Synopsis of the Thesis entitled

MATHEMATICAL STUDY OF MOTION AND CONTROL OF LOW EARTH ORBIT SATELLITES UNDER THE EFFECT OF EARTH'S ZONAL HARMONICS AND ATMOSPHERIC DRAG

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1 Introduction and Historical Background

From the earliest time, man has attempted to understand the universe due to curiosity and practical concern with controlling his environment. Due to this he continuously observes the universe. Observations of the stars and planets of the universe were started by ancient civilizations such as the Babylonians, Egyptians, Greeks, and Chinese. These early astronomers recorded the positions of stars and planets and developed primitive models to explain their motions [1]. In ancient Greece, Claudius Ptolemy proposed a geocentric universe model in the 2nd century AD. According to this system, Earth was at the center, and celestial bodies moved in complex orbits around it. Ptolemy's model was widely accepted for over a millennium [2]. In the 16th century, Nicolaus Copernicus discovered that the Sun was at the center of the solar system and that Earth and other planets revolve around it. This marked a significant paradigm shift in celestial mechanics [3]. In the early 17th century, Johannes Kepler gave three laws of planetary motion based on extensive observations made by Tycho Brahe. Kepler's laws describe the elliptical orbits of planets around the Sun and provide a mathematical framework for understanding celestial mechanics [4]. After the publications of the laws of motion and universal gravitational law by Newton in the book Principia Mathematica in 1687, researchers developed mathematical models for the motion of stars, planets, and satellites, and Era of the celestial mechanics has been started. In the 18 and 19th centuries, Laplace and Langrage developed perturbation theory to study the effects of gravitational interactions between celestial bodies. Perturbation theory allows for the calculation of small deviations from idealized Keplerian orbits caused by gravitational interactions with other bodies [5]. In the 20th century, many space organizations including ISRO were formed and started to conduct research by sending astronauts into space. These agencies also launch artificial satellites into the orbit of the Earth as well as other planets. Artificial satellites are human-made objects intentionally placed into orbit around celestial bodies, typically Earth. These satellites serve various purposes, including communication, navigation, weather observation, scientific research, reconnaissance, and more. These satellites are classified by their orbit type, function, size, and the celestial body they orbit. Based on the orbits satellites are classified as:

(1) **Low Earth Orbit (LEO):** Satellites in LEO typically orbit at altitudes ranging from about 160 k.m. to 2,000 k.m above the Earth's surface. These satellites complete one revolution in 90 to 120 minutes and it is used for Earth observation, weather monitoring, and some communication satellites.

(2) **Medium Earth Orbit (MEO):** Satellites in MEO orbit at altitudes ranging from around 2,000 kilometers to 35,786 kilometers above the Earth's surface. MEO satellites are often used for navigation systems like the Global Positioning System (GPS).

(3) **Geostationary Orbit (GEO):** Satellites in GEO orbit at an altitude of approximately 35,786 kilometers above the Earth's equator. These satellites revolve at the same speed

as the Earth's rotation and they appear stationary concerning fixed points on the Earth's surface. These satellites are commonly used for telecommunications, broadcasting, and weather forecasting of specific location.

(4) **Polar Orbit:** Satellites in polar orbits pass over the Earth's poles on each orbit. These orbits are often used for Earth observation missions, as they provide global coverage over time.

(5) **Sun-Synchronous Orbit (SSO):** Satellites in SSO maintain a constant angle relative to the Sun as they orbit the Earth. This orbit type is commonly used for Earth observation satellites to ensure consistent lighting conditions for imaging.

Due to the usability of LEO satellites, the study of the motion of the LEO satellite and its life span has been a topic of interest to many researchers over the past few decades. When the orbit of the satellite is low Earth orbit (LEO), the perturbation due to the oblateness of Earth and atmospheric drag plays a very important role. Various analytic, semi-analytic, and numerical techniques are adopted for solving perturbed equations of motion. Raj [6] extensively studied the motion of satellites under the oblateness of Earth and also by considering atmospheric drag. They solved the equations of motion by applying KS transformations [7]. King-Hele [8] solved the equations of the motion of a satellite analytically by considering the oblateness of Earth. The motion of satellites in the terrestrial upper atmosphere was studied by Sehnal [9]. Knowles *et. al.* [10] analyzed the effect of geomagnetic storm's driven by a solar eruption on the upper atmosphere of Earth and its effect on the motion of satellites. The dynamics of satellite motion around the oblate Earth using a rotating frame were developed by Yan and Kapila [11]. The Hamilton equations for the motion of satellite under the Earth's oblateness and atmospheric drag were derived and solved using canonical transformation by Khalil [12]. Bezdvěk and Vokrouhlický [13] presented a semi-analytic theory for long-term dynamics of a low Earth orbit of artificial satellites, they considered both the oblateness of Earth and atmospheric drag. Some statistical measures were used by them to compare the observations over the computer efficiency. The resonance in satellite motion under air drag was studied by Bhardwaj and Sethi [14]. Hassan *et. al.* [15] tried to find a solution of equations governing the motion of artificial satellites under the effect of an oblateness of Earth by using KS variables. The authors then applied Picard's iterative method to find the solution. The algorithm is prescribed by the authors depends on the initial guess solution. The differential equations governing the relative motion of the satellite under the oblateness of Earth and atmospheric drag were derived and solved by Chen and Jing [16], the wide application of their work is in satellite attitude control and orbital maneuver for inter-planetary missions. The satellite rotational dynamics was studied and simulated by Lee *et. al.* [18], they used the Lie group variational integrator approach. Reid and Misra [17] studied the effect of aerodynamic forces on the formation flight of satellites. The analytic solution in terms of Keplerian angular elements

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of satellite orbit under atmospheric drag was studied by Xu and Chen [19]. Al-Bermani *et. al.* [20] investigated the effect of atmospheric drag and zonal harmonic J_2 for the near Earth orbit satellite namely *Cosmos*1484. The analytic solution of the motion of the satellite by considering the combined effect of Earth's gravity and air drag was found by Delhaise [21] using Lie transformations. Aghav and Gangal [22] designed and simplified the orbit determination algorithm for low Earth orbit navigation. Sharma *et.al.* studied the motion of artificial satellites under the effect of artificial satellites by considering zonal harmonics J_2 and atmospheric drag. They found that due to these forces, the satellite deviated from the earth and sometimes it hit the surface of the earth [27]. To make the satellite remain in its orbit, one has to control the motion of the satellite by plugging thrusters in various directions.

On the other hand, Mathematical controllability is a fundamental concept in control theory, which deals with the ability to steer or control the behavior of dynamical systems. A system is considered controllable if it is possible to drive it from any initial state to any desired final state within a finite time using suitable control inputs. The concept of mathematical control theory has various applications in almost all fields of science and engineering. The roots of mathematical controllability can be traced back to the mid-20th century, with the work of Richard Bellman, Lev Pontryagin, Rudolf, and Kalman. Richard Bellman's work on dynamic programming in the 1950s laid the groundwork for analyzing the controllability of dynamic systems through optimization techniques [28]. Lev Pontryagin's maximum principle, developed in the 1950s, provided a powerful tool for solving optimal control problems and characterizing controllable trajectories [29]. Rudolf Kalman, along with other researchers, made significant advancements in the analysis of linear time-invariant (LTI) systems and developed controllability criteria based on state-space representations [30]. Thereafter many researchers started to study the controllability of various linear and nonlinear systems. The detailed background of the controllability of linear and nonlinear systems is found in the paper of Levia. Hajovsky [31] used atmospheric drag as a controller to control the trajectory of an artificial satellite. B. Palancz [32] used pole placement to control the trajectory of the artificial satellite. Lamba [33], discussed controllability, observability, and stability problems concerned with artificial satellites using state space method. However, he took a two-dimensional model which leads to sets of four equations in polar form.

2   Mathematical Model of Artificial Satellite

The equation of motion of satellite without any additional perturbing force other than gravitational force between Earth and satellite is given by

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}, \tag{2.1}$$

where $\mu = GM$, G is gravitational constant and M is mass of Earth. In the presence of perturbation, additional perturbing acceleration must be added on the right side of equation (2.1). Since we are considering perturbation due to oblateness of Earth and perturbation due to atmospheric drag, the equation of motion can be written as

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_O + \vec{a}_A, \tag{2.2}$$

where $\vec{a}_O$ is acceleration due to oblateness of Earth and $\vec{a}_A$ is acceleration due to atmospheric drag. The second order equation (2.2) can be written as following set of two first order differential equations

$$\begin{aligned} \dot{\vec{r}} &= \vec{v}, \\ \dot{\vec{v}} &= -\frac{\mu}{r^3}\vec{r} + \vec{a}_O + \vec{a}_A. \end{aligned} \tag{2.3}$$

In the Cartesian co-ordinate system the system of equations (2.3) takes the form,

$$\begin{aligned} \dot{x} &= v_x, \\ \dot{y} &= v_y, \\ \dot{z} &= v_z, \\ \dot{v}_x &= -\frac{\mu x}{r^3} + \vec{a}_{O_x} + \vec{a}_{A_x}, \\ \dot{v}_y &= -\frac{\mu y}{r^3} + \vec{a}_{O_y} + \vec{a}_{A_y}, \\ \dot{v}_z &= -\frac{\mu z}{r^3} + \vec{a}_{O_z} + \vec{a}_{A_z}, \end{aligned} \tag{2.4}$$

where $\vec{a}_{O_x}, \vec{a}_{O_y}$ and $\vec{a}_{O_z}$ are components of acceleration due to oblateness of Earth in the direction x, y and z axis respectively and $\vec{a}_{A_x}, \vec{a}_{A_y}$ and $\vec{a}_{A_z}$ are components of acceleration due to atmospheric drag in x, y and z axis respectively.

The Earth's gravitational potential can be modeled in terms of zonal harmonics Battin([23]). In the expression the value of J_2 zonal coefficient is 400 times higher than other J_n zonal coefficient, $n \geq 3$. Hence we consider only J_2 into account. If these higher order zonal coefficients are neglected and taking the gradient of scalar potential function then the components of acceleration due to oblateness of Earth in the direction of x, y and z direction respectively are,

$$\begin{aligned} \vec{a}_{O_x} &= -\frac{3\mu R^2 J_2 x(x^2 + y^2 - 4z^2)}{2r^7}, \\ \vec{a}_{O_y} &= -\frac{3\mu R^2 J_2 y(x^2 + y^2 - 4z^2)}{2r^7}, \\ \vec{a}_{O_z} &= -\frac{3\mu R^2 J_2 z(3x^2 + 3y^2 - 2z^2)}{2r^7}, \end{aligned} \tag{2.5}$$

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where $R = 6378.1363 \ km$ is radius of Earth, $\mu = GM = 398600.436233 \ km^3/sec^2$ and $J_2 = 1082.63 \times 10^{-6}$.

The acceleration due to atmospheric density is given by

$$\vec{a}_A = -\frac{1}{2}\rho\frac{C_D A}{m}|\vec{v_r}|\vec{v_r}, \tag{2.6}$$

where ρ is atmospheric density, C_D is drag coefficient, A is cross-sectional area of the satellite perpendicular to velocity vector, m is mass of satellite and $\vec{v_r}$ is satellite velocity vector relative to an atmosphere.

We take the simple exponential atmospheric model for which atmospheric density given by,

$$\rho = \rho_{pa}e^{\left[\frac{(r_{pa}-r)}{H}\right]}, \tag{2.7}$$

where ρ_{pa} is the density at initial perigee point, r_{pa} is the initial distance of satellite from Earth's surface, $r = |\vec{r}|$ and H is scale height. The ratio $B^* = \frac{C_D A}{m}$ is called the Ballistic coefficient.

We assume that the atmosphere rotates at the same angular speed as Earth. With this assumption the relative velocity vector is given by Wiesel [24]

$$\vec{v_r} = \vec{v} - \vec{\omega} \times \vec{r}, \tag{2.8}$$

where, $\vec{\omega}$ is the inertial rotation vector of the Earth given by

$$\vec{\omega} = \omega_e \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{2.9}$$

where, $\omega_e = 7.292115486 \times 10^{-5} \ rad/sec$. The cross product of the (2.8) and (2.9) gives three components of the relative velocity vector as

$$\vec{v_r} = \begin{bmatrix} v_x + \omega_e r_y \\ v_y - \omega_e r_x \\ v_z \end{bmatrix}. \tag{2.10}$$

Substituting (2.7), (2.10) and B^* in (2.6), we get the components of acceleration due to atmospheric drag in the direction of x, y and z axis respectively as

$$a_{A_x} = -\frac{\rho_{pa}e^{\left[\frac{r_{pa}-r}{H}\right]}\sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2}\,(v_x + \omega_e r_y)B^*}{2},$$

$$a_{A_y} = -\frac{\rho_{pa}e^{\left[\frac{r_{pa}-r}{H}\right]}\sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2}\,(v_y - \omega_e r_x)B^*}{2}, \tag{2.11}$$

$$a_{A_z} = -\frac{\rho_{pa}e^{\left[\frac{r_{pa}-r}{H}\right]}\sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2}\,v_z B^*}{2}.$$

Substituting (2.5) and (2.11) in (2.4), we get equations of motion of satellite under the oblateness of Earth and atmospheric drag as

$$\dot{x} = v_x,$$
$$\dot{y} = v_y,$$
$$\dot{z} = v_z,$$
$$\dot{v}_x = -\frac{\mu x}{r^3} - \frac{3\mu R^2 J_2 x(x^2 + y^2 - 4z^2)}{2r^7} - \frac{\rho_{pa} e^{[\frac{r_{pa}-r}{H}]} \sqrt{(v_x + \omega_e r_y)^2 + (vy - \omega_e r_x)^2 + v_z^2}\,(v_x + \omega_e r_y)B^*}{2}, \quad (2.12)$$
$$\dot{v}_y = -\frac{\mu y}{r^3} - \frac{3\mu R^2 J_2 y(x^2 + y^2 - 4z^2)}{2r^7} - \frac{\rho_{pa} e^{[\frac{r_{pa}-r}{H}]} \sqrt{(v_x + \omega_e r_y)^2 + (vy - \omega_e r_x)^2 + v_z^2}\,(vy - \omega_e r_x)B^*}{2},$$
$$\dot{v}_z = -\frac{\mu z}{r^3} - \frac{3\mu R^2 J_2 z(3x^2 + 3y^2 - 2z^2)}{2r^7} - \frac{\rho_{pa} e^{[\frac{r_{pa}-r}{H}]} \sqrt{(v_x + \omega_e r_y)^2 + (vy - \omega_e r_x)^2 + v_z^2}\,v_z B^*}{2}.$$

Sharma *et.al.* analyzed the orbit of the satellite under various initial conditions.

3 Controllablity of Dynamical Systems

This section discussed the controllability of linear and nonlinear dinemical systems using the functional analytic approach.

3.1 Linear System

Consider a dynamical system with linear control,

$$x'(t) = A(t)x(t) + B(t)u(t)$$
$$x(0) = x_0 \quad\quad\quad (3.1)$$

Here, for all t the state $x0, x(t) \in \mathbb{R}^n$, $u \in L^2([t_0, t_1], \mathbb{R}^m)$, and $A(t), B(t)$ are matrices of dimensions $n \times n, n \times m$ respectively. Let $\Phi(t, t_0)$ be an transition matrix generated by the homogeneous system $x(t) = A(t)x(t)$, then the solution of (3.1) is given by:

$$x(t) = \Phi(t, t_0)x_0 + \int_0^t \Phi(t, s)B(s)u(s)ds \quad\quad\quad (3.2)$$

Definition 3.1. *The system* (3.1) *is controllable over the interval* $[0, t_1]$ *if for each pair of vectors* x_0 *and* x_1 *in* $\mathbb{R}^n$, *there is a control* $u \in L^2([t_0, t_1], \mathbb{R}^m)$ *such that the solution of* (3.2) *with* $x(0) = x_0$ *satisfies* $x(t_1) = x_1$.

Theorem 3.1. *The following statements are equivalent*

1 *The system* (3.1) *is controllable over the interval* $[0, t_1]$.

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2 The operator $C : L^2([t_0, t_1], \mathbb{R}^m) \times \mathbb{R}^n$ defined by $Cu = \int_0^{t_1} \Phi(t_1, s)B(s)u(s)ds$ is onto.

3 The operator $C* : \mathbb{R}^n \times L^2([t_0, t_1], \mathbb{R}^m)$ (adjoint of C) is one one.

4 The controllability grammian $W : \mathbb{R}^n \times \mathbb{R}^n$ defined by $W = (CC^*)$ is non singular.

5 The control function $u(t) = C^*W^{-1}(x_1 - \Phi(t_1, 0)x_0)$ steers the system from initial state x_0 to desire final state x_1 at $t = t_1$.

3.2 Nonlinear System

Consider a nonlinear system with linear control,

$$x'(t) = A(t)x(t) + B(t)u(t) + f(t, x(t))$$
$$x(0) = x_0 \tag{3.3}$$

Here, for all t the state $x0, x(t) \in \mathbb{R}^n$, $u \in L^2([t_0, t_1], \mathbb{R}^m)$, $f : [0, t_1] \times \mathbb{R}^n \times \mathbb{R}^n$ is nonlinear function, and $A(t), B(t)$ are matrices of dimensions $n \times n, n \times m$ respectively. Let $\Phi(t, t_0)$ be an transition matrix generated by the homogeneous system $x(t) = A(t)x(t)$, then the mild solution of (3.3) is given by:

$$x(t) = \Phi(t, t_0)x_0 + \int_0^t \Phi(t, s)B(s)u(s)ds + \int_0^t \Phi(t, s)f(s, x(s))ds \tag{3.4}$$

Definition 3.2. *The system* (3.3) *is controllable over the interval* $[0, t_1]$ *if for each pair of vectors* x_0 *and* x_1 *in* $\mathbb{R}^n$, *there is a control* $u \in L^2([t_0, t_1], \mathbb{R}^m)$ *such that*

$$x_1 = \Phi(t_1, t_0)x_0 + \int_0^{t_1} \Phi(t_1, s)B(s)u(s)ds + \int_0^{t_1} \Phi(t_1, s)f(s, x(s))ds. \tag{3.5}$$

Given the equation (3.5) the problem of controllability of the nonlinear system (3.3) reduces to the solvability of the equation (3.5).

4 Layout of the Thesis

The thesis entitled "Mathematical Study of Motion and Control of Low Earth Orbit Satellites Under the Effect of Earth's Zonal Harmonics and Atmospheric Drag " is divided into two parts. In the first part, we have discussed the general introduction & historical background, and mathematical preliminaries related to orbital dynamics & mathematical control theory. Research work is discussed in the second part.
Chapter:1 discusses the general introduction and historical background of celestial mechanics.

Chapter:2 discusses preliminaries related to orbital mechanics.

Chapter:3 discussed the preliminaries related to mathematical control theory.

Chapter:4 discussed the controllability analysis of the motion of artificial satellites under the effect of the oblateness of the earth.

Chapter:5 discussed the trajectory controllability of the satellite under the effect of the oblateness of the earth.

5   List of articles published in Journals

1 Controllability Analysis of Motion of Artificial Satellite Under the Effect of Oblateness of the Earth, Tuijin Jishu/Journal of Propulsion Technology, Vol. 44 No. 4 (2023).

2 Trajectory Controllability of Artificial Satellite Under the Effect of Oblateness of Earth, Tuijin Jishu/Journal of Propulsion Technology, Vol. 44 No. 3 (2023).

6   Paper presented in Conferences

1 Presented the paper entitled "Controllability of Motion of Artificial Satellite under the Effect of Zonal Harmonic J_2" in the National Conference on Computational Mathematics organized by the National Institute of Technology, Puducherry, Karaikal on 22-23rd December, 2022.

2 Presented the paper entitled "Trajectory Controllability of Artificial Satellite Under the Effect of Oblateness of Earth" in the International Symposium on Mathematical Analysis of Fractals and Dynamical Systems-2023 organized by Vellore Institute of Technology, Vellore on 24-24th August,2023.

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