Chapter 1

Introduction

An understanding of how the laws of nature work requires a theory of the elementary particles of matter interacting with each other. Equivalently it requires a theory of the basic forces of nature. Four such forces exist, and until recently a different kind of theory was needed for each of them. Two of them, electromagnetism and gravitation are long range; hence they are familiar to us. The remaining forces which are simply called strong forces and weak forces cannot be felt directly as their influence extends only within the nucleus. The strong force binds together the protons and the neutrons in the nucleus and in another context it binds together the quarks which are the constituents of protons and the neutrons. The weak force is responsible for the decay of certain the neutrons.

The long standing problem of physics is to construct a master theory which takes into account all the known forces. One imagines that such a theory would reveal some deep connection between the theories which apparently look so diverse. In recent past one crucial step towards that goal has been taken. The weak force and the electromagnetism can now be understood in terms of a single theory - this unified theory of electroweak interactions form part of the standard model. Thus one thinks that in near future one can formulate a single theory of strong, weak and electromagnetic theory which is popularly named as grand unified theory (GUT).

The first step towards the construction of the unified theory is the demonstration that the weak, the strong and the electromagnetic forces could all be described in terms of the theories of the same kind namely the gauge theories. The three forces remain distinct but they can be seen to operate through the same mechanism. This hints that the grand unified theory which is intended to incorporate (and not supplement) the established theories is also a gauge theory.

In the following section we shall review the status of the standard electroweak theory which we intend to embed in a grand unified version. Latter in the section following it we shall briefly review the rudiments of the prototype grand unified theory namely the minimal SU(5) GUT. Afterwards we shall go on to review the inclusion of left-right symmetry and supersymmetry in the framework of unification.

1.1 Basic ideas of Standard Model

Standard Model(SM) is a gauge theory invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$. The interaction is mediated by eight gluons of the strong interaction sector and the four gauge bosons $W^{\pm}, Z^0$ and γ of the electroweak sector. Here we have labeled the electroweak gauge bosons by their electric charge as they are not eigenstates of $SU(2)_L \times U(1)_Y$ whereas they are eigenstates of electric charge. Which immediately means that in nature $SU(2)_L \times U(1)_Y$ symmetry is broken and $U(1)_{em}$ symmetry is preserved.

The objective is to formulate a renormalized gauge theory of electroweak interactions incorporating massive gauge bosons. This is achieved by spontaneously breaking the gauge symmetry. The experimental data suggests that the interactions are invariant under weak isospin $SU(2)_L$ and the weak hypercharge $U(1)_Y$ transformations.

We first write down the basic interactions in gauge invariant form. First, we write, an isotriplet of weak currents J_μ coupled to the three vector bosons W_μ in an $SU(2)_L$ invariant way,

$$- ig\ J_\mu \,.\, W^\mu = -ig\bar{\chi}_L\gamma_\mu\, T\,.\, W^\mu\chi_L, \tag{1.1}$$

next, a weak hypercharge current coupled to the fourth vector boson B^μ in a $U(1)_Y$ invariant way,

$$- i\frac{g'}{2}\ J_\mu^Y B^\mu = -ig'\bar{\psi}\gamma_\mu\frac{Y}{2}\psi B^\mu. \tag{1.2}$$

The operators T and B are the generators of the $SU(2)_L$ and $U(1)_Y$ transformations.

The left handed fermions form isospin doublets χ_L and the right handed fermions are the isosinglets ψ_R. For example, for the electron and its neutrino we have

$$\begin{aligned}\chi_L = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L &\quad with\quad T = 1/2,\ Y = -1,\\ \psi_R = (\,e^-\,)_R &\quad with\quad T = 0,\quad Y = -2.\end{aligned}$$

Whereas for quarks the assignment is,

$$\chi_L = \begin{pmatrix}u\\ d\end{pmatrix}_L;\qquad \psi_R = u_R\ or\ d_R.$$

The electromagnetic interaction is embedded in such a way that the generator of the $U(1)_{em}$ is related to the generators of $SU(2)_L$ and $U(1)_Y$ by the following relationship,

$$Q = T^3 + \frac{Y}{2}.$$

In other words, the electromagnetic current is a combination of the two neutral currents J_μ^3 and J_μ^Y. The two neutral gauge fields. The interaction in the neutral current sector can be written as

$$- ig\ J_\mu^3 W^{3\mu} - i\frac{g'}{2}\ J_\mu^Y B^\mu = -iej_\mu^{em}A^\mu - \frac{ie}{sin\theta_w\ cos\theta_w}[J_\mu^3 - sin^2\theta_w j_\mu^{em}]Z^\mu. \tag{1.3}$$

We have written the above equation in terms of the two physical fields,

$$\begin{aligned}A_\mu &= cos\theta_w\ B_\mu + sin\theta_w W_\mu^3,\\ Z_\mu &= -sin\theta_w\ B_\mu + cos\theta_w W_\mu^3.\end{aligned}$$

The requirement that electromagnetic interaction must appear in the right hand side of eqn(1.3) has fixed a relationship between (g, g') and (e, θ_w), namely,

$$e = g\ sin\theta_w = g'cos\theta_w.$$

The weak mixing angle $\sin^2\theta_w$ is one of the important parameters of the standard model, which gives us the amount of mixing of the two neutral gauge bosons with photon and Z_μ.

The Higgs mechanism is incorporated to make the electroweak gauge bosons massive. To do this, the scalar doublet ϕ is introduced. The most economical choice is the isospin doublet with hypercharge $Y = 1$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$

The relevant part of the $SU(2)_L \times U(1)_Y$ invariant Lagrangian for the gauge boson masses is,

$$\mathcal{L}_1 = [(i\partial_\mu - g\,T\,.\,W_\mu - g'\frac{Y}{2}B_\mu)\phi]^2 - V(\phi).$$

The [ ]^2 above is defined as [ ]†[ ]. $V(\phi)$ is the Higgs potential given by,

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

When the coefficient of μ^2 becomes negative, the Higgs scalar acquires a vacuum expectation value (vev) to make the gauge bosons massive. The vev ϕ_0 has the following form,

$$\phi_0 = \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}.$$

The gauge boson masses are obtained by shifting the variable, $\phi = \phi' - \phi_0$ the vev ϕ_0 in the Lagrangian $\mathcal{L}_1$ and identifying terms with ϕ_0. The relevant term is,

$$\begin{aligned} &\left|(-ig\frac{\tau}{2}\,.\,W_\mu - i\frac{g'}{2}B_\mu)\phi\right|^2 \\ &= (\frac{1}{2}vg)^2\, W_\mu^+ W^{-\mu} \;+\; \frac{1}{8}v^2\,(W_\mu^3 \quad B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \end{aligned}$$

Now it is easy to see that the gauge bosons $W^\pm$, Z, and the photon A has the following masses,

$$M_W = \frac{1}{2}vg; \quad M_A = 0; \quad M_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}.$$

In the standard model Lagrangian the fermion mass term of the form $m\bar\psi\psi$ is forbidden by gauge invariance. An interesting feature of the standard model is that the same Higgs doublet that gives masses to the gauge bosons is also sufficient to give masses to the fermions. For example, to generate the mass for the electron let us consider the following Yukawa part of the Lagrangian:

$$\mathcal{L}_2 = -h_e\Big[(\,\bar\nu_e \quad \bar e\,)_L \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} e_R + \bar e_R\,(\,\phi^- \quad \phi_0\,)\begin{pmatrix} \bar\nu_e \\ \bar e \end{pmatrix}_L\Big].$$

Now we can read off the mass of the electron when the Higgs field gets a vev,

$$m_e = \frac{h_e v}{\sqrt{2}}.$$

The quark masses are generated in the same way. The only novel feature is that to generate a mass for the upper quark of a quark doublet we must construct a new doublet,

$$\phi_c = -i\tau_2\phi^* = \begin{pmatrix} -\phi^0 \\ \phi^- \end{pmatrix}.$$

Due to the special properties of SU(2) ϕ_c transforms identically to ϕ. Hence this minimal choice of Higgs fields can give mass to both up and down type quarks. However in case of supersymmetry, we shall see that such minimal choice no longer works. One needs in that case two Higgs doublets to give masses to up and down type of quarks.

Standard model describes a tremendous chunk of physics. The strong interactions are responsible for the structure of the nuclei and for the most part of what happens in the high energy collisions. The weak interactions are responsible for the nuclear transmutations. The electromagnetism is of course responsible for, "All of Chemistry and most of Physics". SM is highly successful phenomenologically. It is consistent with all available experimental data. From present experimental inputs both model dependent [2] and independent [3] precision tests have been done and they indicate no deviation from Standard Model.

Despite all these successes the Standard Model cannot be given the status of a complete theory and there is strong motivation for the study of the physics beyond the Standard Model. First, it has a large number of parameters which has to be determined from the experiments (gauge couplings, fermion masses etc). The assignment of quantum numbers is partly arbitrary and it has to be explained from a better theory (Charge Quantization). The Higgs sector is not properly understood as there are problems in the fixed point structure of the ϕ^4 theory (Triviality Problem). There is no fundamental symmetry in the SM which will prevent the Higgs boson from getting large quadratic corrections to its mass from radiative processes (Hierarchy Problem). Now to explain charge quantization one has to embed the SM into a larger group and the fermions in some irreducible representation of the bigger group[4]. Then from the remormalization group analysis one can study the unification of forces [10]. These ideas lead to the study of Grand Unified Theories (GUTS). In a different attempt to explain the hierarchy one may assume that there is a scale Λ above which the Higgs boson is not a fundamental particle. Hence this scale will act as a cut-off scale in the quadratically divergent integrals. This idea lead to the study of the Technicolor and Condensate models[6] where one aims to have a spontaneous symmetry breaking with the gauge interactions alone: there is no elementary scalar with its self couplings and Yukawa couplings. On the other hand there may be fermionic-bosonic symmetry due to which large quadratic divergences cancel upto all orders and one is left with a finite theory. Studies in this direction leads to Supersymmetry. Out of these three major directions to go beyond the standard model Technicolor is less favored by the precision electroweak measurements [3].

1.2 Minimal SU(5) GUT

Grand Unified Theories (GUTs) [7] offer the possibility of a simple but unified description of strong and electroweak interactions. Typically in these models at some high energy all the interactions arise out of a single Lagrangian which is locally invariant under the gauge transformations of a single simple Lie group called the unification group. A large spectrum of GUTs are proposed in the literature which is broadly classified by the unification group. In the minimal $SU(5)$ model all interactions unite at a single step at an energy around 10^{14} GeV therefore predicting the absence of any new physics between the standard electroweak breaking scale(M_z) and the unification scale(M_U) while the $SO(10)$ model admits intermediate breakings of symmetry. On the other hand there are models which are inspired by superstring theories, one of them postulates the exceptional group E_6 as the unifying group. This specific model predicts atleast 12 exotic fermions on top of the 15 standard fermions.

All these theoretically very attractive models have atleast one common prediction namely the decay of proton. There has been a desperate search by the experimentalists to see the signature of proton decay for the last decade and a half. Contrary to the theoretical beliefs proton decay has not been discovered. At present the lower limit of the half life of proton is a whopping 10^{32} years.

The simplest model of unification existing in the literature is the SU(5) GUT [1]. Although the simplest nonsupersymmetric version of SU(5) is ruled out by LEP data and limit on proton lifetime (although the effect of gravity may still make it a viable one) the idea of grand unification is very much alive and supposed to be the most popular extension of standard model. To explain the basic concepts of grand unification we briefly describe the SU(5) GUT which is conceptually easiest to understand.

The major success of the Glashow Weinberg Salam (GWS) model of electroweak interactions has motivated physicists to look for a unified theory of strong and electroweak interactions. According to the present view the strong interaction can be understood in terms of Quantum Chromodynamics (QCD), which is an unbroken gauge theory based on the gauge group $SU(3)_c$. The fundamental representation of this group contains the three colored states of a quark.

The color gauge group $SU(3)_c$ is orthogonal to the WS electroweak symmetry group $SU(2)_L \times U(1)_Y$. Thus a broken gauge theory based on the semi-simple group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with a coupling strengths of g_3, g and g', respectively, for the three factors, can explain the strong and electroweak interactions at low energies. The quantum numbers of the fermions under the group $SU(3)_c \times SU(2)_L \times U(1)_Y$ are given in Table 1.1

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(3, 2, 1/3)$
$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$(1, 2, -1)$
u_L^c	$(\bar{3}, 1, -4/3)$
d_L^c	$(\bar{3}, 1, 2/3)$
e_L^+	$(1, 1, 2)$

Table 1.1: fermions and their $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformation laws

These particles form a family or generation. We have expressed all the fields in a left handed basis, using $\psi_L^c = c\ \bar{\psi}_R^T$. The results for those particles can be extended to other generations also, which will not be discussed here. The right-handed neutrino ν_R or the left handed antineutrino ν_L^c will also be included in some models.

[1]This discussion on minimal SU(5) follows Ref [13]

However, one notices that the theory is not truly unified in the sense that the gauge coupling strengths g_3, g_2 and g_1 are different. Georgi and Glashow first suggested the idea of embedding the the standard group $SU(3)_c \times SU(2)_L \times U(1)_Y$ into a simple group, namely SU(5), such that at very high energies the symmetry group SU(5) breaks down to the standard group. The fermions are contained in the SU(5) representations $\bar{5}$ and 10 respectively. The explicit form of the fermion representations are:

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L .$$

These representations decompose under the standard model gauge group as,

$$\begin{aligned} \bar{5} &= (\bar{3}, 1, \frac{2}{3}) + (1, 2, -1) \\ 10 &= (\bar{3}, 1, -\frac{4}{3}) + (3, 2, \frac{1}{3}) + (1, 1, 2) \end{aligned}$$

The transformation properties of the 24 gauge bosons of the SU(5) model are as follows,

$$24 = (8,1,0) + (1,3,0) + (1,1,0) + (3,2,-5/3) + (\bar{3},2,5/3).$$

The first term represents the gluons and the second and third term represents a linear combination of the $W^{\pm}$, Z boson and the photon. The gauge bosons (3,2,-5/3) and (3,2,5/3) are introduced by the SU(5) GUT and they acquire masses at the scale M_U, i.e, the scale at which the SU(5) symmetry is broken. These gauge bosons couple to diquarks and leptoquarks. They can mediate the processes like $p \longrightarrow e^+\pi^0$ and $n \longrightarrow e^+\pi^-$. The lifetime of such nucleon decay will be given by,

$$\tau_p \sim \alpha_5^{-2} \frac{M_U^4}{m_N^5}, \tag{1.4}$$

where α_5 is the SU(5) coupling constant, m_N stands for the nucleon mass. We shall now estimate M_U following Georgi Quinn and Weinberg [10].

If we use the same normalization conventions for different gauge groups, then the invariance under SU(5) implies that the coupling constants g_5, g_3, g_2 and, g_1 associated with the groups SU(5), SU(3) SU(2) and U(1) respectively are equal when SU(5) is broken at a scale M_U, i.e,

$$g_5(M_U) = g_3(M_U) = g_2(M_U) = g_1(M_U). \tag{1.5}$$

When the SU(5) symmetry is exact the charge Q and the SU(2) and U(1) generators T and T_0 are given by,

$$Q = T_3 + cT_O.$$

The gauge coupling constants g_2 and g_1 must also be related to the usual $SU(2)_L$ and $U(1)_Y$ coupling constants of the WS model, g and g' by,

$$g_2 = g \ \ and \ \ g_1 = c\, g'.$$

Now we can write

$$Tr[Q^2] = (1 + c^2)Tr[T_3^2].$$

For n_F generations of fermions we have,

$$Tr[T_3^2] = 2\ n_f \quad and \quad Tr[Q^2] = \frac{16}{3} n_f$$

This immediately gives us the value of c,

$$c^2 = \frac{5}{3}.$$

Thus we get the value of the Weinberg angle at the scale Q,

$$sin^2\theta_w(Q) = \frac{g'^2(Q)}{g^2(Q) + g'^2(Q)}.$$

Hence, for a scale Q larger than or equal to the unification scale M_U we have,

$$sin^2\theta_w = \frac{c^2}{1 + c^2} = \frac{3}{8}. \tag{1.6}$$

and also,

$$\frac{\alpha}{\alpha_s}(Q) = \frac{e^2/4\pi}{g_3^2/4\pi} = \frac{e^2}{g_3^2} = \frac{g^2 g'^2}{g_3^2(g^2 + g'^2)}.$$

Hence, for a scale Q larger than or equal to the unification scale M_U we have,

$$\frac{\alpha}{\alpha_s} = \frac{3}{8}.$$

Once the value of $sin^2\theta_w$ and $\frac{\alpha}{\alpha_s}$ are calculated in terms of the normalized coupling constants at the SU(5) symmetry level, it is a straight forward job to calculate $sin^2\theta_w$ and $\frac{\alpha}{\alpha_s}$ at the present energies. We use the standard renormalization group equations,

$$\alpha^{-1}(q) = \alpha^{-1}(Q) + 2\ b\ M_{Qq}. \tag{1.7}$$

where in equation (1.7) b is defined as,

$$b = -\frac{1}{16\pi^2}[\frac{11}{3}N - \frac{4}{3}T_R - \frac{1}{6}T_s], \tag{1.8}$$

and, M_{Qq} is defined as $4\pi\ ln\frac{Q}{q}$. The three terms in the r.h.s. of equation (1.8) represent the gauge boson, the fermion and the scalar contributions respectively. For the U(1) groups there is no nonabelian gauge coupling and hence the first term is absent. thus combining the boundary conditions given in equation (1.5) and in in equation(1.6) we can write,

$$\begin{aligned} sin^2\theta_w(Q^2) &= \frac{3}{8}(1 - \frac{55\alpha}{18\pi} ln\frac{M_U^2}{Q^2}), \\ \text{and} \quad \frac{\alpha(Q^2)}{\alpha_s(Q^2)} &= \frac{3}{8}(1 - \frac{11\alpha}{2\pi} ln\frac{M_U^2}{Q^2}). \end{aligned}$$

For $\alpha_s = 0.12$ the value of $sin^2\theta_w$ is predicted to be 0.21 which is lower than the experimentally observed value of 0.23±.002 approximately. The proton lifetime is also predicted to be too low to be consistent with the experimental lower bound on τ_P. Due to these reasons it is believed that the minimal version of the SU(5) model is ruled out. However we shall see in this thesis that including the effects of gravity induced higher dimensional operators one can still make the model consistent with experimental results. Certain other non-minimal extensions of this model like extending it to include new fermions at some intermediate scale can also make the model consistent with experimental results.

1.3 Inclusion of left-right symmetry

One important feature of the minimal SU(5) GUT is that it suggests the existence of a desert above the electroweak scale. Actually, the minimal SU(5) model predicts no new mass scale between the electroweak scale and the unification scale. This depressing feature is absent in more extended models of unification like SO(10) GUT. This model admits intermediate mass scales in which parity is restored.

The nature of the weak interactions appears to be very well described by the celebrated V-A theory, which enjoys sound phenonenological success at the low energies. The standard $SU(2)_L \times U(1)_Y$ gauge theory provides a sound mathematical basis of the V-A theory. An alternative approach to the electroweak interactions has been proposed by Pati and Salam according to which the basic weak Lagrangian is invariant under the space reflections. This involves both V-A as well as V+A interactions. The gauge group of this model is left right symmetric and it is broken to standard model spontaneously by Higgs mechanism.

$$\begin{aligned} SU(2)_L \times SU(2)_R \times U(1)_{B-L} &\xrightarrow{M_R} SU(2)_L \times U(1)_Y \\ &\xrightarrow{M_Z} U(1)_{em}. \end{aligned} \tag{1.9}$$

The electromagnetic charge is related to the diagonal generators of the left-right symmetric model as,

$$Q = T_L^3 + T_R^3 + \frac{B-L}{2}.$$

The weak Lagrangian prior to the symmetry breaking is given by,

$$\mathcal{L}_{weak} = \frac{g}{\sqrt{2}}(J_{\mu L} \, . \, W_L^\mu + J_{\mu R} \, . \, W_R^\mu)$$

here, $J_{\mu L} = J_{\mu R}(\gamma_5 \to -\gamma_5)$. The noninvariance of the vacuum under space reflections results in $M_{W_R} >> M_{W_L}$, and, as a result, all the low energy weak processes appear the same as in the $SU(2)_L \times U(1)_Y$ theory, with small corrections undetectable in experimental results.

In left right symmetric models, since both left handed and right handed helicities of the neutrino are included, the neutrino naturally has a mass. It has been shown that if the neutrino is a Majorana particle, one can obtain the qualitative relation

$$m_{\nu_e} \simeq O(\frac{1}{m_{W_R}})$$

This relation provides a deeper physical insight into the connection between the small neutrino mass and the maximality of parity violation.

SO(10) is the minimal left-right symmetric grand unified theory. Actually, the whole $SU(4)_c \times SU(2)_L \times SU(2)_R$ of Pati and Salam can be embedded into SO(10). The adjoint representation of SO(10) is 45 dimensional, hence there are 45 gauge bosons. We have 21 gauge bosons associated with the group $SU(2)_L \times SU(2)_R \times SU(4)$ and 24 superheavy guage bosons which become massive at the scale of breaking of SO(10).

The symmetry breaking pattern of the SO(10) model is the following,

$$
\begin{aligned}
SO(10) \quad & \xrightarrow[M_U]{} \quad SU(2)_L \times SU(2)_R \times SU(4)_c \quad \equiv G_{PS} \\
& \xrightarrow[M_I]{} \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad \equiv G_{LR} \\
& \xrightarrow[M_R]{} \quad SU(2)_L \times SU(3)_c \times U(1)_Y \quad \equiv G_{std} \\
& \xrightarrow[M_W]{} \quad SU(3)_c \times U(1)_Q
\end{aligned}
\tag{1.10}
$$

The first stage of symmetry breaking at the scale M_U is conventionally done by the the vev of 45 dimensional scalar which has a singlet under the lower symmetry group. In the next stage the breaking at the right handed scale M_R is achieved by the vev of a $SU(2)_R$ triplet field Δ_R which transforms as (1,3,1) under G_{PS} and contained in 126 of SO(10). This scalar also contains the left and handed triplets $\Delta_L = (3,1,1)$ under G_{PS}. Finally in the minimal SO(10) model the 10 dimensional scalar contains the usual $SU(2)_L$ doublet which breaks the electroweak symmetry and gives masses to the scalars.

The mass scales of this symmetry breaking chain can be calculated using the renormalization group equations. The intermediate symmetry breaking scales M_I and M_R reduces the hierarchy problem of the SU(5) GUT. For low M_R there can also be interesting phenomenological consequences.

The fermions transform under the 16 dimensional spinorial representation of SO(10). This multiplet contains the right handed neutrino on top of the standard 15 fermions. Let us at this stage state the transformation properties of the fermions under the group $SU(2)_L \times SU(2)_R \times SU(4)_c$,

$$
\begin{aligned}
\psi_L &= \begin{pmatrix} \nu_L \\ e^-{}_L \end{pmatrix} : (2,1,4) \ ; \ \psi_R = \begin{pmatrix} \nu_R \\ e^-{}_R \end{pmatrix} : (2,1,4) \\
Q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} : (2,1,4) \ ; \ Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} : (2,1,4)
\end{aligned}
\tag{1.11}
$$

Also let us state the scalar fields which may acquire vev,

$$
\begin{aligned}
\phi_1 &\equiv (2,2,1) \ ; \ \tilde{\phi} \equiv \tau_2 \phi_1^* \tau_2 \\
\Delta_L &\equiv (3,1,10) \ ; \ \Delta_R \equiv (1,3,10)
\end{aligned}
$$

The vacuum expectation values of the fields have the following form:

$$
\begin{aligned}
\langle \phi \rangle &= \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \ ; \ \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \\
\langle \tilde{\phi} \rangle &= \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix} \ ; \ \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}
\end{aligned}
$$

Now we are in a position to discuss the physics of neutrino masses in left-right symmetric models. The fermions acquire masses through the Yukawa terms in the Lagrangian when the

Higgs fields acquire vev. The Yukawa part in the Lagrangian written in terms of fermionic and Higgs fields is given by,

$$L_Y = y_1(\bar{f}_L f_R \phi_1) + y_2(\bar{f}_L f_R \phi_2) + y_3(\bar{f}_L^c f_L \Delta_L + \bar{f}_R^c f_R \Delta_R), \tag{1.12}$$

where, y_i (i=1,3) are the Yukawa couplings. With this notation neutrino mass matrix written in the basis (ν_L, ν_L^c) is

$$M = \begin{pmatrix} m_{M_L} & m_D \\ m_D & m_{M_R} \end{pmatrix} \tag{1.13}$$

where m_{M_L} (m_{M_R}) is the left (right) handed Majorana mass term whereas m_D is the Dirac mass term. These terms can be related to the Yukawa couplings and vevs through the following relations;

$$\begin{aligned} m_{M_L} &= y_3 v_L, \\ m_D &= (y_1 + y_2)(k + k'), \longrightarrow ? \quad (y_1 k + y_2 k') \text{ for } T_3 = \tfrac{1}{2} \;\&\; (y_1 k' + y_2 k) \text{ for } T_3 = -\tfrac{1}{2} \\ m_{M_R} &= y_3 v_R. \end{aligned} \tag{1.14}$$

Diagonalizing the mass matrix we obtain the mass eigenvalues. Now let us consider the simplifying assumption that all the Yukawa couplings are of order "h" and the vev k' is much smaller than the *vev* s k. Under this assumption the eigenvalues become,

$$\begin{aligned} m_1 &= y_3 v_R, \\ m_2 &= m_{M_L} - \frac{M_D^2}{m_{M_R}} = y_3 v_L - \frac{h^2 k^2}{y_3 v_R}. \end{aligned}$$

We substitute for v_L from the see-saw condition to get,

$$m_2 = y_3 \frac{\beta k^2}{v_R} - \frac{h^2 k^2}{y_3 v_R}. \tag{1.15}$$

Here β is a function of couplings. We notice that the second term in the right hand side is suppressed by the square of the Yukawa coupling. Due to this the first term dominates. If we want to make the first term small compared to the second we need to fine tune the parameters. Hence one has to fine tune such that $\beta k^2 \simeq 0$ to get acceptable value of the the light neutrino mass.

1.4 Extension to include supersymmetry

Supersymmetry (SUSY) implies that every fermion has a bosoic partner, and vice versa, with the same quantum numbers but with spin differing by one half. Since no such partners have been found in low energy, SUSY has to be broken at the present energy scale but could be restored at and above some higher scale M_{SUSY}.

The primary theoretical motivation for SUSY is that it stabilizes divergent loop contributions to the scalar mass, because fermion and boson loops contribute with opposite signs and largely cancel. This cures the naturalness problem in the SM so long as $M_{SUSY} < O(1TeV)$, where otherwise the Higgs mass would require fine tuning of parameters.

In addition to these general motivations for SUSY there exists several significant motivations for studying SUSY-GUTS. Firstly the ordinary GUTs don't predict satisfactory convergence of the gauge couplings at the scale M_U if there are no intermediate symmetry breaking scale between M_Z and M_U and no new particles (in other words if the couplings converge it does not predict the correct vale of $sin^2\theta_w$ at low energy) however the convergence can be achieved if SUSY partners contribute to the Renormalization Group Equations (RGE). Secondly proton decay is too rapid in non supersymmetric minimal SU(5) GUTs whereas in SUSY GUTs because of the higher value of M_U the it is acceptable. Finally in SUSY-GUT models the Higgs field naturally develops a vacuum expectation value for a heavy top mass.

To understand the basic SUSY algebra, let us consider the following toy example. Here is a field theory where the Lagrangian is given by [2],

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass}, \tag{1.16}$$

where, the terms in the r.h.s. has the following explicit form,

$$\begin{aligned}
\mathcal{L}_{kin} &= \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial_\mu\gamma^\mu\psi + \frac{1}{2}(F^2 + G^2), \\
\mathcal{L}_{mass} &= -m\Big[\frac{1}{2}\bar{\psi}\psi + GA + FB\Big].
\end{aligned}$$

Here A, B, F, G are real scalar fields and ψ is a self conjugate, Majorana spinor field. Since ψ is its own charge conjugate we have,

$$\psi = C\bar{\psi}^T.$$

Wess and Zumino [7] observed that under the transformations,

$$\begin{aligned}
\delta A &= i\bar{\alpha}\gamma_5\psi, \\
\delta B &= -\bar{\alpha}\psi, \\
\delta\psi &= F\alpha - iG\gamma_5\alpha + \partial_\mu\gamma^\mu\gamma_5 A\alpha + i\partial_\mu\gamma^\mu B\alpha, \\
\delta F &= -i\bar{\alpha}\partial^\mu\gamma_\mu\psi, \\
\delta G &= -\bar{\alpha}\gamma_5\partial^\mu\gamma_\mu\psi,
\end{aligned} \tag{1.17}$$

Lagrangian density (1.16) changes by a total time derivative so that the equations of motion are unchanged. We observe that these transformations mix bosons and fermions and so are supersymmetry transformations. The parameter of the transformation, α is thus itself spinorial. Further, to preserve the reality of A and B as well as the Majorana nature of ψ α itself has to have the Majorana nature.

In order to understand the SUSY transformations further, we consider the effect of two successive SUSY transformations with parameters α_1 and α_2.

$$(\delta_2\delta_1 - \delta_1\delta_2)A = -2i\bar{\alpha_1}\gamma_\mu\partial^\mu A\alpha_2. \tag{1.18}$$

In order to find the algebra obeyed by the generator of supersymmetry, we write as usual $\delta = \bar{\alpha}Q$. We note that Q must be a Majorana generator. Now Eqn.(1.18) can be written as,

$$\begin{aligned}
(\bar{\alpha_2}Q\ \bar{\alpha_1}Q - \bar{\alpha_1}Q\ \bar{\alpha_2}Q) &= \bar{\alpha_{1}}_a\alpha_{2b}(\bar{Q}_bQ_a + Q_a\bar{Q}_b)A, \\
&= -2i\bar{\alpha_{1}}_a\alpha_{2b}(\partial_\mu\gamma^\mu A)_{ab}.
\end{aligned} \tag{1.19}$$

[2]The discussion on SUSY algebra closely follows [8]

Here, a, b are spinor indices. To obtain the first equality we have used the fact that the parameter α_{ia} anticommutes with themselves as well as the charges Q_α. Since (1.19) is true for arbitrary values of of the parameters we are led to conclude that the SUSY charges obey the following anticommutation relation,

$$\left[Q_a, \bar{Q}_b\right]_+ = 2(\gamma_\mu P^\mu)_{ab}.$$

Where, P^μ is the translation generator of the Poincare group and the curly brackets denote an anticommutator. This shows that supersymmetry is a space time symmetry and not an internal symmetry. which, in turn, leads us to examine the commutators of Q_a with the generators of the Poincare group. If we identify P^0 with the Hamiltonian, conservation of Q_a implies,

$$\left[Q_a, P^0\right] = 0, \tag{1.20}$$

from covariance,

$$\left[Q_a, P^\mu\right] = 0. \tag{1.21}$$

Notice that the supersymmetry algebra described above contains commutators as well as anticommutators and so is not a Lie algebra. Such algebras are called graded Lie algebras. In fact, Haag, Lopuzanski and Sohnius [9] have shown that the algebra we have derived above is the most general graded Lie algebra compatible with certain rather reasonable physical assumptions.

We note that the the relation (1.20) means that all states but the zero energy one (the vacuum) come in degenerate pairs with one member of the pair being bosonic and the other fermionic. This implies that in a supersymmetric theory, every particle has a partner with the same mass but spin different by $\frac{1}{2}$. Finally we note that supersymmetry being a space time symmetry is independent of any internal symmetry; i.e., the generators of internal symmetry transformations and supersymmetry transformations commute. As a result a particle and its SUSY partner have identical internal symmetry quantum numbers such as electric charge, color, isospin and so on.

In order to construct the Minimal Supersymmetric Standard Model (MSSM), we have to introduce a partner for every particle of the standard model, with a spin differing by $\frac{1}{2}$. Thus the SUSY partners of the matter fermions must have spin zero or spin one. Since the only way we know to consistently introduce interactions of the spin one particles is to make them gauge bosons we are led to consider spin zero bosons as their partners.

In the electroweak symmetry breaking sector, in the standard model, the $SU(2)_L \times U(1)_Y$ symmetry is broken by a single Higgs field. Moreover the same field can give masses to both the upper and lower member of a fermion multiplet. Technically this is possible in SM because both the scalar ϕ and its conjugate ϕ^c can couple to fermions in a gauge invariant way. In SUSY, however, these Yukawa couplings come from the superpotential which in turn cannot depend both on a field and its complex conjugate. As a result, a doublet Higgs scalar can give rise to the mass of either the upper partner or the lower partner of a multiplet but not both.

The MSSM thus contains the particles of the standard model with two Higgs doublets and the spin one bosons which mediate the interactions as shown in the first row of the Table 1.2. The second row contains the super partners of the fields shown in the first row. These are combined to form the super multiplets which are shown in the third row. Thus $L_i(Q_i)$

and $\bar{E}_i(\bar{U}_i, \bar{D}_i)$ are the left handed lepton (quark) and antilepton (antiquark) singlet chiral superfields, where i, refers to the generation index. Similarly $H_{1,2}$ are the chiral superfields representing the the two Higgs doublets and and G is the vector superfield representing the gluon. The Yukawa part of the Lagrangian contains the Higgs coupling terms responsible for the lepton and quark masses.

$$L_Y = h^{\tau}_{ij}(L_i H_1 \bar{E}_j)_F + h^{b}_{ij}(Q_i H_1 \bar{D}_j)_F + h^{t}_{ij}(Q_i H_2 \bar{U}_j)_F. \tag{1.22}$$

ordinaryfields	l_i	$\bar{e}_i$	q_i	$\bar{u}_i$	d_i	h_1	h_2	g	W	Z	γ
superpartners	$\tilde{l}_i$	$\tilde{\bar{e}}_i$	$\tilde{q}_i$	$\tilde{\bar{u}}_i$	$\tilde{\bar{d}}_i$	$\tilde{h}_1$	$\tilde{h}_2$	$\tilde{g}$	$\tilde{W}$	$\tilde{Z}$	$\tilde{\gamma}$
superfields	L_i	$\bar{E}_i$	Q_i	$\bar{U}_i$	$\bar{D}_i$	H_1	H_2	G	..	..	..

Table 1.2: first row - ordinary fields, second row - corresponding superpartners and third row - superfields

In the evolution of the gauge couplings of the MSSM the superpartners also contribute. This changes the expression of the coefficient b of equation (1.8). The new expression for b in the SUSY case is,

$$b = -\frac{1}{16\pi^2}[3N - T_R - T_s]. \tag{1.23}$$

In the LEP experiments at CERN, Geneva, the gauge couplings of the standard model are very precisely measured at the Z-peak. If we evolve the couplings using the supersymmetric beta functions with a symmetry breaking scale of around one TeV the couplings meet at an unique intersection point at the scale of around 10^{16} GeV. This is considered as a hint for the existence of SUSY above the TeV region.

The supersymmetric GUT has another advantage over the nonsupersymmetric GUT. As the unification in the SUSY GUT is achieved at a scale of around 10^{16} GeV the proton decay rate is consistent with the experimental lower bound on proton lifetime. On the other hand the nonsupersymmetric GUT the proton decay is predicted to be too fast.

The supersymmetric GUTs predict the correct electroweak symmetry breaking scale. This happens because the top quark Yukawa coupling contributes to the evolution of the Higgs mass that couples to the top quark. As a result for a heavy top the Higgs mass becomes negative at the electroweak region. It can be shown that for reasonable choices in the parameter space it predicts the correct electroweak symmetry breaking.

Independently of the GUT relations among the Yukawa couplings, h^{τ}, h^b and h^t (for the time being let us consider only the third generation for simplicity) must be bounded from above at the low energies otherwise they would blow up before reaching the unification scale M_U. Using this fact one can obtain an upper bound on the top mass in the SUSY GUTs [10].

$$\begin{aligned} m_t &= h^t v_2, \\ &\lesssim (174GeV)\, h^t_{max}\frac{1}{\sqrt{1+\frac{1}{tan^2\beta}}}, \\ &\lesssim 135, 170, 180, 190\ (GeV)\ \ (for\ tan\beta\ =\ 1, 2, 3, \infty). \end{aligned} \tag{1.24}$$

Here v_2 is the vev of the scalar field H_2 and $tan\beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$. This upper bound is remarkably consistent with the value of the top quark mass predicted from the precision electroweak measurements.

Thus we see that the introduction of SUSY is one of the major candidate theories to solve the gauge hierarchy problem. In the next section we shall introduce another new paradigm of unification models which can also solve the hierarchy problem based on SU(15) and SU(16).

1.5 Low energy unification

One of the most important prediction of GUTs is proton decay. In models such as SU(5) or $SO(10)$, the coupling constants for the SU$(3)_c$, $SU(2)_L$ and $U(1)_Y$ groups evolve to $M_U \sim O(10^{14})$ GeV or higher before they get unified. Proton decay in these models are predicted to occur with a rate of the order of the present experimental limit for such M_U. Recently it has been observed that at least one symmetry breaking chain of a GUT based on the group SU(15) can be unified at a very low energy $M_u \sim O(10^7)$ GeV[17]. Because baryon number B is a gauge symmetry in this model, proton decay can be suppressed, and certain possible Higgs structure has been proposed to this end. This was true for SU(16) GUT[11] also, where it was also known that the proton lifetime can be large. Low energy unification makes these models free from problems of grand unified monopoles[18] and the gauge hierarchy problem is also much less severe.

The drawback of SU(15) and SU(16) GUT compared to SU(5) is that the anomaly cancellation is incomplete without mirror fermions, although the latter could be in principle, replaced by other compensating fermions in higher mass scales in a more general theory based on, e.g., superstrings. If the chiral anomalies are canceled by mirror fermions, it is assumed that such mirror fermions have masses of the order of $O(M_W)$. If they appear as mass degenerate sets, they do not affect the renormalization group determinations of the unification and other mass scales, and more generally need not affect the low energy physics.

The fermions, belong to the 15 dimensional fundamental representation.

$$15_L = (\, u_1, u_2, u_3, d_1, d_2, d_3, \bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{d}_1, \bar{d}_2, \bar{d}_3, e^+, \nu_e, e^- \,)_L \tag{1.25}$$

The 15 of SU(15) decomposes to (12,1)+(1,3) under the group $SU(12)_{quark} \times SU(3)_{lepton}$. Next the subgroup $SU(12)$ breaks to $SU(6)_L \times SU(6)_R \times U(1)_B$ and 12 of $SU(12)$ breaks to (6,1)+(1,6). In these two mass scales the leptoquark and the diquark gauge bosons become massive. Now as the leptoquark and diquarks have different baryon number (note here $U(1)_B$ can be identified as gauged baryon number) they do not mix and hence there is no gauge boson mediated proton decay.

We denote the group SU$(n)^q_L\times$SU$(m)^l_R\times$U$(1)_X$ as n^q_L–m^l_R–1_X where the subscript implies either the charge of the U(1) group or that right (left) handed particles are non-singlets under SU(n) (SU(m)) and the superscript q (l) means that only quarks (leptons) transform under this group. For the breaking $G_\imath \longrightarrow G_{\imath-1}$, the $G_{\imath-1}$ singlet component of the Higgs scalars acquires a vev at a scale $M_\imath$. We consider the pattern $G_1[15] \xrightarrow{\langle\phi_1\rangle} G_2[12^q\text{–}3^l] \xrightarrow{\langle\phi_2\rangle} G_3[6_L\text{–}6_R\text{–}1_B\text{–}3_l] \xrightarrow{\langle\phi_3\rangle} G_4[3_{cL}\text{–}2^q_L\text{–}6_R\text{-}\ \text{-}1_B\text{–}3_l] \xrightarrow{\langle\phi_4\rangle} G_5[3_{cL}\text{–}2^q_L\text{–}3_R\text{–}1_R\text{–}1_B\text{–}2^l_L\text{–}1^l_Y] \xrightarrow{\langle\phi_5\rangle} G_6[3_c\text{–}2_L\text{–}1_B\text{–}1_{Y'}] \xrightarrow{\langle\phi_6\rangle} G_7[3_c\text{–}2_L\text{–}1_Y] \xrightarrow{\langle\phi_7\rangle} G_8[3_c\text{–}1_Q]$, with $\langle\phi_\imath\rangle = M_\imath$.

This scenario of symmetry breaking of the $SU(15)$ GUT has some more interesting features. It is essential for the low energy unification to have the chiral color $SU(3)_{cL} \times SU(3)_{cR}$ group and the quark-lepton un-unified group $SU(2)_L^q \times SU(2)_L^l$ survive till very low energy, for the gauge coupling constants to evolve very fast and get unified at an energy scale $M_U \sim O(10^8)$ GeV. Thus the existence of these groups and the lepto-quarks are some of the essential critereons of the low energy unification, which can be tested in the laboratory in near future. Furthermore, any signature of supersymmetry will rule out the possibility of low energy unification [19]. If we find signatures of the low energy unification and supersymmetry both, then our understanding of the grand unification has to be revised completely.

Bibliography

[1] R. Berbieri, Summary talk given at Recontres de Moriond on Electroweak Interactions and Unified Theories, Les Arcs, France. March 1993, CERN Preprint, CERN-TH-6863/93, (1993).

[2] G. Altarelli, R. Caslbuoni, D. Dominici, F. Feruglio and R. Gatto, Mod Phys Lett **A5**, (1990), 495; Nucl Phys **B342**, (1990), 15.

[3] M. E. Peskin and T. Takauchi, Phys Rev Lett **65**, (1990) 964.
B. Holdom and J. Terning, Phys Lett **B 253**, (1990), 964.
G. Bhattacharya, S. Banerjee, P. Roy, Phys Rev **D45**, (1992), 729.

[4] SU(5) GUT was proposed by H. Georgi and S. L. Glashow, Phys Rev Lett, **32**, (1974), 438.

[5] H. Georgi, H. R. Quinn, S. Weinberg, Phys Rev Lett, **33**, (1974), 451.

[6] See for example, E. Farhi and L. Susskind, Phys Rep. **74c**, (1981), 277.

[7] J. Wess and B. Zummino, Nucl Phys **B 70**, 39, (1970)

[8] For a recent review of MSSM see Lectures by Xerxes Tata, given at Mt. Sorak Symposium. University of Hawai report: UH-511-713-90.

[9] R. Haag, J. Lapuszanski and M. Sohnius, Nucl Phys. **B8**, 61 (1975)

[10] J. L. Lopez in the Proceedings of the International School of Subnuclear Physics 31st course; Erice, Italy, July 4-12, 1993. Texas A and M University report CTP-TAMU-42/93.

[11] J.C. Pati and A. Salam, Phys. Rev. **D 10** (1974) 275; R.N. Mohapatra and J.C. Pati, Phys. Rev. **D 11** (1975) 566.

[12] See for example, A. Masiero in "Grand Unification with and without Supersymmetry and Cosmological Implications", International School for Advanced Studies Lecture Series No. 2, published by World Scientific, (1984), p.1; see also A. Zee (ed.) "Unity of Forces in the Universe", Vol. 1, published by World Scientific, 1982.

[13] U. Sarkar, Ph. D. Thesis, University of Calcutta, (1983).

[14] J.C. Pati and A. Salam, Phys. Rev. **D 10** (1974) 275; R.N. Mohapatra and J.C. Pati, Phys Rev. **D 11** (1975) 566.

[15] T.G. Rizzo and G. Senjanovic, Phys. Rev. **D 24** (1981) Lett. **B 273** (1992), 105

[16] U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. **B 260** (1991), 447
F. Anselmo, L. Ciforelli, A. Zichichi, CERN Preprint, CERN-PPE/92-123, (1992).

[17] P.H. Frampton and B.H Lee, Phys.Rev. Lett **64** (1990) 619.

[18] P. B. Pal, Phys Rev **D44**, 1366, (1992)

[19] B. Brahmachari, U. Sarkar, K. Sridhar, Phys Lett **B297**, (1992), 105