mixed boundary conditions. We establish Cauchy–Schwarz-type inequality to determine a lower bound for the smallest eigenvalues. We give a comparison between the smallest eigenvalues and its lower bounds obtained from the Lyapunov-type and Cauchy–Schwarz-type inequalities. The result shows that Lyapunov-type inequality gives the worse and Cauchy–Schwarz-type inequality gives better lower bound estimates for the smallest eigenvalues. We then use these inequalities to obtain an interval where a linear combination of certain Mittag-Leffler functions has no real zeros.

Keywords Lyapunov inequality · Caputo fractional derivative · Cauchy–Schwarz inequality · Mittag-Leffler function

1 Introduction

The Lyapunov inequality [6] has proved to be very useful in the study of spectral properties of ordinary differential equations. This inequality can be stated as follows [1]:

The nontrivial solution to the boundary value problem $u''(t) + q(t)u(t) = 0$, $a < t < b$, $u(a) = u(b) = 0$, exists, where $q : [a, b] \rightarrow \mathbb{R}$ is a continuous function, then

$$\int_a^b |q(s)| ds > \frac{4}{b-a}.$$

A. Chavada · N. Pathak (✉)
Department of Applied Mathematics, Faculty of Technology and Engineering,
The Maharaja Sayajirao University of Baroda, Vadodara, India
e-mail: nimisha.pathak-appmath@msubaroda.ac.in

A. Chavada
e-mail: anilc-appmath@msubaroda.ac.in



Mathematical analysis of a Pancreatic cancer with Atangana-Baleanu derivative

Anil Chavada¹, Mihir Thakkar² and Nimisha Pathak³

¹*PhD Research Scholar, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

Email: anil.c-appmath@msubaroda.ac.in

²*Master's Student, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

Email: thakkarmihir5@gmail.com

³*Assistant Professor, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

E-mail: nimisha.pathak-appmath@msubaroda.ac.in

Abstract

This article focuses on the development and analysis of a fractional mathematical model for pancreatic cancer using the Atangana - Baleanu fractional derivative. The model includes pancreatic ductal adenocarcinoma and aims to establish the existence and uniqueness of the system of the solution. The study uses a fixed-point theorem and the Lipschitz condition to establish these results. Furthermore, the paper investigates the stability of the model using the Hyers-Ulam criteria. These findings provide insights into the behaviour of the model and its ability to accurately represent pancreatic cancer. Overall, this research provides a valuable contribution to the understanding of pancreatic cancer and its mathematical modelling. The findings of this study could potentially inform the development of new treatments and therapies for pancreatic cancer by providing insights into its behaviour and progression.

Keywords: *Pancreatic cancer, Atangana-Baleanu derivative, Fixed point theorem, Hyers-Ulam stability, Stability analysis*

1. Introduction

Pancreatic cancer is a complex and challenging disease that affects millions of people worldwide. It is the seventh leading cause of cancer-related deaths worldwide. However, its toll is higher in more developed countries. Reasons for vast differences in mortality rates of pancreatic cancer are not completely clear yet, but it may be due to lack of appropriate diagnosis, treatment and cataloging of cancer cases. Because patients seldom exhibit symptoms until an advanced stage of the disease, pancreatic cancer remains one of the most lethal malignant neoplasms [4]. It is a type of cancer that develops in the pancreas, a gland located in the abdomen that produces digestive enzymes and hormones that regulate blood sugar levels. It is known for its aggressive nature and tends to spread rapidly to other parts of the body. It is often difficult to detect in its early stages, as the symptoms may not be noticeable until the cancer has advanced stage.

There are different types of pancreatic cancer, with the most common type being pancreatic ductal adenocarcinoma (PDAC). PDAC arises from the cells lining the ducts that transport digestive enzymes from the pancreas to the small intestine. Other less common types of



Numerical Simulation of Fractional Mathematical Model of Tumor Growth

ANIL CHAVADA¹, NIMISHA PATHAK², BHAVYATA PATEL³ and GEETA CHAUDHARY⁴

¹*PhD Research Scholar, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

Email: anil.c-appmath@msubaroda.ac.in

²*Assistant Professor, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

E-mail: nimisha.pathak-appmath@msubaroda.ac.in

³*PhD Research Scholar, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

E-mail: patel.bhavyata-appmath@msubaroda.ac.in

⁴*Master's Student, Department of Applied Mathematics, Faculty of Technology & Engineering, The Maharaja Sayajirao University of Baroda, India – 390002;*

Email: geetachaudhary960@gmail.com

Abstract

The main objective of this work is to investigate the dynamics and numerical approximations of the recommended arbitrary-order tumor growth model. In this work, we introduce a fractional mathematical model of Tumor growth and its treatment processes for a more agreeable solution by considering some possible factors in the Hahnfeldt et al. model with Caputo fractional derivative operator having the power law kernel. The existence and uniqueness of the arbitrary order system are investigated through the Lipschitz condition. We investigate the numerical solution of the non-linear arbitrary order tumor growth with fractional Euler method. We have calculated the root mean square error (RMSE) for cancers: HCP40, HCT116, HP40 and HSCC1(Head and Neck) with different values of α . For study, the impact of arbitrary order α on the behaviour of dynamics of tumor growth and the numerical simulation are presented for the distinct values of the arbitrary power α . Results indicate that fractional order model provides superior results over classical model.

Keywords: Fractional mathematical model, Tumor growth, Caputo fractional derivative operator, Lipschitz condition.

1. Introduction

Cancer is one of the most frequently encountered diseases that can lead to the end of human life despite the advances in science and medicine. It is a multi-staged disease which occurs because of changes in DNA formation (mutation) of abnormal cells. Cancer become second in terms of rate of mortality in the world today and approximately 9.6 million people died due to cancer in year 2018 [1]. A lot of research is going on to understand this abnormality in behavior of cancer cells. Normal healthy cells do their job systematically and stop cell division. They are replaced by newly formed daughter cells, while on the other hand cancerous cells keep dividing uncontrollably unlike normal cells and work differently. Most cancerous cells form a mass of tissue that is called a tumor. The word tumor comes from the Latin ‘tumor’ referring to the swelling that occurs as a consequence of these abnormal growths and is now used interchangeably with ‘neoplasm’, meaning new or abnormal cell growth. This definition of