Chapter 2

Mathematical Modeling of Power System

2.1 General

The mathematical model of synchronous machine can be described by a set of differential equations representing the dynamics of the machines, exciters and other controls and algebraic equations representing the network relation. The model considered for the stability analysis in this thesis are described below:

2.2 Generator model

Fourth order model of generator [33, 15, 43, 6] has been used in the present study as described below:

The dynamics of the synchronous generator can be represented by the following equations:

$$\dot{\delta} = \omega_B \left(\omega_m - \omega_{m_0}\right) \tag{2.1}$$

$$\dot{\omega_m} = \frac{1}{2H}\left(-k_d\left(\left(\omega_m - \omega_{m_0}\right)\right) + T_m - T_e\right) \tag{2.2}$$

where δ is generator's rotor angle, ω_m the speed deviation, H machine inertia constant, T_m is the Mechanical power input to generator, T_e is the electrical power output of the generator, k_d the damping constant.

The electrical torque equation is represented by following algebraic equation:

$$T_e = E_d^{'} i_d + E_q^{'} i_q + (x_d^{'} - x_q^{'}) i_d i_q \tag{2.3}$$

where i_d and i_q are d-axis and q-axis current respectively, $E_d^{'}$ and $E_q^{'}$ are d-axis and q-axis transient voltage, $x_d^{'}$ and $x_q^{'}$ are d-axis and q-axis transient reactance.

The effect of saliency is considered, the chages in flux linkage of the filed winding have to be accounted for along the d and q axes. Therefore, two more additional state equation (2.4) and (2.5) along with the swing equations (2.1) and (2.2) have to be considered.

$$\dot{E}_q^{'} = \frac{1}{T_{d0}^{'}} \left[\left(-E_q^{'} + \left(x_d - x_d^{'} \right) i_d \right) + E_{fd} \right] \tag{2.4}$$

$$\dot{E}_d^{'} = \frac{1}{T_{q0}^{'}} \left[\left(-E_d^{'} - \left(x_q - x_q^{'} \right) i_q \right) \right] \tag{2.5}$$

where $T_{d0}^{'}$ and $T_{q0}^{'}$ are d-axis and q-axis open circuit time constant, x_d and x_q d-axis and q-axis synchronous reactance,

The line resistance is considering very low, it equal to be zero ohms. The stator d and q axes current and voltage algebraic equations can written as follows:

$$i_d = \frac{E_b cos\delta - E_q^{'}}{\left(x_e + x_d^{'}\right)} \tag{2.6}$$

$$i_q = \frac{E_b sin\delta + E_d^{'}}{\left(x_e + x_q^{'}\right)} \tag{2.7}$$

$$v_q = -x_e i_d + E_b cos\delta \tag{2.8}$$

$$v_d = -x_e i_q - E_b sin\delta \tag{2.9}$$

$$V_t = \sqrt{v_d^2 + v_q^2} \tag{2.10}$$

where x_e is the line reactance, v_d and v_q are d-axis and q-axis voltage, V_t the terminal voltage and E_b infinite bus voltage.

2.3 Excitation System

The IEEE type -ST1 exciter [33, 43] has been considered in this study and equation governing the dynamics is given as follows:

$$\dot{E}_{fd} = -\frac{1}{T_A}E_{fd} + \frac{K_A}{T_A}\left(V_{ref} - V_t\right) \tag{2.11}$$

where E_{fd}= field excitation voltage

K_A =Exciter gain

T_A =Exciter time constant

V_{ref} =Reference voltage setting

V_t =Terminal voltage

2.3.1 Initial Conditions

The power system is described by set of non linear differential equations and is required to be solved numerically. It is assumed that the system is at a stable equilibrium point till t=0. It is necessary to calculate the initial conditions at time t=0 based on power system operating points. Calculation of initial conditions [33, 43] are very important for power system stability analysis. The operating points are calculated from load flow analysis [41, 15].

The initial conditions are calculated from following set of Equations:

Calculation of $\hat{I}_{a0}$ from Equation (2.12).

$$\hat{I}_{a0} = I_{a0}\angle\phi_0 = \frac{P_t - jQ_t}{V_{t0}\angle - \theta_0} \tag{2.12}$$

δ_0 and E_{q0} are calculated from Equation (2.13).

$$E_{q0}\angle\delta_0 = V_{t0}\angle\theta_0 + \left(R_a + jx_q\right) I_{a0}\angle\phi_0 \tag{2.13}$$

The different variables i_{d0}, i_{q0},v_{d0}, E_{fd0}, E'_{q0}, E'_{d0} and T_{e0} are calculated from following Equations:

$$i_{d0} = -I_{a0}sin\left(\delta_0 - \phi_0\right) \tag{2.14}$$

$$i_{q0} = I_{a0} cos\left(\delta_0 - \phi_0\right) \tag{2.15}$$

$$v_{d0} = -V_{to} sin\left(\delta_0 - \phi_0\right) \tag{2.16}$$

$$v_{q0} = V_{to} cos\left(\delta_0 - \phi_0\right) \tag{2.17}$$

$$E_{fdo} = E_{q0} - \left(x_d - x_q\right) i_{d0} \tag{2.18}$$

$$E'_{q0} = E_{fd0} + \left(x_d - x'_d\right) i_{d0} \tag{2.19}$$

$$E'_{d0} = -\left(x_q - x'_q\right) i_{q0} \tag{2.20}$$

$$T_{e0} = E'_{q0} i_{q0} + E'_{d0} i_{d0} + \left(x'_d - x'_q\right) i_{d0} i_{q0} = T_{m0} \tag{2.21}$$

2.4 Linearization and Eigen Properties

2.4.1 Linearization

The dynamic system can be represented in a set of n first order non-linear differential equations [88, 15].

$$\dot{x} = f\left(x, u\right) \tag{2.22}$$

$$y = g\left(x, u\right) \tag{2.23}$$

Where x is the state vector with n state variable, u is input vector to the system, and y is the output vector.

To investigate the small-signal stability at one operating point, equation (2.22) and (2.23) need to be linearized. Assume x_0 is the initial state vector at the current operating point and u_0 is the corresponding input vector. As the perturbation is considered small, the nonlinear function f can be expressed in terms of Taylor's series expansion. With terms involving second and higher order powers of Δx and Δu are neglected, we may write

$$\dot{x}_i = \dot{x}_{i0} + \Delta\dot{x}_{i0} = f_i\left[\left(x_0 + \Delta x\right), \left(u_0 + \Delta u\right)\right] \tag{2.24}$$

$$= f_i(x_0, u_0) + \frac{\partial f_i}{\partial x_1}\Delta x_1 + ... + \frac{\partial f_i}{\partial x_n}\Delta x_n + \frac{\partial f_i}{\partial u_1}\Delta u_1 + ... + \frac{\partial f_i}{\partial x_1}\Delta u_r \tag{2.25}$$

$$x_{i0} = \dot{f_i}(x_{0,}u_0)$$

$$\Delta \dot{x_i} = \frac{\partial f_i}{\partial x_1}\Delta x_1 + ... + \frac{\partial f_i}{\partial x_n}\Delta x_n + \frac{\partial f_i}{\partial u_1}\Delta u_1 + ... + \frac{\partial f_i}{\partial x_1}\Delta u_r \tag{2.26}$$

$$\Delta y_j = \frac{\partial g_j}{\partial x_1}\Delta x_1 + ... + \frac{\partial g_j}{\partial x_n}\Delta x_n + \frac{\partial g_j}{\partial u_1}\Delta u_1 + ... + \frac{\partial g_j}{\partial u_r}\Delta u_r \tag{2.27}$$

Therefore, the linearized forms of Equation (2.26) and (2.27) are written as:

$$\Delta \dot{x} = A\Delta x + B\Delta u \tag{2.28}$$

$$\Delta y = C\Delta x + D\Delta u \tag{2.29}$$

Where A is the state matrix, B the control matrix, C the output matrix, and D is the feed forward matrix. For the stability analysis and eigen value analysis of the synchronous machine, the state matrix A is the most important. This matrix can be represented by Equations (2.30) and (2.31). The block diagram of state space is represented by Figure 2.1.

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & . & . & . & \frac{\partial f_1}{\partial x_n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{\partial f_n}{\partial x_1} & . & . & . & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & . & . & . & \frac{\partial f_1}{\partial u_r} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{\partial f_n}{\partial u_1} & . & . & . & \frac{\partial f_n}{\partial u_r} \end{bmatrix} \tag{2.30}$$

$$C = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & . & . & . & \frac{\partial g1}{\partial x_n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{\partial g_m}{\partial x_1} & . & . & . & \frac{\partial g_n}{\partial x_n} \end{bmatrix} \quad D = \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & . & . & . & \frac{\partial g_1}{\partial u_r} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{\partial g_m}{\partial u_1} & . & . & . & \frac{\partial g_m}{\partial u_r} \end{bmatrix} \tag{2.31}$$

Δx is the state vector of dimension n

Δy is the output vector of dimension m

Δu is the input vector of dimension r

A is the state or plant matrix of size $n \times n$

B is the control or input matrix of $n \times r$

C is the output matrix of size $m \times n$

D is the feedforward matrix $m \times r$

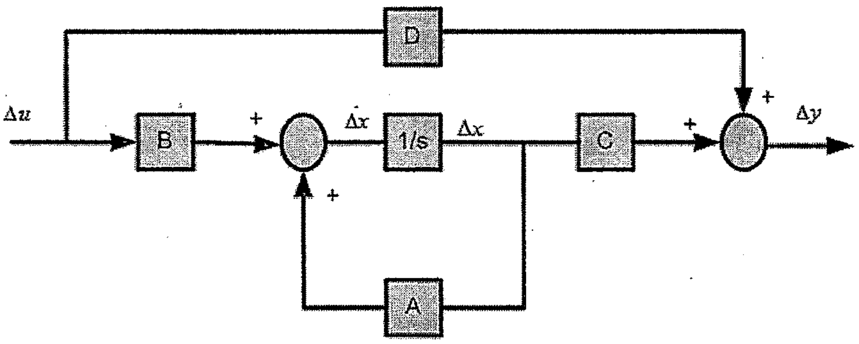

Figure 2.1: Block diagram of State Space Representation

2.4.2 Sate Space Representation

The Laplace transform of the equations (2.26) and (2.27), the state equations (2.32) and (2.33) are presented in the time domain as per follow [65, 88, 97]:

$$s\Delta x(s) - \Delta x(0) = A\Delta x(s) + B\Delta u(s) \tag{2.32}$$

$$\Delta y(s) = C\Delta x(s) + Du(s) \tag{2.33}$$

Rearranging Equation (2.32), we have

$$(SI - A)\Delta x(s) = \Delta x(0) + B\Delta u(s) \tag{2.34}$$

Hence

$$\Delta x(s) = (SI - A)^{-1}\left[\Delta x(0) + Bu(s)\right] \tag{2.35}$$

$$= \frac{adj(SI-A)^{-1}}{det(SI-A)^{-1}}\left[\Delta x(0)+Bu(s)\right] \quad (2.36)$$

and correspondingly,

$$\Delta y(s) = C\frac{adj(SI-A)^{-1}}{det(SI-A)^{-1}}\left[\Delta x(0)+Bu(s)\right]+D\Delta u(s) \quad (2.37)$$

The Laplace transforms of Δx and Δy consist two component, one dependent on the initial conditions and the other on the inputs. These are the Laplace transforms of the free and zero-state components of the state and output vectors. The poles of $\Delta x(s)$ and $\Delta y(s)$ are the roots of the Equation:

$$det(SI-A)=0 \quad (2.38)$$

The values of s which satisfy the above are known as eigen values of matrix A, and Equation is refereed as the characteristics Equation of matrix A.

2.4.3 Eigen Values

The eigen values of a matrix [88] are given by the values of the scalar parameter λ for which there exist non-trivial solutions to the equation

$$A\phi = \lambda\phi \quad (2.39)$$

where

A is an $n \times n$ matrix

$\phi$is an $n \times 1$vector

For the calculation of eigen value, the equation (2.39) may be written in the form

$$(A-\lambda I)\phi = 0 \quad (2.40)$$

For a non-trivial solution

$$det(A-\lambda I)=0 \quad (2.41)$$

Expression of the determinant gives the characteristics equation. The r solution of (2.41) $\lambda = \lambda_1, \lambda_2 \ldots\ldots, \lambda_n$are eigen values of A.

2.4.3.1 Eigen Vector

For any eigen value $\lambda_p$the n-column $\phi_p$which satisfies (2.39) is called the right eigen vector [15] of A associated with the eigen value λ_p.

$$A\phi_p = \lambda_p \phi_p \tag{2.42}$$

$p = 1, 2, \ldots\ldots r$

The right- eigen vector is represented by equation (2.43)

$$\phi_p = \begin{bmatrix} \phi_{1p} \\ \phi_{2p} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{rp} \end{bmatrix} \tag{2.43}$$

Similarly, the r row vector $\psi_p$which satisfies the equation

$$\psi_p A = \lambda_p \psi_p \tag{2.44}$$

$p = 1, 2, \ldots\ldots r$ is called the left eigen vector associated woth the eignvalue λ_p.

$$\psi_p = \begin{bmatrix} \psi_{1p} & \psi_{2p} & \cdot & \cdot & \cdot & \psi_{rp} \end{bmatrix} \tag{2.45}$$

The left and right eigen vectors corresponding to different eigen values are orthoginal, i.e.

$$\psi_q \phi_p = 0 \tag{2.46}$$

$\lambda_p \neq \lambda_q$and

$$\psi_q \phi_p = \alpha_p \tag{2.47}$$

where $\lambda_p = \lambda_q$ and α_p is a non zero constant. To normalized these vector so that

$$\psi_q \phi_p = 1 \tag{2.48}$$

2.4.3.2 Participation Factor

Participation factor [48, 4, 15] is used for identifying the state variables that have significant participation on a selected mode among many modes in a multigenerator power system. Participation matrix (P), which combines the right and left eigen vectors entries and used as measure of the association between the state variables and the modes.

$$P = \begin{bmatrix} P_1 & P_2 & . & . & . & P_r \end{bmatrix} \tag{2.49}$$

with

$$P_p = \begin{bmatrix} P_{1p} \\ P_{2p} \\ . \\ . \\ . \\ P_{rp} \end{bmatrix} = \begin{bmatrix} \phi_{1p}\psi_{p1} \\ \phi_{2p}\psi_{p2} \\ . \\ . \\ . \\ \phi_{1p}\psi_{pr} \end{bmatrix} \tag{2.50}$$

where

ϕ_{KP} =the element on the kth row and pth column of the modal matrix ϕ

$=kth$ entry of the right eigen vector ϕ_p

ψ_{PK}=the element on the pth row and kth column of the modal matrix ψ

$=kth$ entry of the right eigen vector ψ_p

The element P_{KP}=$j_{KP}\psi_{PK}$ is the termed as the participation factor. ϕ_{KP} measures the activity of the variable X_K in the pth mode, and ψ_{PK} gives the weights of contribution of this activity to the mode, the product P_{KP} measures the net participation. The effect of multiplying the element of the left and right eigen vectors makes the P_{KP} dimensionless.

2.4.3.3 Eigen value and Stability

The time dependent characteristics of a mode corresponding to an eigen value λ_i is given by $e^{\lambda_1 t}$ [88, 15]. Hence the stability of the system is determined by the eigen values as follows:

1. A real eigen value corresponds to a non-oscillatory mode. A negative real eigen value describes a decaying mode. The large its magnitude, the faster the decay. A positive real eigen value represents aperiodic instability.

2. Complex eigen values occur in conjugate pairs, and each pair corresponds to an oscillatory mode. The real components of the eigen values give the damping, and the imaginary components present the frequency of oscillation. A negative real part describes a damped oscillation, whereas a positive real part represents oscillation of increasing amplitude.

Thus λ for a complex pair of eigen values is represented by equation (2.51).

$$\lambda = \sigma \pm j\omega \tag{2.51}$$

where σ and ω show the real part and imaginary part of the eigen value.

The frequency of oscillation in Hz is given by equation (2.52), which represents the actual of damped frequency.

$$f = \frac{\omega}{2\pi} \tag{2.52}$$

The equation (2.53) represents the damping factor

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}} \tag{2.53}$$

The damping ratio ζ determines the rate of decay of the amplitude of the oscillation. The time constant of amplitude decay is $\frac{1}{|\sigma|}$.

2.5 Linear model of power system

The single machine infinite bus system (SMIB) is shown in Figure 2.2, where V_t and E_b are generator terminal voltage and infinite bus voltage respectively. The X_e and X_t are transmission line reactance and transformer reactance respectively. The dynamic model of the synchronous machine and exciter described by section 2.2 and 2.3 respectively are linearized about its initial conditions using linerization concept, which is described by section (2.4.1). After linearization of the equations (2.6) and (2.7) of i_d and i_q and substituting these equations in (2.8), (2.9) and (2.10) yield the linearized equations of V_q, V_d, V_t and T_e. The linearized equations are represented by equations (2.54) to (2.61).

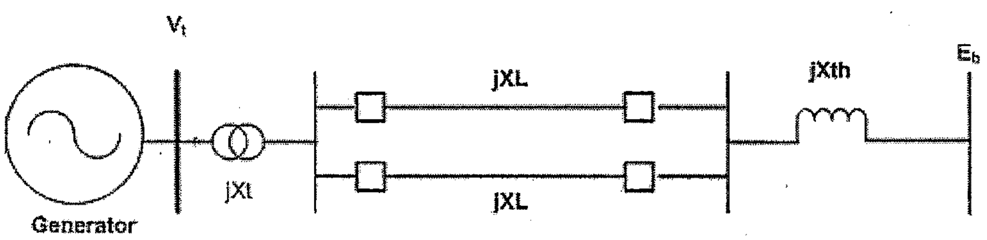

Figure 2.2: Single Machine Infinite Bus System

2.5.1 Calculation of K_1 to K_{10} Constants

The linerized form of Δi_d and Δi_q are represented by equations (2.54) and (2.55).

$$\Delta i_d = P_1 \Delta\delta + P_2 \Delta E_q' \tag{2.54}$$

$$\Delta i_q = P_3 \Delta\delta + P_4 \Delta E_d' \tag{2.55}$$

Where

$$P_1 = \frac{-E_b sin\delta_0}{x_e + x_d'}, \quad P_2 = -\frac{1}{x_e + x_d'}, \quad P_3 = \frac{E_b cos\delta_0}{x_e + x_q'}, \quad P_4 = \frac{1}{x_e + x_q'} \tag{2.56}$$

The linerized equations of Δv_d, Δv_q, V_t and ΔT_e are shown by equations (2.57) to (2.61).

$$\Delta V_d = \left(-E_b cos\delta_0 + P_3 x_e\right)\Delta\delta + x_e P_4 \Delta E_d^{'} \tag{2.57}$$

$$\Delta V_q = \left(-E_b sin\delta_0 + P_1 x_e\right)\Delta\delta - x_e P_2 \Delta E_q^{'} \tag{2.58}$$

$$\Delta T_e = K_1\Delta\delta + K_2\Delta E_q^{'} + K_3\Delta E_d^{'} \tag{2.59}$$

$$\Delta V_t = \frac{V_{d0}}{V_{t0}}\Delta v_d + \frac{V_{q0}}{V_{t0}}\Delta v_q \tag{2.60}$$

$$\Delta V_t = K_8\Delta\delta + K_9\Delta E_q^{'} + K_{10} E_d^{'} \tag{2.61}$$

The fourth-order model of the synchronous machine is considered and K_1 to K_{10} constant are derived by substituting the above equations in linearized form of the machine state equations (2.1), (2.2), (2.4), (2.5) and (2.11). The linearized forms of the machine state Equations are represented by equations (2.62) to (2.66).

$$\dot{\delta} = \omega_b \Delta\omega_m \tag{2.62}$$

$$\Delta\dot{\omega}_m = -\frac{k_d}{2H}\Delta\omega_m + \frac{1}{2H}\Delta T_m - \frac{K_1}{2H}\Delta\delta - \frac{K_2}{2H}\Delta E_q^{'} - \frac{K_3}{2H}\Delta E_d^{'} \tag{2.63}$$

$$\Delta\dot{E}_q^{'} = \frac{1}{T_{d0}^{'}}\left(\Delta E_{fd} - K_5\Delta\delta - \frac{\Delta E_q^{'}}{K_4}\right) \tag{2.64}$$

$$\Delta\dot{E}_d^{'} = \frac{1}{T_{q0}^{'}}\left(K_7\Delta\delta - \frac{\Delta E_d^{'}}{K_6}\right) \tag{2.65}$$

$$\Delta\dot{E}_{fd} = -\frac{K_A}{T_A}\Delta\delta - \frac{K_A K_9}{T_A}\Delta E_q^{'} - \frac{K_A K_{10}}{T_A}\Delta E_d^{'} + \frac{K_A}{T_A}\Delta V_{ref} + \frac{1}{T_A}\Delta_{fd} \tag{2.66}$$

2.5.2 State Space Representation of System

The expression (2.67) is described linearized power system model in state space form $\Delta\dot{x} = A\Delta x + B\Delta u$ and machine constant K_1 to K_{10} are described by equations (2.68) to (2.77). Figure 2.3 represents the block diagram of SMIB.

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega}_m \\ \Delta\dot{E}'_q \\ \Delta\dot{E}'_d \\ \Delta\dot{E}_{fd} \end{bmatrix} = \begin{bmatrix} 0 & \omega_B & 0 & 0 & 0 \\ -\frac{K_1}{2H} & -\frac{D}{2H} & -\frac{K_2}{2H} & -\frac{K_3}{2H} & 0 \\ -\frac{K_5}{T'_{d0}} & 0 & -\frac{1}{T'_{d0}K_4} & 0 & \frac{1}{T'_{d0}} \\ \frac{K_7}{T'_{q0}} & 0 & 0 & -\frac{1}{T'_{q0}K_6} & 0 \\ -\frac{K_AK_8}{T_A} & 0 & -\frac{K_AK_9}{T_A} & -\frac{K_AK_{10}}{T_A} & -\frac{1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega_m \\ \Delta E'_q \\ \Delta E'_d \\ \Delta E_{fd} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \frac{1}{2H} \\ 0 \\ 0 \\ \frac{K_A}{T_A} \end{bmatrix} \begin{bmatrix} 0 & \Delta T_m & 0 & 0 & V_{ref} \end{bmatrix} \tag{2.67}$$

where,

$$K_1 = \frac{\partial T_e}{\partial\delta} = -\left[E'_{do} + \left(\left(x'_d - x'_q\right) i_{q0}\right)\right] \frac{E_{d0}E_b sin\delta_0}{x_e + x'_d} +$$

$$\left[\left(x'_d - x'_q\right) i_{d0} + E'_{q0}\right] \frac{E_b cos\delta_0}{x_e + x'_{dq}} \tag{2.68}$$

$$K_2 = \frac{\partial T_e}{\partial E'_q} = -\left[E'_{do} + \left(\left(x'_d - x'_q\right) i_{q0}\right)\right] \frac{1}{x_e + x'_d} + E'_{q0}\frac{1}{x_e + x'_q} + i_{q0} \tag{2.69}$$

$$K_3 = \frac{\partial T_e}{\partial E'_d} = \left[i_{d0} + \left(\left(x'_d - x'_q\right) i_{d0}\right)\right] \frac{1}{x_e + x'_q} \tag{2.70}$$

$$K_4 = \frac{\partial E'_q}{\partial E_q} = \frac{x_e + x'_d}{\left(x_e + x'_d\right) + \left(x_d - x'_d\right)} \tag{2.71}$$

$$K_5 = \frac{\partial E'_q}{\partial\delta} = \left(x_d - x'_d\right) \frac{E_b sin\delta_0}{x_e + x'_d} \tag{2.72}$$

$$K_6 = \frac{\partial E'_d}{\partial E_d} = \frac{x_e + x'_q}{\left(x_e + x'_q\right) + \left(x_q - x'_q\right)} \tag{2.73}$$

$$K_7 = \frac{\partial E_d^{'}}{\partial \delta} = -\left(x_q - x_q^{'}\right)\frac{E_b cos\delta_0}{x_e + x_q^{'}} \tag{2.74}$$

$$K_8 = \frac{\partial E_{fd}}{\partial \delta} = \frac{V_{d0}}{V_{t0}}\left(-E_b cos\delta_0 + \frac{x_e E_b cos\delta_0}{x_e + x_d^{'}}\right) \tag{2.75}$$

$$K_9 = \frac{\partial E_{fd}}{\partial E_q^{'}} = \frac{V_{q0} x_e}{V_{t0}}\frac{1}{x_e + x_d^{'}} \tag{2.76}$$

$$K_{10} = \frac{\partial E_{fd}}{\partial E_d^{'}} = \frac{V_{d0} x_e}{V_{t0}}\frac{1}{x_e + x_q^{'}} \tag{2.77}$$

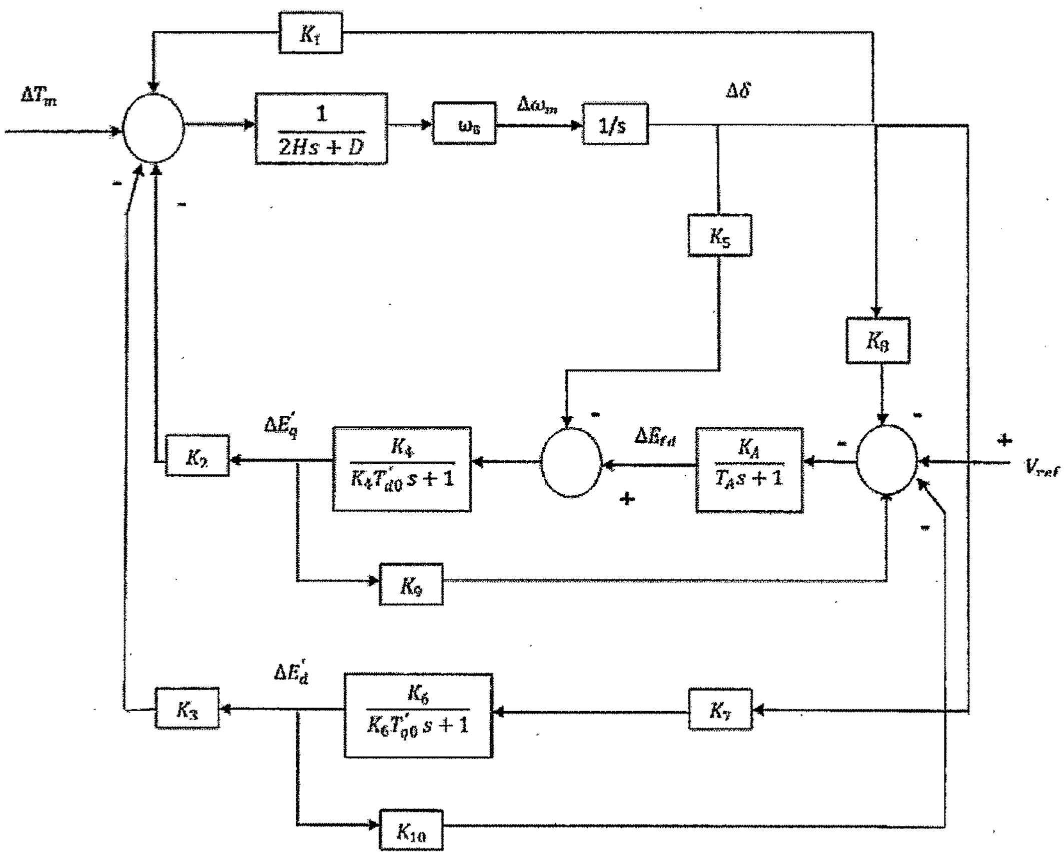

Figure 2.3: Block diagram representaion of SMIB system

2.6 Power System Stabilizer

The single machine infinite bus system with generator connected PSS is shown in Figure 2.4.

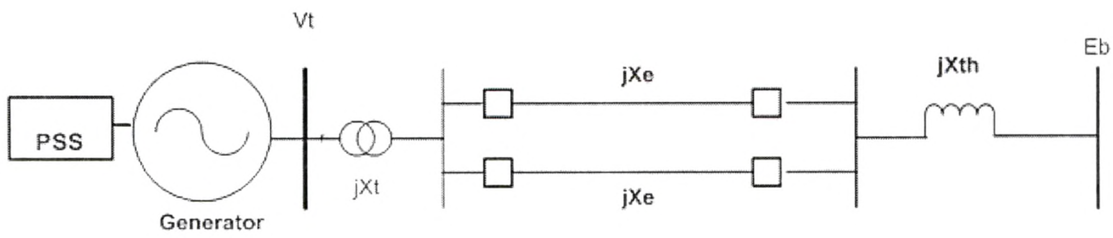

Figure 2.4: SMIB with PSS

2.6.1 Conventional Power system stabilizer

The output response of the PSS is shown as a feedback element from generator speed and is described in the form [56, 33, 15]. The conventional power system stabilizer is described by equation (2.78). The first term in equation (2.78) is a reset term that is used to washout the compensation effect after the time lag T_w. The second term of ΔV_{pss} is a lead compensation pair that can be used to improve the phase lag through the system from V_{ref} to generator shaft speed $\Delta\omega_m$.

$$\Delta V_{pss} = \frac{K_{pss}T_w s}{1+T_w s}\left[\frac{(1+T_1 s)\,(1+T_2 s)}{(1+T_3 s)\,(1+T_4 s)}\right]\Delta\omega_m \tag{2.78}$$

The equation (2.78) is represented by state model of PSS which is shown by Figure 2.5.

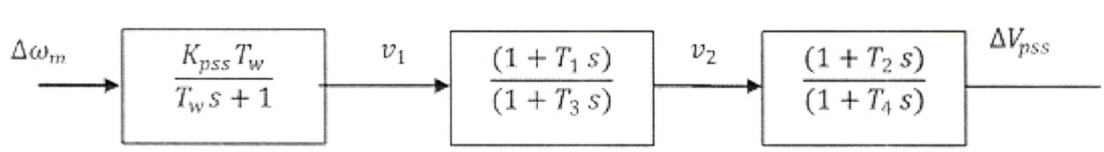

Figure 2.5: CPSS with state variables

The two new state v_1, v_2 and output variable ΔV_{pss} of PSS are included in machine state equations. The CPSS new state equation are described as follow:

$$\Delta\dot{v}_1 = -\frac{K_1K_{pss}}{2H}\Delta\delta - \frac{DK_{pss}}{2H}\Delta\omega_m - \frac{K_2K_{pss}}{2H}\Delta E_q^{'} - \frac{K_3K_{pss}}{2H}\Delta E_d^{'} - \frac{1}{T_W}\Delta v_1 \tag{2.79}$$

$$\Delta \dot{v_2} = -a_1\Delta\delta - a_2\Delta\omega_m - a_3\Delta E_q^{'} - a_4\Delta E_d^{'} - a_5\Delta v_1 + a_6 v_2 \tag{2.80}$$

$$\Delta \dot{v_{pss}} = -b_1\Delta\delta - b_2\Delta\omega_m - b_3\Delta E_q^{'} - b_4\Delta E_d^{'} + b_5\Delta v_1 + b_6\Delta v_2 + b_7\Delta v_{pss} \tag{2.81}$$

2.6.1.1 Model of Power System with CPSS

The Power system stabilizer provides the additional signal to excitation system. After the addition of PSS with excitation system, the exciter equation would be changed. The exciter equation is described by equations (2.82) and (2.83).

$$\dot{E_{fd}} = -\frac{1}{T_A}E_{fd} + \frac{K_A}{T_A}\left(V_{ref} - V_t + V_{pss}\right) \tag{2.82}$$

$$\Delta\dot{E_{fd}} = -\frac{K_AK_8}{T_A}\Delta\delta - \frac{K_AK_9}{T_A}\Delta E_q^{'} - \frac{K_AK_{10}}{T_A}\Delta E_d^{'} + \frac{K_A}{T_A}\Delta V_{ref} - \frac{1}{T_A}\Delta_{fd} + \frac{K_A}{T_A}V_{pss} \tag{2.83}$$

The state model of machine with PSS is described by equation (2.84).

$$A = \begin{bmatrix} 0 & \omega_B & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_1}{2H} & -\frac{D}{2H} & -\frac{K_2}{2H} & -\frac{K_3}{2H} & 0 & 0 & 0 & 0 \\ -\frac{K_5}{T_{d0}^{'}} & 0 & -\frac{1}{T_{d0}^{'}K_4} & 0 & \frac{1}{T_{d0}^{'}} & 0 & 0 & 0 \\ \frac{K_7}{T_{q0}^{'}} & 0 & 0 & -\frac{1}{T_{q0}^{'}K_6} & 0 & 0 & 0 & 0 \\ -\frac{K_AK_8}{T_A} & 0 & -\frac{K_AK_9}{T_A} & -\frac{K_AK_{10}}{T_A} & -\frac{1}{T_A} & 0 & 0 & \frac{K_A}{T_A} \\ -\frac{K_1K_{pss}}{2H} & -\frac{DK_{pss}}{2H} & -\frac{K_2K_{pss}}{2H} & -\frac{K_3K_{pss}}{2H} & 0 & -\frac{1}{T_w} & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & a_5 & a_6 & 0 \\ b_1 & b_2 & b_3 & b_4 & 0 & b_5 & b_6 & b_7 \end{bmatrix}_{8\times 8} \tag{2.84}$$

Where,

$$\dot{x} = \begin{bmatrix} \Delta\dot{\delta} & \Delta\dot{\omega_m} & \Delta\dot{E_q^{'}} & \Delta\dot{E_d^{'}} & \Delta\dot{E_{fd}} & \Delta\dot{v_1} & \Delta\dot{v_2} & \Delta\dot{V_{pss}} \end{bmatrix}^{'}$$

$a_1 = -\frac{T_1K_{pss}}{T_2 2H}, a_2 = -\frac{T_1DK_{pss}}{T_2 2H}, a_3 = -\frac{T_1K_2K_{pss}}{T_2 2H}, a_4 = -\frac{T_1K_3K_{pss}}{T_2 2H}, a_5 = \frac{1}{T_2} - \frac{T_1}{T_2T_w}, a_6 = -\frac{1}{T_2}$

$b_1 = -\frac{T_1T_3K_1K_{pss}}{T_2T_4 2H}, b_2 = -\frac{T_2T_4DK_{pss}}{T_2T_4 2H}, b_3 = -\frac{T_2T_3K_2K_{pss}}{T_2T_4 2H}, b_4 = -\frac{T_1T_3K_3K_{pss}}{T_2T_4 2H}, b_5 = \frac{T_3}{T_2T_4} - \frac{T_1T_3}{T_2T_4 2H},$

$b_6 = \frac{1}{T_4} - \frac{T_3}{T_2T_4}, b_7 = -\frac{1}{T_4}$

2.6.2 Proportional Integral Derivative Power System Stabilizer

One of the most powerful but complex controller mode combines the proportional, integral and derivative mode. This mode eliminates the offset of the proportional mode and provides fast response [20, 88]. In present work PID based PSS is proposed with feedback element from generator speed and is represented by in Laplace form. The Laplace form of PID-PSS is represented by equation (2.85).

$$\Delta V_{pss} = \left[K_p + \frac{K_I}{s} + K_D s \right] \Delta\omega_m \tag{2.85}$$

2.6.2.1 Model of Power System with PID-PSS

The state model of machine with PID-PSS is described by equation (2.86). Figure 2.6 represents block diagram of machine with CPSS and PID-PSS.

$$A = \begin{bmatrix} 0 & \omega_B & 0 & 0 & 0 & 0 \\ -\frac{K_1}{2H} & \frac{D}{-2H} & -\frac{K_2}{2H} & -\frac{K_3}{2H} & 0 & 0 \\ -\frac{K_5}{T'_{d0}} & 0 & -\frac{1}{T'_{d0}K_4} & 0 & \frac{1}{T'_{d0}} & 0 \\ \frac{K_7}{T'_{q0}} & 0 & 0 & -\frac{1}{T'_{q0}K_6} & 0 & 0 \\ -\frac{K_A K_8}{T_A} & 0 & -\frac{K_A K_9}{T_A} & -\frac{K_A K_{10}}{T_A} & -\frac{1}{T_A} & 0 \\ \left(\frac{K_1}{\omega_B} - \frac{K_1 K_{10}}{2H}\right) & \left(K_P - \frac{K_d k_d}{2H}\right) & -\frac{K_2 K_D}{2H} & \frac{-K_3 K_D}{2H} & 0 & 0 \end{bmatrix}_{6\times 6} \tag{2.86}$$

Where,

$$\dot{x} = \left[\begin{array}{cccccc} \Delta\dot{\delta} & \Delta\dot{\omega}_m & \Delta\dot{E}'_q & \Delta\dot{E}'_d & \Delta\dot{E}_{fd} & \Delta\dot{V}_{pss} \end{array} \right]'$$

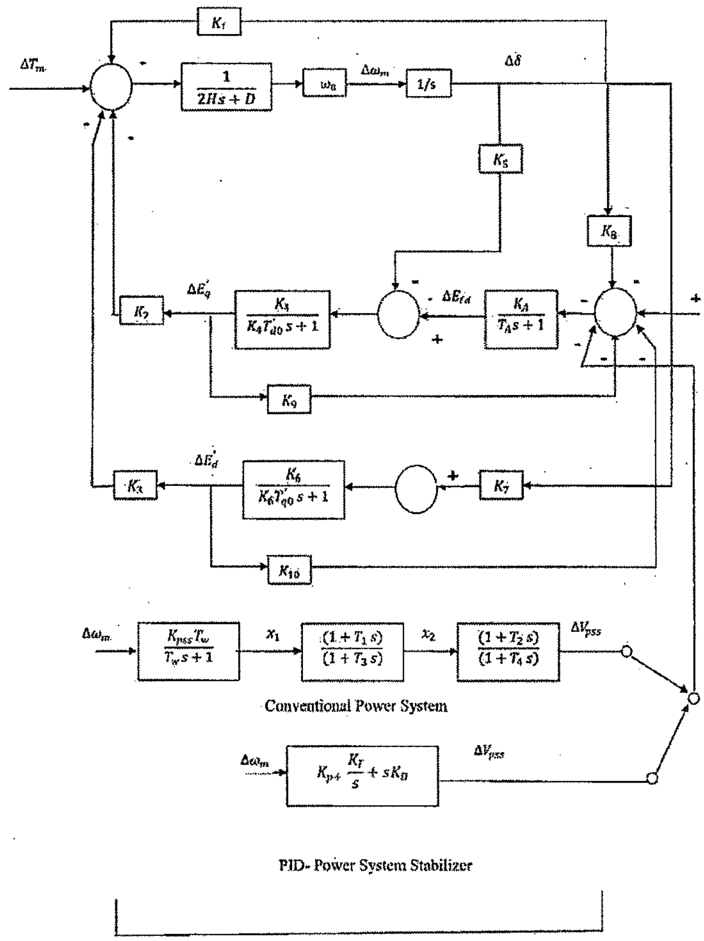

Figure 2.6: Block diagram represntaion of SMIB with PSS

2.7 Thyristor Control Series Capacitor

Thyristor controlled series capacitor provides the fast, continuous and dynamic control of power by varying the apparent reactance of the specific transmission line. The TCSC can enhance the oscillatory stability by damping of oscillation and improves the dynamic and transient stability of the power system [55, 25, 44, 62]. One line diagram of the basic module of the TCSC is shown in Figure 2.7. A TCSC is a parallel combination of a fixed series capacitor and a variable thyristor controlled reactor. The TCSC has two operating ranges around its internal circuit resonance. One is $\alpha_{min} \leq \alpha \leq 180$ where $X_{TCSC(\alpha)}$is capacitive, and other is the $90 \leq \alpha \leq_{Llim}$ where $X_{TCSC(\alpha)}$is inductive. The internal circuit resonance depends on the ratio of inductor and capacitor reactance of TCSC. The steady state relationship between firing angle and the reactance X_{TCSC} can be described by the following equation [44].

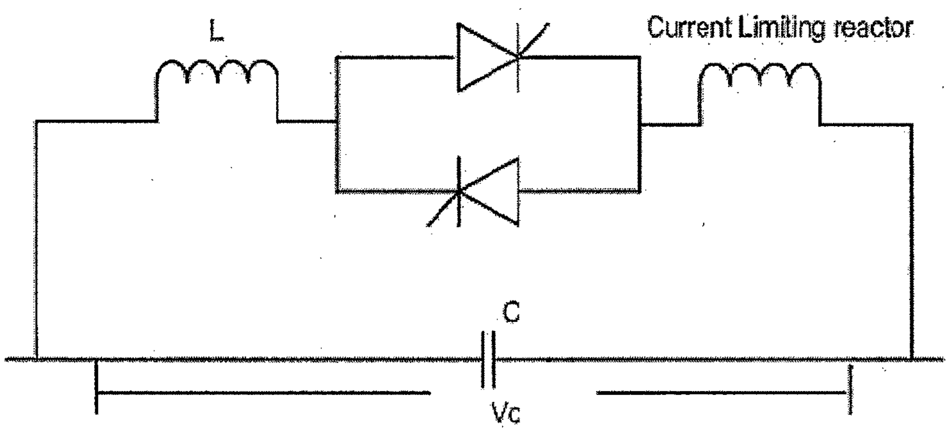

Figure 2.7: Basic module of TCSC

$$X_{TCSC}(\alpha) = X_C - \frac{X_C^2(\mu + sin\mu)}{(X_C - X_L)\pi} + \frac{4X_C^2 cos^2(\mu/2)\left[ptan(\frac{p\pi}{2}) - tan(\pi/2)\right]}{(X_C - X_L)(p^2 - 1)\pi} \tag{2.87}$$

Where X_C is reactance of the fixed capacitor, X_L is inductive reactance of inductor L connected in parallel to capacitor, compensation ratio $p = \sqrt{\frac{X_C}{X_L}}$ and conduction angle of TCSC is $\mu = \pi - \alpha$. The model for the TCSC for the stability study is shown in Figure 2.8 and is based on the variation of the reactance of the TCSC in the capacitive region [38, 44]. In this Figure, X_{mod} is the stability control modulation reactance value as determined by the stability control loop, and X_{ref} denotes the TCSC steady state reactance or set point, whose value is calculated from power flow or steady state control loop. The sum of these two values produce X_{total} which is the final reactance offered by the external control block. This signal is passed through first order lag transfer function and produced the final value of the reactance $X_{TCSC(\alpha)}$.The time constant T_{TCSC} presents the natural response of the device and the delay introduced by the internal control. The limits are given by $X_{Cmin} = X_{TCSC}(180^0) = X_C$ and $X_{cmax} = X_{TCSC}(\alpha_{Cmin})$. Here, the controller is assumed to operate in the capacitive region only, it means $\alpha_{min} \gg \alpha_r$, where $\alpha_r$correspond to the resonant point.

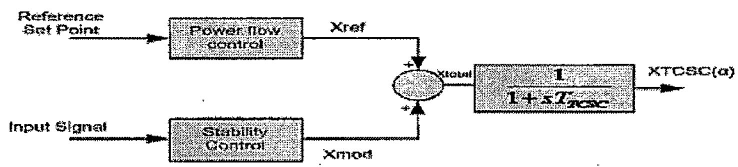

Figure 2.8: Model of TCSC

$$\dot{X}_{TCSC(\alpha)} = \frac{1}{T_{TCSC}} \left(X_{mod} + X_{ref}\right) - X_{TCSC(\alpha)} \tag{2.88}$$

The stability control loop of TCSC can be described by the Equation (2.89) .

$$X_{TCSC(\alpha)} = \frac{K_C T_{w1} s}{1 + T_{w1} s} \left[\frac{(1 + T_{1T} s)(1 + T_{2T} s)}{(1 + T_{3T} s)(1 + T_{4T} s)}\right] \Delta\omega_m \tag{2.89}$$

Where T_{w1} andK_C are time constant and gain of the washout filter. T_{1T}, T_{2T}, T_{3T} and T_{4T} are time constant of the phase compensator, which provides appropriate phase-lead characteristics to compensate for the phase lag between input and output signals.

2.7.1 Model of Power system with Inclusion of TCSC

Figure 2.9 represents the single machine infinite bus system with generator connected PSS and transmission line equipped TCSC.

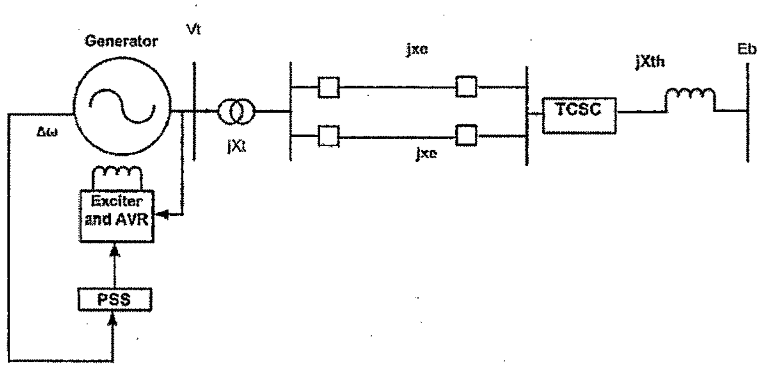

Figure 2.9: SMIB wiht PSS and TCSC

After the inclusion of TCSC in power system model, the line reactance would be changed as $X_{net} = X_e - X_{TCSC}$. The power transfer between the V_t and E_b is written as follows:

$$P_t = \frac{V_t E_b sin\delta_0}{X_{net}} \tag{2.90}$$

The Current and voltage Equations of $i_q, i_{d,} v_q$ and v_d would be changed. Hence electric torque T_e is also changed. The linearized model of the machine is deriveed after the inclusion of TCSC and new set of state Equations $\Delta\dot{\delta}$,$\Delta\dot{\omega}_m$, $\Delta\dot{E}'_q$, $\Delta\dot{E}'_d$, $\Delta\dot{E}_{fd}$ are obtained. Consequently K_1 to K_{10} constants of the power system are recalculated using new line rectance X_{net} and initial conditions. After addition of TCSC, three new state variables and a state variable $X_{TCSC(\alpha)}$ are required to considered for stability control loop of TCSC. For the TCSC stability control loop the equation (2.88) and (2.89) are resolved using four state variables of TCSC model and then included in machine state equation. Figure 2.10 and 2.11 show stability control loop and time delay of TCSC. Equation (2.91) represents power system matrix A with TCSC stability control loop.

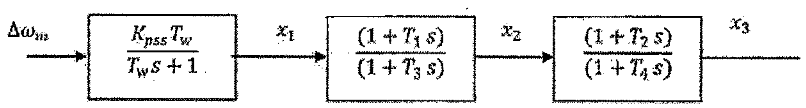

Figure 2.10: TCSC state diagram

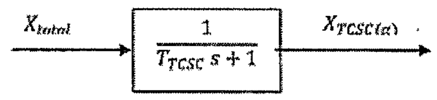

Figure 2.11: TCSC delay Equation

The matrix A is described as follow:

$$A = \begin{bmatrix} 0 & \omega_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_1}{2H} & -\frac{D}{2H} & -\frac{K_2}{2H} & -\frac{K_3}{2H} & 0 & 0 & 0 & 0 & \frac{K_p}{2H} \\ -\frac{K_5}{T'_{d0}} & 0 & -\frac{1}{T'_{d0}K_4} & 0 & \frac{1}{T'_{d0}} & 0 & 0 & 0 & K_q \\ \frac{K_7}{T'_{q0}} & 0 & 0 & -\frac{1}{T'_{q0}K_6} & 0 & 0 & 0 & 0 & K_d \\ -\frac{K_A K_8}{T_A} & 0 & -\frac{K_A K_9}{T_A} & -\frac{K_A K_{10}}{T_A} & -\frac{1}{T_A} & 0 & 0 & 0 & \frac{K_A K_{Ed}}{T_A} \\ c_1 & c_2 & c_3 & c_4 & 0 & c_5 & 0 & 0 & 0 \\ d_1 & d_2 & d_3 & d_4 & 0 & d_5 & d_6 & 0 & 0 \\ e_1 & e_2 & e3 & e_4 & 0 & e_5 & e_6 & e_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_1 & f_2 \end{bmatrix}_{9\times 9} \tag{2.91}$$

Where

$$\dot{x} = \begin{bmatrix} \Delta\dot{\delta} & \Delta\dot{\omega}_m & \Delta\dot{E}'_q & \Delta\dot{E}'_d & \Delta\dot{E}_{fd} & \Delta\dot{x}1 & \Delta\dot{x}_2 & \Delta\dot{x}_3 & \Delta X_{TCSC} \end{bmatrix}'$$

2.7.2 Model of Power System with Inclusion of TCSC and PSS

The two state variables and ΔV_{pss} of CPSS, and three state variables and $X_{TCSC(\alpha)}$ of TCSC are included in power system model. After the inclusion of state variables of PSS and TCSC, the power system model is converted in 12×12 matrix form. Equation (2.92) represents power system matrix A with PSS and TCSC stability control loop. Figure 2.12 represents the block diagram of power system with TCSC and ΔV_{pss} signal coming from PSS.

$$\begin{bmatrix} 0 & \omega_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_1}{2H} & -\frac{D}{2H} & -\frac{K_2}{2H} & -\frac{K_3}{2H} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_p}{2H} \\ -\frac{K_5}{T'_{d0}} & 0 & -\frac{1}{T'_{d0}K_4} & 0 & \frac{1}{T'_{d0}} & 0 & 0 & 0 & 0 & 0 & 0 & K_q \\ \frac{K_7}{T'_{q0}} & 0 & 0 & -\frac{1}{T'_{q0}K_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_d \\ -\frac{K_AK_8}{T_A} & 0 & -\frac{K_AK_9}{T_A} & -\frac{K_AK_{10}}{T_A} & -\frac{1}{T_A} & 0 & 0 & \frac{K_A}{T_A} & 0 & 0 & 0 & \frac{K_AK_{fEd}}{T_A} \\ -\frac{K_1K_{pss}}{2H} & -\frac{DK_{pss}}{2H} & -\frac{K_2K_{pss}}{2H} & -\frac{K_3K_{pss}}{2H} & 0 & -\frac{1}{T_w} & 0 & 0 & 0 & 0 & 0 & \frac{K_CK_{pss}}{2H} \\ a_1 & a_2 & a_3 & a_4 & 0 & a_5 & a_6 & 0 & 0 & 0 & 0 & \frac{K_CK_{pss}T_1}{2HT_2} \\ b_1 & b_2 & b_3 & b_4 & 0 & b_5 & b_6 & b_7 & 0 & 0 & 0 & \frac{K_CK_{pss}T_1T_3}{2HT_2T_4} \\ c_1 & c_2 & c_3 & c_4 & 0 & c_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ d_1 & d_2 & d_3 & d_4 & 0 & d_5 & d_6 & 0 & 0 & 0 & 0 & 0 \\ e_1 & e_2 & e3 & e_4 & 0 & e_5 & e_6 & e_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_1 & f_2 & 0 & 0 & 0 \end{bmatrix} \tag{2.92}$$

Where,

$$K_p = \frac{\partial T_e}{\partial X_{TCSC}}, \quad K_q = \frac{\partial E'_q}{\partial X_{TCSC}}, \quad K_d = \frac{\partial E'_d}{\partial X_{TCSC}}, \quad K_{fed} = \frac{\partial E_{fd}}{\partial X_{TCSC}}$$

$$c_1 = \frac{-K_CK_1}{2H}, \quad c_2 = -\frac{K_CD}{2H}-, \quad c_3 = -\frac{K_CK_2}{2H} \quad c_4 = -\frac{K_CK_3}{2H}, \quad c_5 = -\frac{1}{T_{w1}}$$

$$d_1 = -\frac{T_{1T}K_1K_C}{T_{2T}2H}, \quad d_2 = -\frac{T_{1T}DK_C}{T_{2T}2H}, \quad d_3 = -\frac{T_{1T}K_2K_C}{T_{2T}2H}, \quad d_4 = -\frac{T_{1T}K_3K_C}{T_{2T}2H},$$

$$d_5 = \left[\frac{1}{T_{2T}} - \frac{T_{1T}}{T_{2T}T_{w1}}\right], \quad d_6 = -\frac{1}{T_{2T}}$$

$$e_1 = -\frac{T_{1T}T_{3T}K_1K_C}{T_{2T}T_{4T}2H}, \quad e_2 = -\frac{T_{1T}T_{3T}DK_C}{T_{2T}T_{4T}2H}, \quad e_3 = -\frac{T_{1T}T_{3T}K_2K_C}{T_{2T}T_{4T}2H}; \quad e_4 = -\frac{T_{1T}T_{3T}K_3K_C}{T_{2T}T_{4T}2H}$$

$$e_5 = \left[\frac{T_{3T}}{T_{2T}T_{4T}} - \frac{T_{1T}T_{3T}}{T_{2T}T_{4T}T_{w1}}\right], \quad d_5 = \left[\frac{1}{T_{4T}} - \frac{1}{T_{2T}}\right], \quad d_6 = -\frac{1}{T_{4T}}$$

$$f_1 = -\frac{1}{T_{TCSC}}, \quad f_2 = \frac{1}{T_{TCSC}}$$

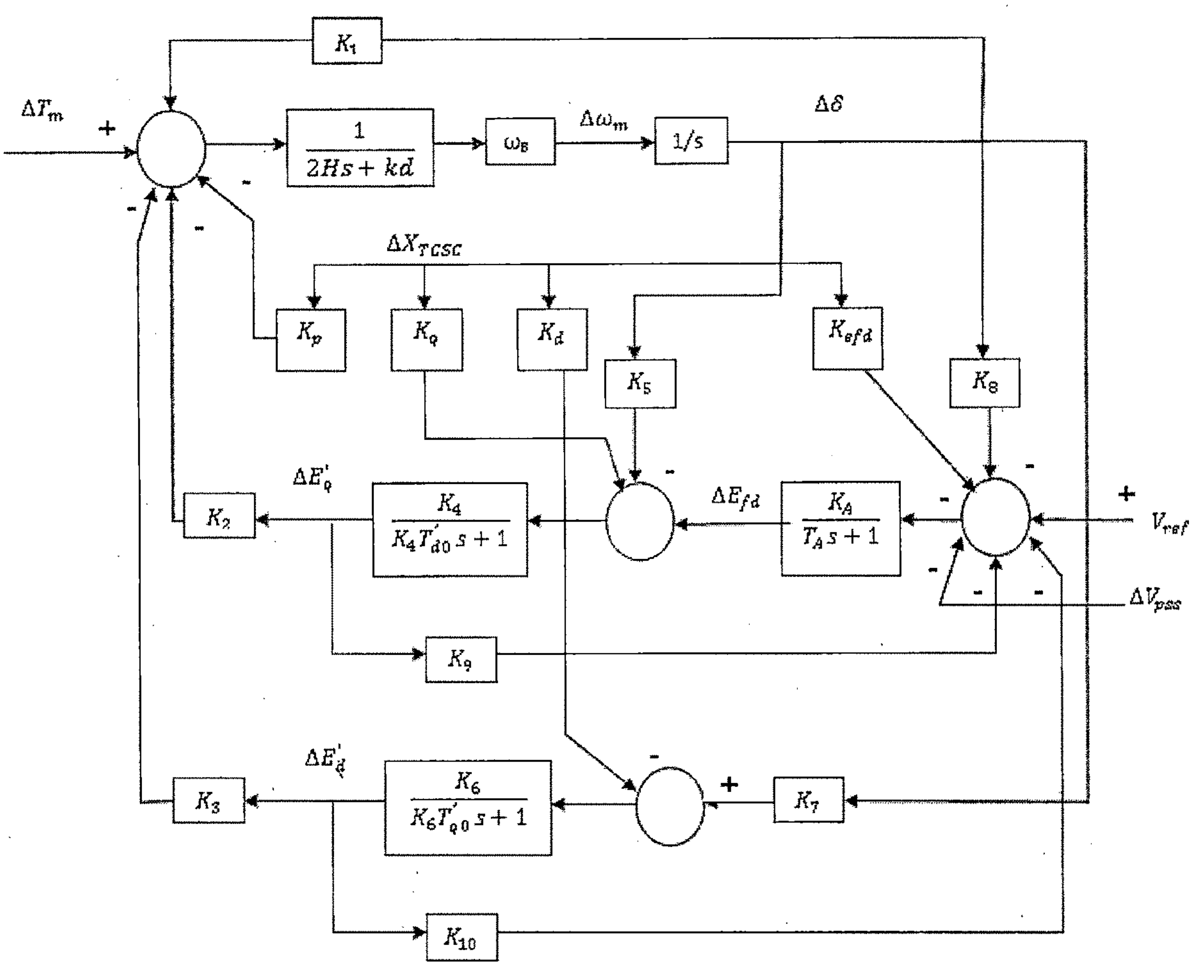

Figure 2.12: Block diagram of System with PSS and TCSC

2.8 Conclusion

For designing of computational intelligent techniques based PSS and TCSC, the mathematical model of the power system with different damping controllers are required. Calculation of eigen values, damping factors and participation of rotor mode are very important parameters for depth analysis of small signal stability issues. The transient stability analysis can be done by dynamical model of the power system.

This chapter has presented the fourth order non linear mathematical model of the power system with IEEE-ST1 excitation system. The systematic procedure for conversion of non linear model into linear model with both PSS, TCSC and simultaneous designing of PSS and TCSC have been discussed. Using Taylor's series method, a new fourth order linearized model of the power system with exciter has been derived and the equations of machine constant K_1 to K_{10} have been calculated. The linerized mathematical model and state space form of power system with conventional power system stabilizer and a new PID- power system stabilizer have been described. The linearized state space form of power system with individual TCSC and simultaneous CPSS and TCSC have been also derived. The bock diagram representation of system with PSSs and TCSC has been included.