Chapter 1

General Introduction and Historical Backgroud

Contents

This chapter deals with the general introduction and historical background of celestial mechanics, artificial satellites, classification of artificial satellites, importance of artificial satellites, forces acting on artificial satellites and mathematical control

theory. The motivation for research is subject under consideration is also included in this chapter. The layout of thesis is described at the end of this chapter.

1.1 Celestial Mechanics

Celestial mechanics is a field of study that deals with the motions of celestial bodies such as planets, moons, stars, and other objects in the universe. It seeks to understand and predict their movements using principles of physics, mathematics, and observation. This branch of science has played a crucial role in shaping our understanding of the cosmos and has profound implications for fields ranging from astronomy to space exploration. The study of celestial mechanics dates back to antiquity when early civilizations observed the regular patterns of motion exhibited by celestial bodies.

Ancient civilizations such as the Babylonians, Egyptians, and Greeks made significant contributions to our understanding of the solar systems. The Babylonians were among the first to develop sophisticated methods for predicting celestial events such as lunar eclipses and the motion of the planets. They created detailed astronomical records known as ephemerides, which recorded the positions of celestial bodies at different times of the year [21]. Greek astronomers such as Aristotle and Ptolemy furthered the study of celestial mechanics by proposing theories to explain the motions of the planets. Aristotle's geocentric model placed the Earth at the center of the universe, with the planets and stars orbiting around it in perfect circles. Ptolemy refined this model with his theory of epicycles, which accounted for the observed retrograde motion of the planets [26]. The Renaissance period witnessed a paradigm shift in our understanding of the cosmos with the work of Nicolaus Copernicus. In his groundbreaking book "De Revolutionibus Orbium Coelestium" (On

the Revolutions of the Celestial Spheres), published in 1543, Copernicus proposed a heliocentric model of the solar system with the Sun at the center and the planets orbiting around it in elliptical orbits. This marked a significant departure from the geocentric model of the ancient Greeks and laid the foundation for modern celestial mechanics [13]. Johannes Kepler, a German mathematician and astronomer, made further advancements in celestial mechanics with his three laws of planetary motion, published in the early 17th century [14]. The crowning achievement of celestial mechanics came with the publication of Sir Isaac Newton's "Philosophiæ Naturalis Principia Mathematica" (Mathematical Principles of Natural Philosophy) in 1687. In this monumental work, Newton formulated his laws of motion and universal gravitation constant G, which provided explanation for the motion of celestial bodies [41]. The advancement in technology in 20th and 21th centuries leads to sophisticated telescopes, spacecrafts, and computers enhanced our understating of celestial mechanics. Albert Einstein's theory of general relativity, published in 1915, revolutionized our understanding of gravity and its effects on the motion of celestial bodies. General relativity provides a more accurate description of the gravitational interaction between massive objects, especially in extreme conditions such as near black holes and neutron stars [17]. The exploration of space has provided invaluable data for studying celestial mechanics.

1.2   Artificial Satellites

Artificial satellites are crucial tools that humanity employs to enable communication, navigation, Earth observation, scientific research, and national security. Since the launch of the first artificial satellite, Sputnik 1, by the Soviet Union in 1957, satellites have become indispensable to modern life. This section and subsections contain the

importance of artificial satellites, the different types of satellites, the forces acting on them, and the effects of Earth's oblateness, drawing on insights from literature and research in the field.

1.2.1 Importance of Artificial Satellites

Artificial satellites are important in the following:

(1) Communication satellites are instrumental in facilitating long-distance communication. They relay signals between ground stations, enabling global telecommunications networks, internet connectivity, television broadcasting, and satellite radio services. Satellites such as Intelsat, SES, and Inmarsat provide essential communication links for businesses, governments, and individuals worldwide [19].

(2) Navigation satellites play a crucial role in modern navigation systems, providing accurate positioning, timing, and velocity information. Global navigation satellite systems (GNSS) such as GPS, GLONASS, Galileo and BeiDou enable precise navigation and timing services for aircraft, ships, vehicles and handheld devices, facilitating efficient transportation, logistics and emergency response operations [23].

(3) Earth observation satellites monitor and analyze the Earth's surface, atmosphere and oceans, providing valuable data for environmental monitoring, resource management, disaster response and scientific research. Satellites such as Landsat, MODIS, and Sentinel capture high-resolution images and data on land cover, vegetation, climate patterns, natural disasters, environmental changes, aiding policymakers, scientists and conservationists in making

informed decisions [10].

(4) Satellites enable scientific research and exploration beyond Earth's atmosphere, providing unique opportunities to study the universe, explore other planets and celestial bodies, and conduct experiments in microgravity [37].

(5) Satellites play a critical role in national security and defense, providing surveillance, reconnaissance, communication and navigation capabilities for military and intelligence purposes. Military satellites such as reconnaissance satellites, spy satellites and early warning satellites enable governments to monitor potential threats, gather intelligence, and coordinate military operations with precision and efficiency, enhancing national security and defense preparedness.

1.2.2   Types of Artificial Satellites

Depending on the orbit of the satellites, they are classified as follows [39]:

(1) Geostationary satellites orbit the Earth at an altitude of approximately 35,786 kilometers (22,236 miles) above the equator, completing one orbit in 24 hours and appearing stationary relative to a fixed point on the Earth's surface. Geostationary satellites are ideal for communication, broadcasting, and meteorological applications, providing continuous coverage of specific regions of the Earth.

(2) Low Earth orbit (LEO) satellites orbit the Earth at altitudes ranging from a few hundred kilometers to approximately 2,000 kilometers (1242 miles), completing one orbit in approximately 90 minutes. LEO satellites are used for a variety of purposes, including communication, navigation, Earth observation, and scientific research. Examples of LEO satellites include the International

Space Station (ISS), Earth observation satellites, and satellite constellations such as the Starlink constellation.

(3) Medium Earth orbit (MEO) satellites orbit the Earth at altitudes ranging from approximately 2,000 kilometers to 35,786 kilometers (22,236 miles). They are typically inclined or have elliptical orbits. MEO satellites are used primarily for navigation and positioning applications, providing regional or global coverage with higher accuracy and reliability than LEO satellites. Examples of MEO satellites include navigation satellites in the GPS, GLONASS, and Galileo constellations.

(4) Highly elliptical orbit (HEO) satellites orbit the Earth in highly elliptical orbits, with apogees (highest points) extending far beyond geostationary altitude and perigees (lowest points) dipping into low Earth orbit. HEO satellites are used for communication, navigation, scientific research applications, providing extended coverage of high-latitude regions and polar regions. Examples of HEO satellites include Molniya satellites used in Russian communication networks.

(5) Polar orbit satellites orbit the Earth in near-polar orbits, passing over the North and South Poles on each orbit and providing global coverage of the Earth's surface. Polar orbit satellites are used primarily for Earth observation, scientific research applications, capturing images, and providing data over different regions of the Earth with varying spatial and temporal resolutions. Examples of polar orbit satellites include weather satellites, environmental monitoring satellites and reconnaissance satellites.

1.2.3   Forces Acting on Artificial Satellites

Various forces are acting on the Artificial satellites. Some of them are as follows
[14]:

(1) The gravitational force exerted by the Earth is the primary force acting on
    artificial satellites, causing them to orbit the Earth in elliptical, circular, or
    geostationary orbits. The magnitude of the gravitational force depends on the
    mass of the Earth, the mass of the satellite and the distance between them.

(2) Atmospheric drag, caused by interactions between the satellite and the Earth's
    atmosphere, exerts a decelerating force on satellites in low Earth orbit, causing
    their orbits to decay over time. Atmospheric drag depends on factors such as
    the density of the atmosphere, the velocity of the satellite, the cross-sectional
    area and shape of the satellite.

(3) Solar radiation pressure, caused by the momentum transfer from solar radia-
    tion to the satellite's surface, exerts a small but significant force on satellites,
    particularly those with large surface areas or highly reflective surfaces. So-
    lar radiation pressure can affect satellites' orientation, stability, especially in
    geostationary and high-altitude orbits.

(4) Magnetic forces, resulting from interactions between the satellite's magnetic
    field and the Earth's magnetic field, can perturb the motion of satellites in
    certain orbits, particularly polar and high-altitude orbits. Magnetic forces can
    affect the orientation, attitude, and trajectory of satellites, requiring active
    control strategies to compensate for their effects.

(5) The Earth's oblateness, or equatorial bulge, results from its rotation, causing it
    to be slightly flattened at the poles and bulging at the equator. This oblateness

affects the gravitational field experienced by satellites, introducing small perturbations in their orbits and causing them to progress over time. The effects of Earth's oblateness must be accounted for in satellite orbit determination and control to ensure accurate and reliable satellite operations.

(6) The gravitational force exerted by celestial bodies like the Sun and the Moon significantly influences the motion of artificial satellites orbiting the Earth. Understanding these forces is crucial for satellite trajectory planning, orbit maintenance and overall mission success.

The Sun, being the most massive object in the solar system, exerts a dominant gravitational force on all objects within the solar system, including artificial satellites. Despite its enormous distance from the Earth compared to the Moon, the Sun's mass compensates for its distance, making its gravitational influence considerable.

Similarly, the Moon also exerts a gravitational force on satellites. Though less massive than the Sun, its proximity to Earth results in a significant gravitational influence. The Moon's gravitational pull causes tides on Earth and affects the orbits of satellites, particularly those in low Earth orbit (LEO).

When an artificial satellite orbits Earth, it experiences gravitational forces from both the Sun and the Moon. These forces cause perturbations in the satellite's orbit, leading to changes in its trajectory over time. Engineers and scientists must account for these influences when designing satellite missions to ensure accurate orbit predictions and maneuver planning[5, 40, 51].

1.3   Control Theory

Control theory is a branch of engineering and mathematics concerned with the behavior of dynamical systems. It deals with the design and analysis of systems that dynamically change over time, to influence their behavior to achieve desired objectives. Control theory finds applications in a wide range of fields, including aerospace engineering, robotics, economics, and biology, etc. This section provides an overview of control theory, its historical development, key concepts and applications.

The origins of control theory can be traced back to ancient civilizations, where basic control mechanisms were used to regulate the flow of water, control the movement of machines and maintain stability in various systems. Examples include the water clocks of ancient Egypt, the automata of ancient Greece and the water-driven escapement mechanism in medieval clocks. The Industrial Revolution marked a significant milestone in the development of control theory, as the need for automated control systems grew with the advent of mechanized production processes. In the 18th and 19th centuries, inventors and engineers began to develop mechanical devices such as governors, steam engines, and feedback control systems to regulate the speed, pressure, and temperature of industrial machinery. The mid-20th century saw the emergence of cybernetics as a multidisciplinary field that sought to understand and control complex systems. Cybernetics introduced the concept of feedback control, where the output of a system is measured and used to adjust its input, thereby regulating its behavior. This idea laid the foundation for modern control theory and has since become a central concept in the design of control systems.

Mathematical control theory emerged in the mid-20th century as a rigorous mathematical framework for analyzing and designing control systems. Building on the foundations of classical mechanics, differential equations and linear algebra, math-

ematical control theory provided powerful tools for modeling, analyzing and opti-
mizing the behavior of dynamical systems [20].

1.3.1   Key Concepts in Control Theory

(1) Dynamical Systems: A dynamical system is a mathematical model that de-
scribes the evolution of a system over time. It consists of a set of state variables
that represent the system's internal state and a set of equations that govern
how these variables change over time. Dynamical systems can be continuous
or discrete, linear or nonlinear and deterministic or stochastic, depending on
their properties and behavior.

(2) Control Systems: A control system is a dynamical system that is designed to
influence the behavior of another system, called the plant, by applying control
inputs or actions. Control systems can be classified into two main types: open-
loop and closed-loop (or feedback) control systems. In open-loop control, the
control inputs are predetermined based on a model of the plant and do not
depend on the system's output. In closed-loop control, the control inputs are
adjusted based on feedback from the system's output, allowing for greater
flexibility and robustness in controlling the system.

(3) Feedback Control: Feedback control is a fundamental concept in control the-
ory, where the output of a system is measured and used to adjust the control
inputs, thereby regulating the system's behavior. Feedback control systems
consist of four main components: the plant, the sensors, the controller and the
actuator. The sensors measure the system's output, the controller computes
the control inputs based on the measured output, a reference signal and the
actuator applies the control inputs to the plant.

(4) Stability and Performance: Two key objectives in control system design are stability and performance. Stability refers to the ability of a control system to maintain a desired equilibrium or trajectory in the presence of disturbances or uncertainties. Performance, on the other hand, refers to how well a control system achieves its specified objectives, such as tracking a reference signal, rejecting disturbances and minimizing errors.

(5) Optimization and Control Synthesis Control theory also deals with optimization and control synthesis, where the goal is to design control systems that optimize certain performance criteria subject to constraints. This involves formulating mathematical optimization problems, such as optimal control, robust control, and model predictive control, and solving them to obtain optimal control policies or strategies.

1.3.2   Applications of Control Theory

Control theory is applicable in a wide range of fields, including aerospace engineering, robotics, automotive systems, power systems, biomedical engineering and economics. Some examples of control systems include aircraft autopilots, industrial robots, cruise control systems in automobiles, power grid stabilization systems and insulin pumps for diabetes management. Control theory also plays a crucial role in emerging technologies such as autonomous vehicles, unmanned aerial vehicles (UAVs), and smart grid systems.

1.3.3 Challenges and Future Directions

Despite its successes, control theory faces several challenges, including the increasing complexity and interconnectedness of modern systems. This include for real-time decision-making, adaptation and the limitations of existing mathematical models and algorithms. Future directions in control theory include the development of advanced control techniques such as adaptive control, distributed control, and networked control, as well as the integration of control theory with other fields such as machine learning, optimization, and information theory.

Hence, Control theory is a rich and diverse field that has made significant contributions to science, engineering, and technology. From its humble beginnings in ancient times to its modern-day applications in advanced control systems, control theory continues to evolve and expand, driven by the quest to understand and manipulate the behavior of dynamical systems. In the chapters that follow, we will delve deeper into the theory, analysis, and design of control systems, exploring the mathematical principles, algorithms, and applications that underpin this fascinating discipline.

1.4 Motivation for Present Work

Artificial satellites play a crucial role in modern society, facilitating communication, navigation, weather forecasting, scientific research, and national security. The ability to precisely control the motion of these satellites is essential for ensuring their proper functioning, optimizing their performance and mitigating the risks of collision or orbital decay. This section explores the motivations behind the study of the controllability of artificial satellite motion, drawing on insights from literature and

research in the field [39, 52, 1, 27, 22].

1.4.1   Importance of Controllability in Satellite Motion

(1) Precision Orbit Determination: Accurate orbit determination is essential for the operation of satellite-based systems such as global navigation satellite systems (GNSS), Earth observation satellites and space-based telecommunications networks. By precisely controlling the motion of satellites, engineers can ensure that they maintain their intended orbits and provide reliable positioning, navigation, and timing services to users on the ground.

(2) Collision Avoidance: With the increasing proliferation of satellites in Earth's orbit, the risk of collisions between spacecraft has become a major concern for space agencies and satellite operators. By actively controlling the motion of satellites, operators can maneuver them away from potential collision hazards, thereby reducing the likelihood of costly and potentially catastrophic accidents in space.

(3) Orbital Maintenance and Debris Mitigation: Satellites are subject to various perturbations, including atmospheric drag, solar radiation pressure and gravitational interactions with other bodies, which can cause their orbits to decay over time. By implementing control strategies such as station-keeping maneuvers and orbit-raising burns, operators can extend the operational lifespan of satellites and minimize the accumulation of space debris in critical orbital regions.

(4) Mission Flexibility and Optimization: The ability to control the motion of satellites enables operators to adapt to changing mission requirements, respond to emerging threats or opportunities, and optimize the performance of

satellite-based systems. By dynamically adjusting the orbits, attitudes, and trajectories of satellites, operators can maximize their utility, resilience, and cost-effectiveness over their operational lifetimes.

1.4.2   Challenges in Satellite Motion Control

(1) Nonlinear Dynamics: The motion of artificial satellites is governed by complex nonlinear dynamics, including gravitational forces, atmospheric drag, solar radiation pressure, and magnetic interactions. Modeling and predicting the behavior of satellites in such dynamic environments pose significant challenges for control engineers, requiring sophisticated mathematical models and numerical simulations to accurately capture the underlying physics.

(2) Uncertain Environments: Satellites operate in environments characterized by high levels of uncertainty, including unpredictable atmospheric conditions, fluctuations in solar activity and variations in gravitational fields. Uncertainties in environmental parameters can degrade the performance of control algorithms and increase the risk of mission failure or loss of spacecraft, highlighting the importance of robust control techniques and adaptive strategies for satellite motion control.

(3) Limited Resources: Limited onboard resources, including computational power, energy storage, and propellant reserves often constrain satellite-based control systems. Optimizing the allocation of these resources to achieve desired control objectives while minimizing operational costs and maximizing mission longevity is a fundamental challenge in satellite motion control, requiring careful trade-offs between performance, efficiency and sustainability.

(4) Communication and Telemetry: Real-time communication and telemetry links

between ground stations and satellites are essential for monitoring the status of spacecraft, transmitting control commands and receiving telemetry data for feedback control. However, communication delays, bandwidth limitations, and signal degradation can introduce latency and uncertainty into the control loop, affecting the responsiveness & stability of satellite motion control systems.

1.4.3   Research Directions and Future Perspectives

(1) Advanced Control Techniques: Recent advances in control theory, optimization algorithms and machine learning techniques hold promise for improving the controllability of satellite motion in uncertain environments. Robust control methods, adaptive control strategies, and reinforcement learning algorithms offer new opportunities for enhancing the performance, reliability and autonomy of satellite control systems.

(2) Distributed Control Architectures: Distributed control architectures, where control algorithms are decentralized and distributed across multiple spacecraft or ground stations, can enhance the resilience and scalability of satellite-based systems. By leveraging distributed sensing, computation, and actuation capabilities, these architectures enable coordinated motion control, fault tolerance and collaborative decision-making in complex and dynamic environments.

(3) Integrated Mission Planning: Integrated mission planning frameworks that combine satellite motion control with mission scheduling, task allocation, and resource management can optimize the utilization of satellite fleets and enhance mission flexibility and responsiveness. By dynamically adapting control strategies based on mission objectives, environmental conditions, and system constraints. These frameworks enable adaptive, intelligent control of satellite

motion in response to changing mission requirements and operational scenarios.

Hence, the study of the controllability of artificial satellite motion is motivated by the critical importance of precise orbit determination, collision avoidance, orbital maintenance and mission optimization in satellite-based systems. Despite the challenges posed by nonlinear dynamics, uncertain environments, limited resources, communication constraints, recent advances in control theory, optimization algorithms, distributed architectures offer new opportunities for enhancing the controllability, reliability and autonomy of satellite motion control systems. By addressing these challenges and embracing emerging research directions, we have discussed

- Controllability analysis of the motion of artificial satellite under the effect of the oblateness of Earth

- Trajectory controllability of artificial satellite under the effect of the oblateness of Earth

in this research work.

1.5   Layout of the Thesis

The thesis entitled "Mathematical Study of Motion and Control of Low Earth Orbit Satellites Under the Effect of Earth's Zonal Harmonics and Atmospheric Drag " is divided into two parts. In the first part, we have discussed the general introduction, historical background and mathematical preliminaries related to orbital dynamics & mathematical control theory. Research work is discussed in the second part.

Chapter:1 discusses the general introduction and historical background of celestial mechanics, Artificial satellites, Control theory and Motivation to conduct the research followed by the Layout of the thesis

Chapter:2 discusses preliminaries related to the mathematical modeling of two-body problem, various perturbing forces acting on the motion of the artificial satellite and orbital elements.

Chapter:3 discussed the preliminaries related to Morden control theory of linear as well as nonlinear systems.

Chapter:4 discussed the controllability analysis of the motion of artificial satellites under the effect of the oblateness of the earth.

Chapter:5 discussed the trajectory controllability of the satellite under the effect of the oblateness of the earth.

The bibliography is provided after Chapter:5.