

Abstract

This Ph.D. thesis delves into the mathematical analysis of diverse epidemiological models, employing various fractional derivative operators. Epidemiological models serve as indispensable tools for unraveling the intricate dynamics of disease spread and evolution. The overarching objective is to leverage advanced mathematical techniques grounded in fractional calculus to model and analyze various epidemiological phenomena. The research utilizes Caputo, Caputo-Fabrizio, and Atangana-Baleanu fractional derivative operators in the mathematical modelling of epidemiological systems. These operators facilitate the formulation of dynamic equations that capture the nuanced behavior of studied diseases, introducing a layer of complexity by accounting for memory and long-range dependencies observed in real-world epidemiological processes.

Firstly, we investigate a Cauchy-Schwarz-type inequality for a fractional Sturm-Liouville boundary value problem involving a Caputo derivative of order α , where $1 < \alpha \leq 2$. This analysis is instrumental in determining the smallest eigenvalue for the fractional boundary value problem. Secondly, we present three distinct fractional mathematical models elucidating Breast cancer dynamics, each employing various fractional derivative operators. The first model employs the Caputo fractional derivative operator to scrutinize Breast cancer growth and control through chemotherapy treatment, considering three control parameters: monoclonal antibody drug, keto diet, and z-control. In the second study, Breast cancer dynamics are explored using the Caputo-Fabrizio fractional derivative operator with real data, quantifying the dynamical behavior of different phases of Breast cancer. The aim is to understand how fractional order influences Breast cancer behavior and how chemotherapy rate impacts patient outcomes. The third investigation focuses on the development, analysis, and simulation of fractional mathematical models, incorporating Caputo, Caputo-Fabrizio-Caputo, and Atangana-Baleanu-Caputo fractional derivative

operators. These models are validated using reported cases of stage-IV Breast cancer among female patients in Saudi Arabia from 2004 to 2016. Thirdly, two novel mathematical models of Chikungunya virus dynamics are formulated, analyzed, and simulated, employing Caputo fractional derivatives. In the subsequent study, we introduce and analyze a fractional mathematical model assessing cancer risk associated with smoking habits, employing the Atangana–Baleanu–Caputo fractional derivative operator. Furthermore, the effectiveness of the Caputo fractional-order mathematical model for hard water treatment is assessed in reducing waterborne disease incidence. In another investigation, fractional-order mathematical models for Pancreatic cancer are introduced and analyzed, utilizing both Caputo and Atangana-Baleanu fractional derivative operators. Lastly, an arbitrary-order tumor growth model is explored using the Caputo fractional derivative operator.