CHAPTER - II

CHAPTER – II
REVIEWS OF RELATED LITERATURE

2.0 INTRODUCTION

From the worldwide, volume of researches available in Mathematics education is quite large and varied (in both quality and applicability). For the present research study, the researcher has outlined many of the research study reviews and their findings concerned with many of the essential learning and academic requirements in Mathematics which have been sorted and segregated in fair manner as below. Perhaps, some of the research results taken up for this study may seem dated or with respect to the perspectives of particular state or country but was included because it contributes in some manner with reference to the current situation and concerns as well found useful for the present research study in terms to get valuable insights which may help for the improvements at any stage of present research in any manner. Further, reviews of related literature are elaborated among the sections which are categorised in five parts as shown in the following table-2.1.

Table – 2.1:
Distribution Of Reviewed Research Studies

Sr. No.	Category and Area of Mathematics	Years of the studies (From – To)	No. of Studies
I	**Reviews on Misconceptions in learning of Mathematics**	**1979 - 2012**	**Total = 24 (03+21=24)**
a)	Arithmetic	1991 - 2012	05
b)	Measurement	1979 - 2000	03
c)	Geometry	1988 - 2000	06
d)	Algebra And Calculus	1997 - 2012	05
e)	Probability And Statistics	1990 - 2000	02
II	**Reviews on other Attributes concerned with Mathematics Education**	**1975 - 2013**	**Total = 42**
a)	Attitudes	1979 - 2005	06
b)	Problem Solving	1985 - 2004	05
c)	Interactions And Responses	2000 - 2005	02
d)	Manipulative	2000 - 2013	07
e)	Constructivism	1999 - 2010	08

f)	Assessment And Achievement	1975 - 2012	14
III	**Reviews on Instructional Strategies For Mathematics Education**	**1978 - 2010**	**Total = 10**
IV	**Reviews on Conceptual Understanding or In-depth learning in Mathematics**	**1980 - 2013**	**Total = 06**
V	**Reviews on SOLO Taxonomy**	**1987 - 2012**	**Total = 24**
Total studies			**106**

The researcher had made best possible efforts to collect present research related literatures as well studies and many of these are incorporated here in this chapter-II. It may be possible that many more studies might be available but it is possible that the researcher might have not been come across. The Researcher for the present study had found a report drafted by Jhonson (2000) that is a research study based on the (literature) reviews of the various research studies and literatures and concluded with the critical findings on teaching and learning of Mathematics at Washington. Thus, for this present study also, many of the findings from the said report have been found appropriate to take into consideration and included in the summary manner in these sections or parts presented below.

2.1 PART- I: REVIEWS ON MISCONCEPTIONS IN LEARNING OF MATHEMATICS

Following are the reviews collected for some of the commonly identified learning difficulties, misconceptions or misunderstandings with findings and suggestions derived from the research studies conducted on the various areas or concepts of the Mathematics.

Paria (1999) had attempted to search the origin of errors committed by the higher secondary students in some selected topics. It was found that the main errors identified were conceptual and computational and computational difficulty in the selected topics. Students faced difficulty in applying the laws of indices. The errors originated due to certain teacher and learner factors. The teachers were often unaware of the necessary and sufficient background knowledge of the students before teaching a particular Mathematical topic. Student often failed to remember formulae, key concepts and other relation of earlier topics. This ignorance prevented them from understanding the current topics properly. Sometimes students were not conversant with or did not know the theory, basic principles and the operations, and often make mistakes in applying them.

George (2003) investigated into the Mathematical Backwardness and its Remediation in Goa. The focus of the study was on construction of a diagnostic test in Mathematics for standard VII, identification of the causes of backwardness in Mathematics and to formulate remedial programs for the selected case studies. The sample was selected from a population of forty-one schools of Ponda Taluka. Of these schools, ten were government schools; one missionary school and remaining were privately run management schools. The study involved several samples for various purposes at various stages mainly for Standardization of test, Diagnosis and case study. Tools used in the study were Standardized Mathematics Achievement EST, Diagnostic Test, Cattle's Cultural Fair Intelligence Test Scale three Form-A, Raven's standard Progressive Matrices, Interview Schedules, Home Background and other details Questionnaire for Backwardness. Study includes both qualitative and quantitative data. The findings of the study were: (i) Mean for entire sample as well as Highest score of entire sample was much lower than the norms itself. (ii) Percentages of correct response on diagnostic test revealed areas of backwardness. (iii) Case study findings were such that the expectations of students from Mathematics teacher and kind of teacher behaviors appreciated by the students were posing questions about prevalent teacher practices. (iv) Remedial programme showed improvement in terms of attitude and performance.

Yasoda (2009) conducted a study on the problems in teaching and learning mathematics. The objectives of the study were, (1) To identify the difficulty areas in secondary level mathematics as perceived by the pupils and teachers. (2) To identify the problems faced by the pupils in learning mathematics and by the teachers in teaching mathematics. (3) To study the attitudes of pupils towards learning mathematics and of teachers towards teaching the subject.(4) To study the variation in the problems and attitudes of the pupils of sub groups depending upon their personal and demographic variables. (5) To suggest the suitable strategies for the improvement of teaching-learning mathematics at the secondary level. The findings of the study were in VIII class text book the chapters **'commercial mathematics'** and

‘mensuration’ are the most difficult chapters for the students whereas for the teachers along with the above two chapters **‘triangles and polygons’** and **‘circles and concurrent lines of triangles’** are respectively are **most difficult chapters**. Students are **facing problems in understanding the mathematical language**, symbols and relation between different concepts in mathematics.

2.1.1 ARITHMETIC

Nalayini (1991) studied about the effectiveness of Using Number games to teach Arithmetic at Primary level. The sample comprised students of classes I to IV of kendriya Vidyalaya, Coimbatore. In each class, the experimental group consisted of 50 students and the control group of 25 students. It was found that neither the educational level nor the economic status of parents influenced the arithmetic growth score of the pupils. It was also conclude that number games motivated children to develop the computational skills.

Kapur & Rasario (1992) conducted intervention strategies for students with problems in learning Arithmetic. The sample consisted of twenty five students in the age group of eight to eleven years of class four, having significant problems in learning Arithmetic. Tools used in the study were the Wesheler Intelligence Scale for children and a short form of Arithmetic test based on Schonell Diagnostic Arithmetic Test. It was found that: (i) despite having average intellectual abilities and having regular classroom coaching, many students fail to perform well in Arithmetic. (ii) Students with problems in learning can be helped through remedial education which has varied instructional objectives.

Subramaniam & Singh (1996) studied the mistakes committed by students in the application of different mathematical skills and developing preventive and remedial teaching strategies using metacognitive approach for qualitative improvement in teaching of Mathematics. The data were collected from eight government primary schools in the districts (Sehore and Bilaspur districts of MP). Each school was visited for three consecutive days. On the first day, test in Mathematics was administered to children. These children were interviewed on the second day. On the last day, mistakes committed by children were identified, analyzed and classified through a workshop and the recorded diagrams were scanned. Finally a compendium of mistakes was prepared. The major findings were: (i) The students committed six types of mistakes in addition, eight types of mistakes in subtraction, ten types of mistakes in multiplication and six types of mistakes in division. (ii) Some students felt that due to confusion between multiplication and addition signs, forgetfulness of the procedures, lack of opportunity to write on the note book etc, they committed mistakes in the test. (iii) Poor concept of carrying over, zero & multiplication, introvert behaviour, lack of writing skills, etc were observed as possible causes of mistakes committed by students. (iv) The teachers of the schools cited home environment, SES, physical facilities in the school, extra workload on teachers, lack of interest, motivation and discipline, large size of class, general promotion policy, etc responsible for the poor performance of children in the test.

Johnson (2000) drafted a report as 'Teaching and Learning Mathematics: Using Research to Shift From the "Yesterday" Mind to the "Tomorrow" Mind', summarised the reviews revealed from various research studies were conducted to investigate about **misconception/s in the areas of Arithmetic of Mathematics**. Findings were outlined in the report as: Students' **conceptual misunderstandings** of decimals lead to the adoption of rote rules and computational procedures that often were incorrect. This adoption occurs despite a natural connection of decimals to whole number, both in notation and computational procedures (English & Halford, 1995). Students with a weak understanding of place value have a difficult time understanding decimals. For example, students will mentally separate a decimal into its whole number part and its pure decimal part, such as rounding 148.26 to 150.3 (Threadgill-Sowder, 1984). Or, students will assume that "more digits" implies that a number is larger, such as 0.1814 being larger than 0.385 and 0.3 (Hiebert and Wearne, 1986). Students do not make good use of their understandings of rational numbers as a starting point for developing an understanding of ratio and proportion (Heller et al., 1990). Students with good understandings of the part/whole interpretation of a fraction still can have difficulty with the concept of fraction equivalence, confuse quantity notions with proportionality, possess limited views of fractions as numbers, and have cognitive difficulty relating fractions to division (Kerslake, 1986). Students' intuitive understanding of the concept of infinity remains quite stable over the middle grades and is relatively unaffected by mathematics instruction (Fischbein et al., 1979). Many teachers have a surface level understanding of fractions and decimals, with the result being that students are engaged in learning activities and discussions that are misleading and prompt misconceptions such as “multiplication makes bigger” and “division makes smaller” (Behr et al., 1992).

Mirirai et al. (2012) conducted a study on ‘Teaching Fractions at Ordinary Level: A Case Study of Mathematics Secondary School Teachers in Zimbabwe’. The aim of the study was to investigate the teaching of fractions by Ordinary level mathematics teachers at Radcliff high school in Zimbabwe. The main objective of the study was to establish how teachers teach the concept of fraction and to find out why they teach in the manner they do. A case study research design was used and purposive sampling was implemented from the whole population of mathematics teachers at the school to select the sample for the study. The sample consisted of three "O" level mathematics teachers. Documentary analysis, lesson observations and interviews were used to collect data to answer the research questions. The collected data was qualitatively interpreted and analyzed. The results of the study revealed that teachers use traditional methods which are anchored on practice of problem tasks, exemplification (teaching by giving examples), drill and teaching of rules and algorithms in the teaching of fractions. Post data interpretation led to the findings that teachers promote procedural understanding of the concept of fraction. The way teachers teach is heavily influenced by their beliefs on teaching. Teachers believe that giving worked examples and having students to follow rule based procedures will enhance the students' problem solving capabilities. As justification of their strategy, teachers cited shortage of time to prepare for the lessons, examination driven curriculum and limited resources. Basing on these findings, the researchers recommend staff development workshops and seminars to equip teachers with skills which will enable them to employ child centred teaching strategies that may result in the conceptual understanding of fractions.

2.1.2 MEASUREMENT

Hildreth (1979) examined the measurement strategies used by “good estimators,” then suggested that students need to learn and practice these strategies: 1) Simple comparison: Ask students to think of a “known” object that is both familiar to them and about the same size as the new object. 2) Bracketing: Ask students to think of two “known” objects such that when they are compared to the new object, one is just slightly smaller and the other is slightly bigger. 3) Chunking: Ask students to partition the new object into parts (not necessarily equal) where they know the measure of these parts. 4) Unitizing: Ask students to create a unit that can be mentally reproduced to form a partition of the new object. 5) Rearrangement: Ask students to mentally cut and rearrange an object to make estimation easier (especially for area situations). 6) Error reduction: Ask students to identify and discuss systematic errors that can occur in an estimation strategy, then create techniques for compensating for these errors.

Figueras & Waldegg (1984) investigated about the understanding of measurement concepts and techniques of middle school students, with these conclusions: 1) In increasing order of difficulty: Conservation of area, conservation of length, and conservation of volume; 2) Measurement units are used incorrectly by more than half of the students; 3) Students are extremely mechanical in their use of measuring tools and counting iterations of equal intervals; 4) Students find areas and volumes by counting visual units rather than using past "formula" experiences, even if the counting process is tedious or complex; 5) Student performance on measurement tasks decreases significantly when the numbers involved are fractions. The researchers suggested that "a fixed measuring system is introduced far too early in the curriculum of elementary school, thus creating a barrier to the complete understanding of the unit concept".

Johnson (2000) summarised the reviews revealed from various research studies conducted on **misconception in the area of Measurement of Mathematics** reported as: Students are fluent with some of the simple measurement concepts and skills they will encounter outside of the classroom (e.g. recognizing common units of measure, making linear measurements), but have great difficulty with other measurement concepts and skills (e.g., perimeter, area, and volume) (Carpenter et al., 1981). Students initially develop and then depend on physical techniques for determining volumes of objects that can lead to errors in other situations. For example, students often calculate the volume of a box by counting the number of cubes involved. When this approach is used on a picture of a box, students tend to count only the cubes that are visible. The counting strategy also fails them if the dimensions of the box are fractions (Hart, 1981a). Also, Students at all grade levels have great difficulties working with the concepts of area and perimeter, often making the unwarranted claim that equal areas of two figures imply that they also have equal perimeters. Perhaps related to this difficulty, many secondary students tend to think that the length, the area, and the volume of a figure or an object will change when the figure or object is moved to another location.

2.1.3 GEOMETRY

Rawool (1988) studied the conceptual maturity of students belonging to the age group 11 to 14 in non-metric geometry. The sample of the study consisted of 50 students. The data were collected using three tests. The first test consisted of the task of classification, the second of the task of drawing geometrical figure as per the given description, third involved the task of describing a verbally for given geometrical figure. The major finding of the study was the evidences showed that students were familiar with the terminology, assumptions and figural and concrete representation related to the non-metric geometrical concepts, but they failed to use these concepts at the “understanding” and “applications” levels. The students failed to use geometrical terms, assumptions and figural representations rigorously and failed to deduce relationships in the geometrical context with the different concepts they added their own ideas and formulated their own assumptions, which were not accepted by the geometrical structure.

Dutta (1990) had studied the diagnosis and prevention of learning disabilities in the reasoning powers of the students in Geometry. The researcher found that the disabilities were there because the teaching of Geometry was geared to the needs of the most able students, there were no experiments to strengthen the teaching of Geometry; and the relation of Geometry and physical space was not explored. Also found that the use of audio-visual materials leads to greater interest as well clearer understanding and longer retention of Geometrical concepts. Finally, recommended as the teaching of Geometry has been a subject of debate.

Gurusamy (1990) attempted to diagnose the errors committed by students of class IX in solving problems in geometry and has develop a remedial package. The case study method was used by the investigator. Tool used for data collection was a questionnaire. The major finding was, the level of performance of the students in the post-test was found to be high after the implementation of the remedial programme.

Clements & Battista (1992) conducted a research study in terms to identify the errors made by students in Geometry and it found as students' misconceptions in geometry lead to a "depressing picture" of their geometric understanding. Some examples are: 1) An angle must have one horizontal ray. 2) A right angle is an angle that points to the right. 3) A segment must be vertical if it is the side of a figure. 4) A segment is not a diagonal if it is vertical or horizontal. 5) A square is not a square if the base is not horizontal. 6) Every shape with four sides is a square. 7) A figure can be a triangle only if it is equilateral. 8) The angle sum of a quadrilateral is the same as its area. 9) The area of a quadrilateral can be obtained by transforming it into a rectangle with the same perimeter. Also some findings stated in the report as, both teachers and students use an imprecise language that directly impacts the students' developmental progress in geometric understanding. In turn, teachers must help students distinguish between the mathematical use of a term and its common interpretation (e.g. plane). Finally, the geometric meaning underlying a student's geometric language may differ considerable from what a Mathematics teacher might think is the student's meaning.

Haralambos (2000), also studied about the conceptual errors made by the students in the Geometry and also examined how students conceptualize various geometric concepts on tenth-grade geometry. It provides the suggestion of additional strategies for the improvement of the teaching and learning of geometric proofs. Further results of the research indicated that students write proofs that are better organized through shared knowledge than the proofs presented in the textbooks.

Johnson (2000) reported from the reviews on the **misconceptions about the aspects of shapes and dimensions of Geometry** as: Students have difficult time communicating visual information, especially if the task is to communicate a 3-D environment (e.g., a building made from small blocks) via 2-D tools (e.g., paper and pencil) or the reverse (Ben-Chaim et al., 1989). A computer environment can generate multiple representations of a shape that help students generalize their conceptual image of that shape in any size or orientation (Shelton, 1985). Both teachers and students use an imprecise language that directly impacts the students' developmental progress in geometric understanding. In turn, teachers must help students distinguish between the mathematical use of a term and its common interpretation (e.g. plane). Finally, the geometric meaning underlying a student's geometric language may differ considerable from what a mathematics teacher might think is the student's meaning (Clements & Battista, 1992). Students have a difficult time using the word **"similar"** and its mathematical meaning correctly. Too often, the word is used loosely by students and teachers to mean "approximately the same," which led to a subsequent classification of rectangles of most dimensions as being similar (Hart, 1981c).

Johnson (2000) reported from the reviews on the **misconceptions about the relationships or transformation aspects of Geometry** as: Students could perform successfully on various assessments in a geometry class, yet hold several false beliefs. Examples of these false beliefs are (1) that "geometric form" is preferred over "geometric substance," (2) that a geometry problem not solved in a few minutes is unsolvable, and (3) that geometry (or mathematics) is a collection of facts established by others that "are inaccessible to them except by memorizing" (Schoenfeld, 1988). Students build false interpretations of geometric terms from their exposure to a limited number of static pictures in texts. For example, many students claim that two lines cannot be parallel unless they are the same length or are oriented vertically or horizontally (Kerslake, 1981). Some students think that the sides of a triangle change length when the triangle is rotated in a plane (Kidder, 1976).

2.1.4 ALGEBRA AND CALCULUS

Jayasree (1997) identified the difficulties experienced by the pupils of standard VIII in expanding algebraic expression using identities with the help of a diagnostic test. The study revealed that the level of attainment is poor in the case of classification of open and closed sentences, finding the always-true sentences and product numbers using identities. The study also revealed that there is no mastery of the rules of signs and many pupils do not seem to have a clear grasp of identities.

Johnson (2000) concluded the finding from the research studies on the **misconceptions in Algebra** as: Students experience many difficulties if they persist in viewing algebra as "generalized arithmetic". Some pertinent **algebraic misconceptions or inconsistencies** identified by research studies are: (1) Arithmetic and algebra use the same symbols and signs but interpret them differently. For example, an equal sign can signify "find the answer" and express equality between two expressions (Booth, 1988; Matz, 1982). (2) Arithmetic and algebra use letters differently. For example, students can confuse the expressions 6 m with 6m, where the first represents 6 meters (Booth, 1988). (3) Arithmetic and algebra treat the juxtaposition of two symbols differently. For example, "8y" denotes a multiplication while "54" denotes the addition 50+4. Another example is the students' inclination that the statement "2x=24" must imply that x=4. (Chalouh & Herscovics, 1988; Matz, 1982). (4) Students have cognitive difficulty accepting a procedural operation as part of an answer. That is, in arithmetic, closure to the statement "5+4" is a response of "9," while in algebra, the statement "x+4" is a final entity by itself (Booth, 1988; Davis, 1975). (5) In arithmetic word problems, students focus on identifying the operations needed to solve the problem. In algebra word problems, students must focus on representing the problem situation with an expression or equation (Kieran, 1990).

Johnson (2000) concluded the finding from the research studies on the **misconceptions in Algebra with reference to the 'Relations, Representation & Operations'** in Algebra as: Schoenfeld & Arcavi (1988) argue that "understanding the concept [of a variable] provides the basis for the transition from arithmetic to algebra and is necessary for the meaningful use of all advanced mathematics." The concept of a variable is "more sophisticated" than teachers expect and it frequently becomes a barrier to a student's understanding of algebraic ideas (Leitzel, 1989). For example, some students have a difficult time shifting from a superficial use of "a" to represent apples to a mnemonic use of "a" to stand for the number of apples (Wagner & Kieren, 1989). Students have difficulty representing and solving algebraic word problems because they rely on a direct syntax approach which involves a "phrase-by-phrase" translation of the problem into a variable equation (Chaiklin, 1989; Hinsley et al., 1977). An example of this difficulty is the common reversal error associated with the famous "Students-and-Professors" problem **:** *Write an equation using the variables S and P to represent the following statement: "There are 6 times as many students as professors at this university." Use S for the number of students and P for the number of professors.* A significant number of adults and students (especially engineering freshmen at MIT) write the reversal "6S=P" instead of the correct expression "S=6P." Clement et al. (1981) suggest that the reversal error is prompted by the literal translation of symbols to words, where S is read as "students" and P as "professors" rather than S as "the number of students" and P as "the number of professors." Under this interpretation, the phrase "6 students are equal to 1 professor" becomes a ratio. Students over generalize while simplifying expressions, modelling inappropriate arithmetic and algebra analogies. Using the distributive property as the seed, students generate false statements such as a+(bxc)=(a+b)x(a+c) ; $\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$ and $(a+b)^2 = a^2+b^2$ (Matz, 1982; Wagner & Parker, 1993). An emphasis on the development of "operation sense" is necessary to prepare students for their introduction to algebraic reasoning. A suggested approach is the use of word problems and computational problems as contexts for both constructing and enhancing the meanings for the four basic operations (+, −, x, ÷) (Schifter, 1999).

Johnson (2000) concluded the finding from the research studies on the **misconceptions in Algebra with reference to the 'Graphical representation'** in Algebra as: Students may be able to solve traditional problems using both **algebraic and graphical representations**, yet have minimal understanding of the relationships between the two representations (Dreyfus & Eisenberg, 1987; DuFour et al., 1987). Students have difficulty accepting the fact that there are more points on a graphed line than the points they plotted using coordinates. This is known as the continuous vs. discrete misconception. Some students even contend that no points exist on the line between two plotted points, while other students accept only one possible such point, namely the mid-point (Kerslake, 1981). Middle school students find constructing Cartesian graphs difficult, especially with regard to their choice of a proper scaling, positioning the axes, and understanding the structure involved (Leinhardt et al., 1990). Students misinterpret time-distance graphs because they confuse the graph with the assumed shape of the road. Also, students do not necessarily find it easier to interpret graphs representing real-world contexts when compared to graphs representing "symbolic, decontextualized" equations (Kerslake, 1977). Students have a difficult time interpreting graphs, especially distance-time graphs. Intuitions seem to override their understandings, prompting students to view the graph as the path of an actual "journey that was up and down hill" (Kerslake, 1981).

Johnson (2000) concluded the finding from the research studies on the **misconceptions in Calculus** as: The concept of a function is the "single most important" concept in mathematics education at all grade levels (Harel and Dubinsky, 1992). Students have trouble with the language of functions (e.g., image, domain, range, pre-image, one-to-one, onto) which subsequently impacts their abilities to work with graphical representations of functions (Markovits et al., 1988). Students experience difficulty with functions often because of the different notations. For example, Herscovics (1989) reported that in his research study, 98 percent of the students could evaluate the expression a+7 when a=5 when only 65 percent of this same group could evaluate f(5) when f(a)=a+7. In Dreyfus' (1990) summary of the research on students' working to understanding functions, three problem areas are identified **:** (i) The mental concept that guides a student when working with a function in a problem tends to differ from both the student's personal definition of a function and the mathematical definition of a function. (ii) Students have trouble graphically visualizing attributes of a function and interpreting information represented by a graph. (iii) Most students are unable to overcome viewing a function as a procedural rule, with few able to reach the level of working with it as a mathematical object. Students can learn to interpret the elements of a matrix and do matrix multiplication, but their understandings are very mechanical (Ruddock, 1981). Students experience difficulties understanding and working with the concept of a limit. The underlying problems are (1) the use of common words as mathematical language (e.g., a speed limit is something that cannot be exceeded), (2) the different mathematical interpretations for different contexts (e.g., limit of a sequence, limit of a series, or limit of a function), and (3) the student's false assumption that everything can be reduced to a formula (Davis and Vinner, 1986).

Garo (2008) compared algebraic achievement and educational practices of 9th graders in Albania and the U.S., as well as identifying those educational practices that appear to be significant predictors for algebraic achievement of students in each country. In April and May 2007, about 242 of 9th grade American students from four high school in Grand Forks of the state of North Dakota (U.S.) and 219 students from four high schools in Durres (Albania), participated in the study. The data collection instruments consisted of an achievement test and a student questionnaire. The test adopted a Texas Publicity Released Standardized Test. It was focused on the algebra 1 knowledge covered by schools of the two countries during the academic year 2006-07. While one part of the U.S. sample (145 students) did not use calculators on the test, the other part

(97 students) used calculators. The entire sample of Albania students did not use calculators on the test. The questionnaire attempted to measure students perceptions of educational practices exercised in classrooms and communities of each country. The study did not found significant difference in the overall algebraic achievement of students in the two countries. Albanian students significantly outperformed American students in the specific domains of knowledge and applying. The difference in the specific domain of reasoning was not significant. American calculator users significantly outperformed by Albanian in the cognitive domains of applying and reasoning but significantly outperformed in the cognitive domain of knowing. The study also found significant differences in many instructional and non-instructional practices used in the cultures of these countries. Some practices such as taking multiple-choice tests, spending time with friends, re-teaching of a topic and self competence in mathematics appear as significant predictors of achievement of students in both countries.

Ross et al. (2011) reported in the research study as, a central goal of secondary mathematics is for students to learn to use powerful algebraic strategies appropriately. Research has demonstrated student difficulties in the transition to using such strategies. The study examined strategies used by several thousand 8th, 9th, and 10th-grade students in five different school systems over three consecutive years on the same algebra problem. The research study also analyzed connections between their strategies and their success on the problem. The findings suggest that many students continued to struggle with algebraic problems, even after several years of instruction in algebra. Students did not reflect the anticipated growth toward the consistent use of efficient strategies deemed appropriate in solving this problem. Instead a surprisingly large number of students continued to rely on strategies such as guessing and checking, or offered solutions that were unintelligible or meaningless and not useful to the researchers. Even those students who used algebraic strategies consistently did not show the anticipated improvement of performance that would be expected from several years of continuing to study Mathematics.

Noss et al. (2012) conducted study on "The Design of a System to Support Exploratory Learning of Algebraic Generalisation". This study charts the design and application of a system to support 11-14 year old students' learning of algebraic generalisation, presenting students with the means to develop their understanding of the meaning of generality, see its power for mathematics and develop algebraic ways of thinking. The study focuses squarely on design, while taking account of both technical and pedagogical issues and challenges and provides an account of how we have designed and built a system with a very close fit to our knowledge of students' difficulties with the subject matter. Also, reported the challenges involved in building a system that is both intelligent and exploratory, a learning environment in which both student and teacher are supported without explicit tutoring.

2.1.5 PROBABILITY AND STATISTICS

Sarala (1990) has analyzed the conceptual errors of secondary schools students in learning of selected areas in modern Mathematics. The sample consisted of 800 pupils selected by the stratified sampling procedure from the secondary schools in the Trivendram revenue district. The tools used were diagnostic tests in sets, Trigonometry and in Statistics; the non-verbal test of intelligence by Nafde, personal data sheet. The major findings were the numbers of errors were quite large, and these errors were influenced by sex, locality of the school, management of the school, intelligence, study habits and socio- economic-status. The errors decrease with intelligence.

Johnson (2000) concluded the finding from the research studies on the **misconceptions in Probability with reference to the 'Chance/s' & 'Data Analysis'** in Probability as: Students estimating the probability of an event often ignore the implications of the sample size. This error is related to an operational misunderstanding of the law of large numbers (Kahneman & Tversky, 1972). Students have poor understandings of fundamental notions in probability: the use of tree diagrams, spinners using the area model, random vs. nonrandom distributions of objects, and the general idea of randomness itself. To overcome these understandings, students need more exposure to the ratio concept, the common language of probability (e.g., "at least," "certain," and "impossible"), and broad, systematic experiences with probability throughout their education (Green, 1983, 1988). Students tend to categorize events as equally likely because of their mere listing in the sample space. An example is the student who claims that the probability of rolling a prime number is the same as the probability of rolling a composite number on a single role of a single die (Lecoutre, 1992). Appropriate instruction can help students overcome their probability misconceptions. Given an experiment, students need to first guess the outcome, perform the experiment many times to gather data and then use this data to confront their original guesses. A final step is the building of a theoretical model consistent with the experimental data (Shaughnessy, 1977; DelMas & Bart, 1987).

Johnson (2000) reported the finding from the research studies on the **misconceptions in Probability with reference to the 'Predictions & Inference'** in Probability as: Students often compute the probabilities of events correctly but then use incorrect reasoning when making an inference about an uncertain event. The problem is the students' reliance on false intuitions about probability situations that overpower their mathematical computations (Garfield & Ahlgren, 1988; Shaughnessy, 1992). Students often will assign a higher probability to the conjunction of two events than to either of the two events individually. This conjunction fallacy occurs even if students have had course experiences with probability. For example, students rate the probability of "are 55 and having a heart attack" as more likely than the probability of either "being 55" or "having a heart attack." An explanation for the error is that students may confuse the conjunction form (e.g., "being 55 and having a heart attack") with the conditional form (e.g., "had a heart attack given that they are over 55") (Kahneman and Tversky, 1983). Student misconceptions of independent events in probability situations can be impacted by exposure to real-world experiences that help the students: (1) Realize that dependence does not imply causality (e.g. oxygen does not cause life yet life depends on oxygen to keep breathing). (2) Realize that it is possible for mutually exclusive events to not be complementary events. (3) Realize the distinction between contrary events & contradictory events (Kelly & Zwiers, 1988).

Johnson (2000) summarised the finding from the research studies about the **misconceptions in Statistics** as: Students can calculate the average of a data set correctly, either by hand or with a calculator and still not understand when the average (or other statistical tools) is a reasonable way to summarize the data (Gal., 1995). Introducing students prematurely to the algorithm for averaging data can have a negative impact on their understanding of averaging as a concept. It is very difficult to "pull" students back from the simplistic "add-then-divide" algorithm to view an average as a representative measure for describing and comparing data sets. Key developmental steps toward understanding an average conceptually are seeing an average as reasonable, an average as a midpoint, and an average as a balance point (Mokros and Russell, 1995). Students and adults hold several statistical misconceptions that researchers have shown to be quite common: 1) Students assign significance incorrectly to any difference in the means between two groups. 2) They believe inappropriately that variability does not exist in the real world. 3). They place too much confidence (unwarranted) in results based on small samples. 4) They do not place enough confidence in small differences in results based on large samples. 5) They think incorrectly that the choice of a sample size is independent of the size of the actual population (Landewehr, 19889). Computer environments help students overcome **statistical misconceptions** by allowing them to control variables as they watch a sampling process or manipulate histograms (Rubin & Rosebery, 1990).

2.1.6 SUMMARY OF REVIEWS IN PART-I

As reviews in part-I are segregated according to the areas or concepts of Mathematics. Paria (1999) conducted study to investigate the origin of errors committed by the higher secondary students in some selected topics while George (2003) and Yasoda (2009) made research study to find out the reasons for the backwardness and the problems with the teaching – learning of the Mathematics.

In efforts to investigate about the misunderstandings or misconceptions or errors or mistakes committed by the students in various area are as: Nalayani (1991); Kapur & Rasario (1992); Subramaniam & Singh (1996); Mirirai et al (2012) conducted the studies in Arithmatics in terms to study problems, misunderstanding or errors and effectiveness of the teaching-learning. Hildreth (1979) and Figueras & Waldegg (1984) conducted the research studies in the area of Measurement of Mathematics. Rawool (1988); Dutta (1990); Gurusamy (1990); Clements & Battista (1992) and Haralambos (2000) carried out the research studies in Geometry. While Jayasree (1997); Garo (2008); Ross et al. (2011) as well Noss et al. (2012) took up the researches in the area of Algebra and Calculus. Sarala (1990) reviewed and major of the reviews found in Johnson (2000) about Probability and Statistics.

As, Johnson (2002) reviewed many research studies in the areas as well sub-areas of the Arithmetic, Measurement, Geometry, Algebra and Calculus as well in Probability and Statistics where reviews derived from many research studies as well from various literatures are concluded with very specifically identified errors committed in general by the students.

About the methodologies, Nalayini (1991) experimented through Number games with 50 and 25 primary school students were divided into two groups. Kapur & Rasario (1992) also experimented with twenty five students of grade-IV with the help of tools Wesheler Intelligence Scale for children and a short form of Arithmetic test based on Schonell Diagnostic Arithmetic Test. Subramaniam & Singh (1996) used Achievement test and interviews were conducted to collect the data from the students of eight government primary schools. Mirirai et al. (2012), a case study research design was used and purposive sampling was implemented for Mathematics teachers to investigate the teaching of fractions. Hildreth (1979) examined the Measurement Strategies among the students and study the errors made for Estimation. Figueras & Waldegg (1984) experimented with the measurement tools among the middle school students. Rawool (1988) studied about the conceptual maturity of the 50 students of upper primary school level in non-metric geometry through three tests as (i) the task of classification, (ii) the task of drawing geometrical figure as per the given description and (iii) the task of describing a verbally for given geometrical figure. Gurusamy (1990) used case study method to diagnose the errors committed by students of class IX in solving problems in geometry and has develop a remedial package. Questionnaire was used to collect data. Haralambos (2000) studied about the conceptualization of geometric concepts and the geometric proofs by Xth grade students. Jayasree (1997) used diagnostic test to study the difficulties experienced in using identities for algebraic expression by the students of grade –VIII. Garo (2008) compared algebraic achievement and educational practices of 9th graders from two countries. An achievement test and a student questionnaire used to collect data from 242 students from U.S. and 219 students from Albania. The test adopted a Texas Publicity Released Standardized Test administered during academic year 2006-07 where 145 students from U.S. sample and 97 students from Albania sample used Calculators. American calculator users significantly outperformed by Albanian. Ross et al. (2011) conducted the study for consecutive three years on the same algebra problem to examine the strategies used by several thousand students of grade VIII, IX and X and also analyzed connections between their strategies and their success on the problem. Noss et al. (2012) studied about the learning of algebraic generalisation by the students of the upper primary level. Sarala (1990) analysed about the conceptual errors made by the sample of 800 students from secondary school level selected by the stratified sampling procedure. Two tools diagnostic tests and the non-verbal test of intelligence by Nafde, personal data sheet was used to collect the data. Diagnostic tests were administered in Sets, Trigonometry and in Statistics.

While looking to the findings or conclusions drawn from various studies as, Nalayini (1991) concluded that number games motivated children to develop the computational skills. Kapur & Rasario (1992) outlined as despite having average intellectual abilities and having regular classroom coaching, many students fail to perform well in Arithmetic and it can be helped through remedial education which has varied instructional objectives. Subramaniam & Singh (1996) reported about the mistakes committed generally by the students were in basic operations, confusion between the signs of addition and multiplication. Other responsible factors for the poor performance cited by the teachers were home environment, physical facilities, large size of classes, extra workload on teachers and along with lack of interest, motivation and discipline. Mirirai et al. (2012) resulted about the factors responsible for the poor performance in teaching as shortage of time to prepare for the lessons, examination driven curriculum and limited resources. The researchers recommend staff development workshops and seminars to equip teachers with skills which will enable them to employ child centred teaching strategies that may result in the conceptual understanding of fractions. Figueras & Waldegg (1984) stated about the concept of Measurement as “a fixed measuring system is introduced far too early in the curriculum of elementary school, thus creating a barrier to the complete understanding of the unit concept”. Dutta (1990) reported as the use of audio-visual materials leads to greater interest, clearer understanding and longer retention of Geometrical concepts. The teaching of Geometry has been a subject of debate. Gurusamy (1990) has shown positive favor towards the remedial programme. Clements & Battista (1992) interpreted as the geometric meaning underlying a student’s geometric language may differ considerable from what a Mathematics teacher might think is the student’s meaning. Haralambos (2000) indicated that students write proofs that are better organized through shared knowledge than the proofs presented in the textbooks. Ross et al. (2011) concluded from the research study that even those students who used algebraic strategies consistently did not show the anticipated improvement of performance that would be expected from several years of continuing to study Mathematics. Noss et al. (2012) indicated that the challenges involved in building a system that is both intelligent and exploratory, a learning environment in which both student and teacher are supported without explicit tutoring. Sarala (1990) outlined the major findings were the numbers of errors are quite large, and these errors are influenced by sex, locality of the school, management of the school, intelligence, study habits and socio- economic-status. The errors decrease with intelligence.

Johanson (2000) had conducted the study about literature reviews on the misconceptions or difficulties occurred generally in the teaching-learning within various areas of Mathematics. It is briefly highlighted here. For Arithmetic, the study revealed the findings as: Students' conceptual misunderstandings of decimals lead to the adoption of rote rules and computational procedures that often are incorrect and they do not make good use of their understandings of rational numbers as a starting point for developing an understanding of ratio and proportion. Students' intuitive understanding of the concept of infinity remains quite stable over the middle grades and is relatively unaffected by mathematics instruction. Many teachers have a surface level understanding of fractions and decimals, with the result being that students are engaged in learning activities and discussions that are misleading and prompt misconceptions such as "multiplication makes bigger" and "division makes smaller". For Measurement, Students at all grade levels have great difficulties working with the concepts of area and perimeter. Perhaps related to this difficulty, many secondary students tend to think that the length, the area, and the volume of a figure or an object will change when the figure or object is moved to another location. For the Geometry, it is reported as: A computer environment can generate multiple representations of a shape that help students generalize their conceptual image of that shape in any size or orientation. Students have a difficult time using the word "similar" and its mathematical meaning correctly. Too often, the word is used loosely by students and teachers to mean "approximately the same," which led to a subsequent classification of rectangles of most dimensions as being similar. In Algebra, difficulties encountered as: Yet, the concept of a variable is "more sophisticated" than teachers expect and it frequently becomes a barrier to a student's understanding of algebraic ideas. Students have difficulty representing and solving algebraic word problems because they rely on a direct syntax approach which involves a "phrase-by-phrase" translation of the problem into a variable equation. For Probability and Statistics, Students have poor understandings of fundamental notions in probability: the use of tree diagrams, spinners using the area model, random vs. nonrandom distributions of objects, and the general idea of randomness itself. Students can calculate the average of a data set correctly, either by hand or with a calculator and still not understand when the average (or other statistical tools) is a reasonable way to summarize the data.

2.2 PART-II: REVIEWS ON OTHER ATTRIBUTES CONCERNED WITH MATHEMATICS EDUCATION

2.2.1 ATTITUDES

Jain (1979) studied significant correlates of high school failures in Mathematics and English with special reference to Jammu division. For the data collection the measuring devices used were Humanities Group Test of General Mental Ability, Numerical Ability Test, Abstract Reasoning Test, Mathematical Ability Test, a Scale to Assess Attitude towards Mathematics and a Questionnaire on various factor associated with Mathematics learning. The major finding was that the factors played a vital role in learning Mathematics were intelligence, abstract reasoning, numerical ability, and knowledge of Mathematical concepts, rules and principles, attitude towards Mathematics.

Rosaly (1992) has found that the attitude of high school students towards the learning of Mathematics and their achievement in Mathematics are highly correlated and that urban boys & girls have a more positive attitude than rural boys & girls.

Karen (1998) studied student attitudes toward mathematics projects. The purpose of this study as to examine if mathematics anxiety, learning preference, exposure to projects, the teacher, gender, and ethnicity are related to student attitude toward projects. This study tested 17 hypotheses and both qualitative and quantitative methods were employed. The 304 students in the study completed an attitude survey, a mathematics anxiety survey, and a learning style inventory. The qualitative portion of the study revealed that sixty one percent of students had a positive attitude toward projects and were willing to take another project class. Over forty percent of students stated that they enjoyed working groups. Students also disliked many things, including: negative group experiences, the extent of writing, the amount of work involved, and a desire for more time when working on a project. Students recommended that more time be given in class to work on projects, and that projects be consistently interesting and creative. Over fifty percent of students stated that their attitudes toward projects were also affected by the teacher, the group and the content of the project.

Johnson (2000) summarised the reviews of the related research studies about the **Attitudes of the students towards Mathematics and Mathematics learning** as: A student's attitude toward mathematics is not a one-dimensional construct. Just as there are different types of mathematics, there potentially are a variety of attitudes towards each type of mathematics (Leder, 1987). Students develop positive attitudes toward mathematics when they perceive mathematics as useful and interesting. Similarly, students develop negative attitudes towards mathematics when they do not do well or view mathematics as uninteresting (Callahan, 1971; Selkirk, 1975). Furthermore, high school students' perceptions about the usefulness of mathematics affect their decisions to continue to take elective mathematics courses (Fennema & Sherman, 1978). The development of positive mathematical attitudes is linked to the direct involvement of students in activities that involve both quality mathematics and communication with significant others within a clearly defined community such as a classroom (van Oers, 1996). One out of every two students thinks that learning mathematics is primarily memorization (Kenny & Silver, 1997).

Johnson (2000) summarised the reviews of the related research studies about the **Attitudes of the teachers towards Mathematics and Mathematics teaching** as: The attitude of the mathematics teacher is a critical ingredient in the building of an environment that promotes problem solving and makes students feel comfortable to talk about their mathematics (Yackel et al., 1990). Mathematics teachers have little understanding of the actual beliefs of students relative to their intrinsic motivation in mathematics classrooms. Thus, teachers build mathematics lessons based on their personal conceptions of intrinsic motivation, which may not be appropriate in every situation. When given techniques for both giving attention to and being able to predict student beliefs, mathematics teachers are able to "fine-tune their instruction to turn kids on to mathematics" (Middleton, 1995). Teaching children to both set personal learning goals and take responsibility for their own learning of mathematics leads to increased motivation and higher achievement in mathematics (DeCharms, 1984). A meta-analysis of 26 research studies concludes that there is a consistent, negative correlation between mathematics anxiety and achievement in mathematics. This correlation is consistent across all grade levels, both gender groups, and all ethnic groups (Ma, 1999). The most effective ways to reduce mathematics anxiety are a teacher's use of systematic desensitization and relaxation techniques (Hembree, 1990).

Çetin et al. (2005) investigated through their research on 'Study on 8th Grade Students' Thoughts about the Mathematics Course' that the thoughts of the 8th grade students in Turkey on the mathematics course and the relations between the mathematics courses and other variables such as the students' origins, gender and the mathematics scores students achieved. Based on the results, it was concluded that there is no evidence reflecting a relation between students' origins and thoughts, but there is a relation between their thoughts, their gender and their mathematics scores. The study was conducted in the schools of Eskisehir, which is a city in Turkey, 15 % of the primary schools in Eskisehir were selected randomly. The questionnaire used for collecting data was distributed in the last two weeks of the period during the education year 2001-2002. In the analysis of the data, frequency, percent and chi-square test were employed as statistical method. As result out of the 831 respondent, 14% of the respondents consider Maths as an enjoyable course, while 7% respondents state their dislike. 56% prefer the choice of "sometimes I like, sometimes I have difficulty in understanding." 23% of the students state that it is a course that they usually have difficulty in understanding.

Mriano (2005) investigated the relationship between students attitude towards learning mathematics and mathematics achievement with respect to gender in 10^{th} grade students in Amman Jordan. Three instruments were used in the study to collect data: A mathematics achievement test (MAT), consisting of questions selected by the researcher from the (2004) Ohio Graduation Practice Test, An attitude questionnaire (ATM) developed by Taylor (1997) from the Aiken scale (1976), and student interview developed by the researcher. The overall results of the study indicate that there was significant difference in attitude and achievement between male and female students in the 10_{th} grade ($F=10.3$, $P<0.01$).It also shows, using a Pearson correlation that there is a significant correlation between the two dependent variables attitude toward learning mathematics and mathematics achievement.

2.2.2 PROBLEM SOLVING

According to Fuson (1992c), Interpretations of the term “problem solving” in Mathematics vary considerably, ranging from the solution of standard word problems in texts to the solution of non-routine problems. In turn, the interpretation used by an educational researcher directly impacts the research experiment undertaken, the results, the conclusions, and any curricular implications. Also Brown & Walter (1983, 1993) stated as Problem posing is an important component of problem solving and is fundamental to any mathematical activity. While, National Research Council (1985) in their extensive review of research on the problem solving approaches of novices and experts, concluded that the success of the problem solving process hinges on the ‘problem solvers’ representation of the problem’**.** Students with less ability tend to represent problems using only the surface features of the problem, while those students with more ability represent problems using the abstracted, deeper-level features of the problem. The recognition of important features within a problem is directly related to the “completeness and coherence” of each problem solvers’ knowledge organization. Also, Krishnan (1990) had found that there is no significant relationship between ‘Identification of Problem Solving Strategies’ (IPSS) and either ‘Application of Problem Solving Strategies’ (APSS) or ‘Achievement of Problem Solving in Mathematics’ (APSM), though the last two are significantly correlated. The essential problem in school Mathematics is how to teach problem solving strategies to students. So that they may becomes efficient problem solvers.

Viswanathan (1997) studied the effect of diagnostic error learning strategy on the achievement of slow learners of standard IX in Mathematics. Experimental method with equated group design was adopted for the study. Both the groups, the experimental and the controlled group, consisted of 150 subjects each. The tools used were a diagnostic test in Mathematics for standard IX pupils, Achievement test in Mathematics for standard IX by C.P.Sreekantan Nair and Viswanathan K. S., Raven’s progressive Matrices for measuring intelligence of slow learners of IX standard, and attitude scale towards problem solving. The data were analyzed with the help of t-test. The learners treated with diagnostic error learning strategy when compared with those taught using conventional method. Slow learners of experimental group performed better in retention than those in the control group.

Johnson (2000) reported about many of the research study reviews on **other responsible and related attributes of Mathematics Education.** Problem solving is one of the major attribute as concerned with present study also, so reviews are comprised in the paragraph as: Problem posing is an important component of problem solving and is fundamental to any mathematical activity (Brown and Walter, 1983, 1993). A problem needs two attributes if it is to enhance student understanding of mathematics. First, a problem needs the potential to create a learning environment that encourages students to discuss their thinking about the mathematical structures and underlying computational procedures within the problem's solution. Second, a problem needs the potential to lead student investigations into unknown yet important areas in mathematics (Lampert, 1991). While solving mathematical problems, students adapt and extend their existing understandings by both connecting new information to their current knowledge and constructing new relationships within their knowledge structure (Silver & Marshall, 1990). Mathematics teachers can help students use problem solving heuristics effectively by asking them to focus first on the structural features of a problem rather than its surface-level features (English & Halford, 1995; Gholson et al., 1990). Explicit discussions of the use of heuristics provide the greatest gains in problem solving performance, based on an extensive meta-analysis of 487 research studies on problem solving. However, the benefits of these discussions seems to be deferred until students are in the middle grades, with the greatest effects being realized at the high school level. As to specific heuristics, the most important are the drawing of diagrams, representing a problem situation with manipulative objects, and the translation of word situations to their representative symbolic situations (Hembree, 1992). Mathematics teachers who help students improve as problem solvers tend to ask frequent questions and use problem resources other than the mathematics textbook. Less successful teachers tend to demonstrate procedures and use problems taken only from the students' text-book (Suydam, 1987).

Yeo (2004), studied on 'Secondary-2 Students' Difficulties in Solving Non-Routine Problems'. As part of a study on Mathematical problem solving of secondary-2 (13- to 14-yearsold) students in Singapore, 56 Secondary-2 students from ten secondary schools participated in this study. The purpose of this study is to explore difficulties faced by 56 Secondary-2 students when solving problems. These interviews were analysed using the structure derived from Newman (1983) and Ransley (1979). From the interviews the difficulties experienced by Secondary-2 students who were prevented from obtaining a correct solution were: (a) lack of comprehension of the problem posed, (b) lack of strategy knowledge, (c) inability to translate the problem into Mathematical form, and (d) inability to use the correct Mathematics.

2.2.3 INTERACTIONS AND RESPONSES

Johnson (2000) concluded the finding from the research studies on the **Interactions take place in the teaching-learning processes** as: When teachers increase their **wait times,** the length of the student responses increases, the numbers of student responses increases, the apparent confidence of students in their responses increases, the number of disciplinary interruptions decreases, the number of responses by less able students increases, and students seem to be more reflective in their responses (Rowe, 1978). This study was done in a science classroom, but its results may be applicable to a mathematics classroom as well. The teacher has mathematical understandings that allow them to "see" mathematical objects or concepts in ways that learners are not yet ready to "see" themselves. The result is that teachers often "talk past" their students, unless they "see" through their students' understanding (often peculiar) and make the necessary adjustments. Classroom miscommunication is well documented by researchers in many areas of mathematics: number sense and place value, basic operations, decimals and fractions, variables, and geometric proofs (Cobb, 1988). Teachers need to build an atmosphere of trust and mutual respect when turning their classroom into a learning community where students engage in investigations and related discourse about mathematics (Silver et al., 1995). An easy trap is to focus too much on the discourse process itself; teachers must be careful that "mathematics does not get lost in the talk" as the fundamental goal is to promote student learning of mathematics (Silver & Smith, 1996). Students writing in a mathematical context help improve their mathematical understanding because it promotes reflection, clarifies their thinking, and provides a product that can initiate group discourse (Rose, 1989). Furthermore, writing about mathematics helps students connect different representations of new ideas in mathematics, which subsequently leads to both a deeper understanding and improved use of these ideas in problem solving situations

(Borasi & Rose, 1989; Hiebert & Carpenter, 1992). Teachers need to do more than ask questions in a mathematics classroom, as the cognitive level of the questions being asked is very important. Though the research is quite depressing in regard to teachers' use of questioning, it is quite consistent: (1) 80 per cent of the questions asked by mathematics teachers were at a low cognitive level (Suydam, 1985). (2) During each school day, there were about five times more interactions at low cognitive levels as than at high cognitive levels (Hart, 1989). (3) Low cognitive level interactions occurred about 5.3 times more often than high cognitive level interactions (Fennema & Peterson, 1986). (4) In an average of 64.1 interactions in a 50-minute class period, 50.3 were low level cognitive interactions, 1.0 involved high-level cognitive interactions, and the remaining interactions were not related to mathematics (Koehler, 1986). High-confidence students have more interactions about mathematics with their teachers than low-confidence students and these interactions tend to be on a higher cognitive level. Mathematics teachers perhaps are unconsciously sending a message to the low-confidence students that they also have less ability in mathematics and thus should expect less of themselves mathematically (Reyes, 1980).

Manullang (2005) conducted study on **'**Quality of Teaching and Learning Interaction for Mathematics Teachers: A Case Study'. This study attempted to find out a correlation among known variables in relation to the development and improvement of the quality of teaching and learning interaction for Mathematics teachers. The population of the research was all the 34 Mathematics teachers at Dairi Regency, North Sumatra, Indonesia. By applying the statistical computation on the correlation of the variables such as teachers' educational level, teaching experience, teachers' training as predicting variables and professional attitude, and quality of teaching and learning interaction as intervening variables while Mathematics was used as a dependent (criterion) variable, it was found out that there is a significant correlation of the variables of teachers' educational level, teaching experience, and professional attitude with the quality of the teaching and leaning interaction. The findings suggest that Mathematics teachers should improve their knowledge about the course, other related institutions should be involved in supervising the interaction, cooperation with institutions producing teacher graduates should be developed, and the training programs should be revaluated in terms of efficiency, effectiveness and investment.

2.2.4 MANIPULATIVE

According to Moyer, Niezgoda, & Stanley (2005), research suggests that students may also develop more complex understandings of concepts when using virtual manipulatives. Some research reviews given below.

Marsh & Cook (1996) studied the use of Cuisenaire rods as a support for solving word problems with three third grade students with learning disabilities. The students were not only more successful at selecting the correct operation when using the manipulatives but continued to improve after the manipulatives were withdrawn.

Johnson (2000) summarised based on the 60 study reviews of the related research studies on **the Manipulative** and derived the conclusions as: Sowell (1989) concluded that "mathematics achievement is increased through the long-term use of concrete instructional materials and that students' attitudes toward mathematics are improved when they have instruction with concrete materials provided by teachers knowledgeable about their use". Mathematics teachers need much more assistance in both how to select an appropriate manipulative for a given mathematical concept and how to help students make the necessary connections between the use of the manipulative and the mathematical concept (Baroody, 1990; Hiebert & Wearne, 1992). Teachers sometimes overestimate the value of manipulatives because they as adults are able to "see" the mathematical concepts or processes being represented. Children do not have this "adult eye" (Ball, 1992). Many secondary students are at a developmental level that necessitates experiences with both concrete and pictorial representations of mathematical concepts (Driscoll, 1983). Manipulative should be used with beginning learners, while older learners may not necessarily benefit from using them (Fennema, 1972). Timing is the key. Once students have learned a rote procedure, it is quite difficult for students to acquire a conceptual understanding of that procedure. Thus, teachers need to focus each student's initial instruction on using manipulatives to build a solid understanding of the concepts and processes involved (Wearne & Hiebert, 1988a).

Cass et. al (2003) used case-study methods to investigate the effectiveness of teaching perimeter and area concepts using manipulative (geo-boards). Study participants were three fourth grade students with learning disabilities, all of whom improved in their ability to solve these geometric problems.

Steen, Brooks & Lyon (2006) compared the academic achievement of a group of first grade students who used virtual manipulative for practice in geometry instruction (treatment group) to another group who did not (control group). A total of 31 students were randomly assigned to either the treatment or control group. Achievement was measured by the Grade One and Grade Two assessments provided by the classroom textbook's publisher. The treatment group improved significantly on both the Grade One and Grade Two tests, while the control group showed significant improvement only on the Grade One test. The treatment teacher also noted that her students showed increased motivation and increased time on task.

Suh & Moyer (2007) compared the use of concrete and virtual manipulative in third grade students studying algebraic thinking. Both types of manipulative were associated with higher achievement and increased flexibility in representing algebraic concepts. Also reported as, Manipulative by itself have no inherent meaning. It is important for teachers to make this meaning explicit and help students build connections between the concrete materials and the abstract symbols that they represent. This holds true for both concrete and virtual manipulative, but virtual manipulative often have this type of structure built in. Many virtual manipulative activities give students hints and feedback, something that the more traditional concrete manipulative cannot do without teacher assistance. For example, using Tangrams (http://www.cited.org/ & http://nlvm.usu.edu/en/nav/frames_asid_268_g_1_t_3.html? open=activities) students can virtually copy a design made from pattern blocks, and when a block is near a correct location, it will snap into place. This virtual manipulative also includes a hint function that will show the correct location of all the blocks. Additionally, virtual manipulative often provide explicit connections between visual and symbolic representations, a feature which was found to benefit learning.

Ramani & Patadia (2012) conducted a study on "Development and Try-out of the Programmed Learning Material in Mathematics for class XI students studying in schools affiliated to Gujarat Secondary and Higher Secondary Education Board

(GSHSEB)”. The objectives of the study were l). To develop programmed learning material in mathematics for XI standard students. 2). To implement the developed programmed learning material in mathematics to the XI Std. students studying in one of the English Medium Schools following the syllabus of GSHSEB. 3). To study the effectiveness of the; developed programmed learning material. The methodology of the study was posttests only control group design, groups were matched using comparable mean, standard deviation and correlated t-test was used for data analysis. The sample size consisted of fourteen, students of XI standard. PLM was found to be effective in teaching probability to XI standard science streams students as the achievement test score of experimental group students was found significantly higher than the achievement test score of the control group students.

Yuliang (2013) has studied effect of multimedia to improve Math learning. This quasi-experimental study was to design, develop, and implement one multimedia math lesson in third grade to improve students' math learning. The non-equivalent control group design was used. The experimental group had 11 third grade students and the control group had 15 third grade students in an African American predominated elementary school in the Midwest of USA. The independent variable was the multimedia math lesson and the dependent variable was students' math performance. It was hypothesized that the (a) teacher and students scored favorably about the multimedia math lesson, (b) students were very attentive to multimedia math instruction, and (c) the students scored statistically higher on the posttest at the end of the intervention in the experimental group than in the control group. The findings have theoretical and practical international implications for K-12 education.

2.2.5 CONSTRUCTIVISM

Ridlon (1999) studied the effect of problem centered learning on the Mathematics of Sixth graders. This study described the effects of a problem centered approach to Mathematics on the attitudes, actions, and achievement of sixth grade students at a middle school in the South-east. Two groups of sixth graders were randomly selected to participate in a nine-week study. Both classes had students of varying ability and diverse demographics. The regular classroom teacher taught the control group of 25 students using a traditional textbook and methods. The researcher presented the experimental group of 27 students with meaningful problem tasks that were solved in small groups and then presented to the class of validation. Data sources from both groups included a pre-test and post-test, students and parent surveys, student writings, and observations by the researcher and regular teacher who were both present at all times. Additional data were collected from the experimental group through interviews of students and their parents, student journals, and student work. A quantitative and qualitative analysis of all the results showed that problem centered learning was indeed effective in the opinion in the involved stakeholders. Students came to view Mathematics in a more positive light, enjoyed the class, and felt they had learned more than usual. They believed attitude and achievement were measurable increased. The test scores gave strong evidence to support these convictions because the problem centered group had a highly significant increase in achievement compared to the traditional curriculum. Thus, problem centered learning appeared promising and worth further investigation.

Johnson (2000) summarised the reviews about **the Constructivism as the student centred approach in Mathematics** as: The role of teachers and instructional activities in a constructivist classroom is to provide motivating environments that lead to mathematical problems for students to resolve. However, each student will probably find a different problem in this rich environment because each student has a different knowledge base, different experiences, and different motivations. Thus, a teacher should avoid giving problems that are "readymade" (Yackel et al., 1990). A fundamental principle underlying the constructivist approach to learning mathematics is that a student's activity and responses are always rational and meaningful to themselves, no matter how bizarre or off-the-wall they may seem to others. One of the teacher's responsibilities is to determine or interpret the student's "rationality" and meaning (Labinowicz, 1985; Yackel et al., 1990). **Scaffolding** is a metaphor for the teacher's provision of "just enough" support to help students progress or succeed in each mathematical learning activity. Elaborating on this metaphor, Greenfield (1984) suggests as "The scaffold, as it is known in building construction, has five characteristics: it provides support, it functions as a tool; it extends the range of the worker; it allows the worker to accomplish a task not otherwise possible; and it is used selectively to aid the worker where need be". Mathematics teachers must engage in "close listening" to each student, which requires a cognitive reorientation on their part that allows them to listen while imagining what the learning experience of the student might be like. Teachers must then act in the best way possible to further develop the mathematical experience of the student, sustain it, and modify it if necessary (Steffe and Wiegel, 1996). Young children enter school with a wide range of self-generated algorithms and problem solving strategies that represent their a priori conceptual understandings of mathematics. Classroom instruction too often separates the child's conceptual knowledge from the new procedures or knowledge they construct because the "students' informal ways of making meaning are given little attention" (Cobb, Yackel, and Wood, 1991). From multiple research efforts on creating a constructivist classroom, Yackel et al.(1990) concluded that "not only are children capable of developing their own methods for completing school mathematics tasks but that each child has to construct his or her own mathematical knowledge. That is mathematical knowledge cannot be given to children. Rather, they develop mathematical concepts as they engage in mathematical activity including trying to make sense of methods and explanations they see and hear from others." In her survey of the research on arithmetic-based learning, Fuson (1992b) concluded that students can "learn much more than is presented to them now if instruction is consistent with their thinking." Teachers can improve their students' ability to construct and evaluate mathematical proofs if they (1) transfer to students the responsibility of determining the truth value of mathematical statements and (2) integrate their teaching of proof constructions into other mathematical content rather than treat it as a separate unit (Balacheff, 1988).

Makanong (2000) investigated student's mathematics problem solving processes and to compare the Mathematics problem solving processes and achievement levels between students being taught Mathematics based on constructivist theory and those being taught Mathematics based on traditional teaching on Thailand. The constructivist Teaching Based Model was developed based on a framework of constructivist theory and was composed of three teaching steps: Construction of cognitive conflict; reflection and discussion; and occlusion of the results from cognitive conflict; reflection and discussion; and conclusion of the results from cognitive restructuring. Lesson plans for constructivist teaching were constructed based on the constructivist Teaching Based Model. Activities in these lesson plan focused on student's existing knowledge, Collaborative working, real word context, use of manipulative, cognitive conflict, reflective teaching approach and Lash's translation model. The study is a quasi-experimental research using pretest-posttest non-equivalent control group design. Four classes comprising 164 of 9th –grade students were involved. Students in the experimental group were taught 9th grade algebra based on constructivist teaching and these in the control group were taught based on traditional teaching. Four types of instrument the Mathematics problem solving process test, the Mathematics achievement test, the interview task problems and the classroom observation protocol were employed to collect data. Data were analyzed quantitatively and qualitatively. The quantitative data indicated no significant differences of mathematics problem solving process and achievement between students in the two treatment group. The qualitative data indicated that constructivist teaching was a promising approach capable getting students more involved in learning Mathematics. Students in this study who learned Mathematics based on constructivist teaching tried harder, as measured by the interview task problems, than those who learned based on traditional teaching. A variety of institutional methods for teaching algebraic problem solving and applying mathematics to the outside world were explored.

Ross (2006) conducted a study entitled “The Effects of Constructivist Teaching Approaches on Middle School Students’ Algebraic Understanding.” The goal in mathematics has shifted towards an emphasis on both procedural knowledge and conceptual understanding. The importance of gaining procedural knowledge and conceptual understanding is aligned with Principles and Standard for School Mathematics (National Council of Teachers of Mathematics, 2000) which encourages fluency, reasoning skills and ability to justify decisions. Possession of only procedural skills will not prove useful to students in many situations other than on tests (Boaler, 2000). Teachers and researchers can benefit from this study, which examined the effects of representations, constructivist approaches and engagement on middle school students’ algebraic understanding. Data from an algebra pretest and posttest as well as 16 algebra video lessons from an NSF-IERI funded project, were examined to determine occurrences of indicators of representations, constructivist approaches, and engagement as well as student understanding. A mixed methods design was utilized by implementing multilevel structural equation modeling and constant comparison within the analysis. Calculation of descriptive statistics and creation of bar graphs provided more detail to add to the findings from the components of the statistical test and qualitative comparison method. The results of the final structural equation model revealed a model that fit the data with a non-significant model $p > 0.01$. The new collectively named latent factor of constructivist approaches with the six indicators of enactive representations, encouragement of student independent thinking, creation of problem-centered lessons, facilitation of shared meaning, justification of ideas and receiving feedback from the teacher was shown to be a significant predictor of procedural knowledge ($p < 0.05$) and conceptual understanding ($p < 0.10$). The indicators of the original latent factor of constructivist approaches were combined with one indicator for representations and two indicators for engagement. Constant comparison revealed similar findings concerning correlations among the indicators as well as effects on student engagement and understanding. Constructivist approaches were found to have a positive effect on both types of student learning in middle school Mathematics.

Kamp (2007) examined the impact of standards based method (JBHM) and traditional method on student Mathematics achievement. The research design was a quasi-experimental design, with 65 students of seventh-grade participating. Group A received a traditional method of instruction through the use of mathematics applications and connections course 2 and group B received a standard based method of instruction through the use of JBHM Achievement connections. The test instrument administered for the pre-test and post-test was the PLATOedu test. An analysis of the pre-test and post-test scores was conducted. T-tests were run to examine the differences between pre-test and post-test scores and gender based on the method of instruction. Analysis of variance (ANOVA) was computed to examine difference in performance based on class period representation. A paired t-test was computed to examine differences between the pre-test and post-test scores after students were exposed to a method of instruction. The findings showed that there were no statistical differences in student achievement between both the methods. The students taught by both the instruction, increased mathematics outcomes. However, the students taught using JBHM achievement connections standards based method of instruction had a higher mean score and a greater degree of gain between pre-test and post-test scores than the students taught using Mathematics Applications and Connections, course 2 traditional method of instruction.

Fengfeng (2008) has examined the use of educational computer games in a summer Math program to facilitate 4th and 5th graders' cognitive math achievement, meta-cognitive awareness, and positive attitudes toward math learning. The results indicated that students developed more positive attitudes toward math learning through five-week computer math gaming, but there was no significant effect of computer gaming on students' cognitive test performance or meta-cognitive awareness development. The in-field observation and students' think-aloud protocol informed that not every computer math drill game would engage children in committed learning. The study findings have highlighted the value of situating learning activities within the game story, making games pleasantly challenging, scaffolding reflections, and designing suitable off-computer activities.

Warren (2008) conducted a study entitled "A comparative study of traditional vs. constructivist teaching method used in algebra classes for pre-service elementary teachers." Since, the publication of *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus* researchers have suggested the use of a constructivist based instruction that views mathematics as both an individual and collective activity. Although constructivist based instruction has been an area of focus of mathematics education since 1990, college professors continue to use traditional instruction in pre-calculus mathematics classrooms. Responding to the concern for improving student performance, the National Council of Teachers of Mathematics (NCTM) developed guidelines for school mathematics. However, research has shown that although mathematics teachers are aware of the NCTM guidelines, implementation has not followed. One approach to implementation of reform has been constructivist based instruction, an alternative pedagogy to traditional instruction. Studies have highlighted advantages of constructivist based instruction in relation to positively impacting attitudes and feelings toward mathematics, but studies focused on mathematics achievement have not been undertaken. Thus, the purpose of this study was to describe the effect of constructivist instruction on the mathematics achievement of intermediate algebra students at a private college in Arizona. Achievement was measured using two teacher developed tests. The study was guided by the following research question: What is the effect of constructivist based instruction on the mathematics achievement of intermediate algebra students? This study compared students from 1999-2002 who were taught with traditional instruction and students taught after 2002 who were taught with constructivist instruction. A parametric t-test for unpaired independent samples was used to analyze the data for significant differences. The results indicated that, overall the mathematics achievement of the treatment group was not significantly higher than that of the comparison group. It was concluded that the adoption of a constructivist methodology does not negatively impact test scores.

Lopez et al. (2010) conducted a study entitled "Analysis of the effects of a Constructivist-Based Mathematics Problem Solving Instructional Program on the achievement of Grade Five Students in Belize, Central America." This study examined whether social constructivist activities can improve the mathematical competency of grade five students in Belize, Central America. The sample included 342 students and eight teachers from two rural and urban schools. A switching replication design was employed enabling students in the experimental groups to be taught using social constructivist activities for 12 weeks and the controls exposed to similar instructional practices from weeks 7 to 12. Students' performance was assessed using pre-test, post-test 1 and 2 with an internal consistency of 0.89, 0.90 and 0.93 respectively. As revealed by the repeated measures ANOVA within subjects analysis, there were significant differences among the pre-test and post-test 1 and 2 results. That is, students in the control groups, who were instructed using a procedural approach from weeks 1 to 6, demonstrated higher gains than the experimental groups who were immersed in social constructivist activities. Furthermore, when the control groups became immersed in similar activities from weeks 7 to 12, they continued to outperform the experimental groups who were exposed to social constructivist activities alone. Hence, due to this unexpected result, the aim of this thesis became to explain why these results came about and what implications for teaching were highlighted by the consideration. Besides the quantitative results highlighted above, qualitative data was also obtained as part of the study. For example, students were videoed within constructivist math groups and their performance analysed using Pirie and Kieren's (1994) Model of Growth for Mathematical Understanding. The data from the video recording revealed that use of one step math problems did not enabled students to restructure their thinking to solve innovative problems. Data from semi-structured interviews also revealed that some students lacked basic math skills and were not exposed or guided to solve complex problems. Besides the need for careful examination of social constructivist activities on performance, this thesis underscores the importance of relevant teaching and learning activities, the important role of teachers during social constructivist activities and the need to identify suitable forms of assessment to measure performance.

2.2.6 ASSESSMENT AND ACHIEVEMENT

Lalithama (1975) has found some factor affecting achievement of secondary school pupils in mathematics. The study was conducted on 732 pupils of standard IX selected on stratified random basis. The tools used were a Standardized Achievement Test in Mathematics, a Study Habit Inventory, an Interest Inventory, a Socio Economic Status Scale and the Raven's Standard Progressive Matrices. The study revealed that the achievement in mathematics positively related to intelligence, interest in mathematics, study habits and SES. Studying lesson daily, studying mathematics by writing, repetition in learning, over learning, private tuition etc. influence the achievement of mathematics.

Sharma (1978) studied the achievement in Mathematics by pupils of secondary school with particular reference to the state of Assam. The study was confined to the area of arithmetic and algebra. The sample included 1295 pupils from ten schools. A Battery of Sequential Achievement Tests was constructed for standard five to ten. An analysis of syllabus, textbooks, school records and board's records was also done. The major defects were the lack of drilling and knowledge of fundamentals and the inability to transform verbal statements into Mathematical statements. All the pupils acquired knowledge and skill better than understanding and application of different topics, there was undue emphasis on the mechanical learning of Mathematics. Some major factor responsible for low achievement in Mathematics were the imparting of limited knowledge, blind use of rules, heavy syllabus, defective textbooks, lack of the natural urge among pupils to learn mathematics, insufficient drill work at the primary stage and absence of the methodical approach of the classroom teaching.

Manika (1983) found about the acquisition of concept in Mathematics of pupils at primary school level and its relation to some personal and environmental variables of the pupils. The data were collected from 524 pupils from municipal school, grant-in-aid schools and private schools of urban areas from grade I through grade V. Tools employed in the study were Raven's coloured progressive matrices test, abstraction and generalization test/black test and Mathematical concept test. The study found that the majority of pupils who were promoted to the next grade did not show acquisition of concepts of the lower grade. Mathematical concepts developed better with pupils good in language. For the better development and the acquisition of concepts, individualized instruction was found useful.

Chitkara (1985) studied the effectiveness of different strategies of Teaching on Achievement in Mathematics. In the study a pre test, post test experimental design was followed. The strategies of teaching varied in three ways (a) lecture-discussion (b) inductive-drill and (c) auto-instruction group discussion. The variable of intelligence had three levels – low average, and above average. A sample of 300 students was randomly selected from grade IX students if four schools of Chandigarh. The students were divided into three groups of 100 each. One group was taught Mathematics through lecture discussion and the second group was taught Mathematics through inductive – drill and third group was taught Mathematics through auto-instruction group discussion. The data collected through pre-test post-test were analyzed through four ways (3 X 2 X 2 X 3) analysis of variance. The major findings were: i). All the three strategies were found to be equally effective in terms of achievement in Mathematics disregarding levels of intelligence, sex and personality type. ii). Lecture discussion strategy was found in favour of average ability students as they scored significantly higher than above – average and below average groups. iii). Inductive – drill and auto instruction group discussion was more suited to the students having above intelligence than average and below-average intelligence.

Doshi (1989) has studied the positive relationship between achievements in Mathematics and cognitive preference styles. For all the students the questioning style is the last, while for the majority of arts and commerce students, the recall style is the first. No significant relationship is found between cognitive preference styles and Mathematics. It is an open question worth investigation whether by changing teaching strategies we can change the cognitive preference style and whether this can lead to significantly improved learning of Mathematics.

Chel (1990), the study attempted to diagnose and suggest remediation of underachievement in the compulsory mathematics of the madhyamik examination in West Bengal with the objectives: (i) To identify different kinds of difficulties related to underachievement of students in mathematics from classroom observation of mathematics lessons, (ii) to seek out the types of errors which are identified from the performances of the students in their answer scripts, (iii) to find out the factors, according to the opinion of students, teachers and guardians, that are responsible for underachievement in mathematics at secondary school level, (iv) to know the extent to which the procedure of evaluation is responsible for underachievement, (v) to know the reinforces and noises in communicating mathematical principles to learning, (vi) to find out the remediation .programme that should be suggested for students, teachers and other for obtaining better achievement in mathematics at secondary level, and (vii) to find out what should be the role of the authority or management in implementing the remedial programme. The sample comprised urban, semi-urban and rural students of Classes VI to X of West Bengal. The case study method was used in collecting the data. The statistics used to treat the collected data were mean and rank differences correlation. Major Findings were reported as: (1) The main difficulties faced by students included, concept gaps, confusion in understanding mathematical language, stereotype way of presenting contents and lack of openness in teaching. (2) The major mistakes found in the performances of students and teacher trainees in the areas include mathematisation of verbal problems, interpretations of mathematical results and learning new topics in mathematics. (3) Underachievement was caused due to lack of understanding of the mathematical concepts of the earlier stage, and the abstract nature of mathematics. (4) Errors are caused due to the versatility and variability of contents. (6) Reinforces in the channel of learning were readiness, interest, active involvement, use of effective materials of instruction and learning efficiency.

Baskaran (1991) studied the relationship among achievement motivation, attitude towards problem-solving and achievement in mathematics. The sample was selected by stratified sampling technique. The researcher prepared a tool with three parts in it having Achievement Motivation Inventory Test as the first part, Attitude scale as the second part and Achievement test in mathematics for standard tenth as the final part. There was a positive relation between the attitude towards problem solving and achievement in mathematics. Urban and rural students did not differ in their achievement motivation and attitude towards mathematics. Urban and rural students differ significantly in their mathematics achievement.

Rajyaguru (1991) studied the achievement in mathematics, personal characteristics and environmental characteristics of over-achievers and underachievers. The sample of the study consists of 1093 students which were selected through stratified proportionate sampling. The subjects were selected from six urban, six semi-urban and thirteen rural schools. The tools used were Desai-Bhatt Group Test for intelligence, Bhavsar Numerical Aptitude Test, Mathematics Achievement Test developed by investigator, Mathematics Anxiety Scale by Patel J. Z., Study Habit Inventory by Patel B. V., Mathematics Aptitude Scale by Desai, H. G., Interview Schedule and Rotter's Locus of control Scale adopted by Bhogayata. The study revealed that there was positive and significant correlation between (a) intelligence and achievement in Mathematics, (b) achievement in Mathematics and numerical aptitude, (c) intelligence and numerical aptitude. Overachievers and underachievers did not differ in intelligence, numerical aptitude and locus of control. Overachievers had better study habits, more positive towards mathematics and less mathematics anxiety.

Sashidharan (1992) investigated about learning intellectual skill as an educational outcome in relation to student entry characteristics and quality of instruction. The major findings were: The prevailing promotion policy gives opportunities to children to attain tenth class even though they cannot perform basic operations in mathematics. The initial deficiencies have long term damaging effect because the content of education is organized in such a way that learning in each class is depending on prior learning.

Srivastava (1992) studied the learning outcomes in terms of objectives in Mathematics. The sample consisted of one thousand and thirty students selected at random by multi-stage random sampling technique. The tools used in the study were:

an Achievement Test in Mathematics, the Socio-economic Status Scale by S.P.Kulshreshta, and the Culture Fair Test of Intelligence (Form A) by R. B. Cattle. Intelligence and socio-economic status both were such which contributed significantly and positively to the development of learning outcomes in Mathematics in terms of knowledge, understanding, application, and skill. Male and female students belonging to high socio-economic status group were better in all the four types of learning outcomes in comparison to low socio-economic status group.

Sharma (1999) studied the effect of mathematical instructions on students' performance interactions in mathematics. The researcher followed the following stages while developing mathematical exercise on the proposed topics. Stage-I: Preparing of the program, Stage-II: Writing the exercise, Stage-III: Try-out for Modification, and Stage-IV: Evaluation of the programme. The program developed by the researcher was evaluated on the basis of the try-out in terms of: (i) Error rate; (ii) Sequence progression; (iii) Criterion test findings and (iv) Exercises Analysis. Unit wise error rate, sequence programme chart, criterion test findings and exercises analysis were prepared. Experimental findings in terms of error rate, criterion test and significance of difference between mean show that: (1) 85% of the learners were able to perform sums correctly, (2) 85% of the learners were able to perform 85% of the items of criterion test correctly, (3) Researcher also found that there was a significant difference in mathematics attainment of the learners studying through mathematical instructions as compared to conventional instructions.

Johnson (2000) reported about reviews of many research studies and literatures **about performance based Assessment** and concluded with the findings as: Conceptual understanding of the four basic operations incorporates **connections among representations**—concrete, pictorial, symbolic, and real-world. Assessments in mathematics classrooms need to focus on these connections as they have great influence on students' ability to use their conceptual understanding in problem situations (Huinker, 1990). Clements (1999), Stacey & Macgregor (1999a), and Murphy (1999) stated that at the present time, the assessment in algebra is still focusing on getting a correct answer, symbol manipulations, rote skill and little or no application of algebraic concepts in the problem situation. As Stacey & MacGregor (1999a) viewed that although the students have apparently learned algebra, in reality they find algebra difficult and do not know how to apply it. They still can't see the way to use what they learnt about algebra and it is still seen separately. Therefore, more complex algebraic problem items that demonstrating the efficiency and power of algebraic solving ability should be constructed and frequently used in examination and classroom practice.

Johnson (2000) reviewed many research studies and literatures **about Assessment through Open-ended Mathematical problems** and concluded with the findings as: Secondary students "learn usable knowledge and skills more effectively and efficiently through experiences" with open-ended mathematics problems than with traditional goal-specific problems or exercises. When solving goal-specific mathematics problems, students use strategies that successfully solve the specific problem at hand but are "less effective for making connections among concepts and procedures for organizing knowledge."When solving open-ended mathematics problems, students create, adapt and use solution strategies that "make important relationships more salient. Thereby helping students to develop knowledge that is better organized and skills that are more usable" (Sweller and Levine, 1982; Sweller, Mawer, and Ward, 1983; Owen and Sweller, 1985). The use of open-ended problems in a mathematics classroom is a teacher's "best chance" to assess a student's level of understanding or development of meaning in the mathematics (Davis, 1978). Open-ended assessment tasks require students to communicate in a mathematical context that reveals both the level and the quality of their understanding of mathematics (Magone et al., 1994). A variety of alternative assessments in mathematics must be used to generate the information a mathematics teacher needs to determine what his/her students are thinking, how his/her students are reasoning, and what the next instructional steps should be (Thompson & Senk, 1993; Gay & Thomas, 1993). From research work with performance-based assessments over a decade, Aschbacher (1991) concludes that they must have these key features: (1) Students need to be asked to produce, do, or create something that requires higher level thinking or problem solving skills. (2) Students need to respond to assessment tasks that are meaningful, challenging, engaging, and instructional. (3) Students should face assessment tasks set in real-world contexts or close models. (4) Students' process behavior must be assessed equally along with a product. (5) Criteria and standards for performance need to be public knowledge and made known to students in advance. Teachers tend to plan in terms of classroom learning activities rather than in terms of content performance outcomes (Clark and Yinger, 1979).

Patel (2007) developed a programme for enhancing achievement of the students of class X in Mathematics. The tools used were data collection were information schedule for students, questionnaire for students, teachers and parents, unit tests and achievement test. It was a single group pre-test and post-test design. Multi-stage cluster sampling was used and the size of the sample was seventy students. The programme for locating the weaknesses related to the perquisite for teaching each unit and remediating it prior to teaching was developed and implemented. It was found using t-test that the programme was effective and students were able to score high in the achievement test.

Patel (2012) has studied Academic Achievement of students in Mathematics of standard IX in relation to some psycho-social factors. The sample of the study consisted of 1486 students of Standard IX and sample selected by multistage sampling technique. Data were analyzed through Mean, Median, Standard Deviation, Correlated-t, Percentile and T. The study revealed that there was no significant effect of gender, school type and area on achievement in Mathematics. But there is significant effect of cast, intelligence and SES on achievement in Mathematics.

2.2.7 SUMMARY OF REVIEWS IN PART- II

As summarizing about the methodologies as: Jain (1979) studied with major concerns with high school failures and attitude towards Mathematics. For the data collection the measuring devices used were Humanities Group Test of General Mental Ability (Joshi), Numerical Ability Test, Abstract Reasoning Test, Mathematical Ability Test, a Scale to Assess Attitude towards Mathematics and a Questionnaire on various factor associated with Mathematics learning. Karen (1998) examined the mathematics anxiety, learning preference, exposure to projects, the teacher, gender, and ethnicity are related to student attitude toward projects. An attitude survey, a mathematics anxiety survey, and a learning style inventory conducted on 304 students. Found from the study as over fifty percent of students stated that their attitudes toward projects were also affected by the teacher, the group and the content of the project. Çetin et al. (2005) conducted survey to study about the 8th grade students' thoughts about Mathematics course using questionnaire. For which 15% primary schools constituted with 831 respondents. In the analysis of the data, frequency, percent and chi-square test were employed as statistical method. It is found as 7% respondents state their dislike and 56% prefer the choice of "sometimes I like, sometimes I have difficulty in understanding." Mriano (2005) studied about attitude of grade-X students using the instruments A mathematics achievement test (MAT), consisting of questions selected by the researcher from the (2004) Ohio Graduation Practice Test, An attitude questionnaire (ATM) developed by Taylor (1997) from the Aiken scale (1976) and student interview developed by the researcher. The study indicates that there was significant difference in attitude and achievement between male and female students in the grade-X. National Research Council (1985) conducted extensive review of research on the problem solving approaches of novices and experts. Viswanathan (1997) conducted experimental study on 150 slow learners of grade-IX for Mathematics to study the effect of diagnostic error learning strategy. Four tools were used as, a Diagnostic test, Achievement test by C.P. Sreekantan Nair, Viswanathan K. S., Raven's progressive Matrices for measuring intelligence and attitude scale towards problem solving. t-test were used to analyze the data. Yeo (2004) conducted interviews for 56 students of Secondary-2 (13 to 14 years old) to study the Difficulties in Solving Non-Routine Problems. These interviews were analysed using the structure derived from Newman (1983) and Ransley (1979). Manullang (2005), a case-study was conducted on 34 teachers to study the teaching and learning interactions for Mathematics. Correlation were analysed among various variables with the quality of teaching and learning interactions. Marsh & Cook (1996) studied about the use of Cuisenaire rods as manipulative for solving word problems with three third grade students with learning disabilities. Cass et. al (2003) investigated the effectiveness of teaching perimeter and area concepts using manipulatives (geoboards) for primary students. Steen, Brooks & Lyon (2006) experimented on 31 students about virtual manipulative for the geometric instructions. Ramani & Patadia (2012) used PLM for 14 students of grade-XI. Data were used collected using achievement test and were analyzed using mean, standard deviation and correlated t-test. Yuliang (2013) studied the effect of multimedia as manipulative to improve Math learning. Now about the Constructivism, Ridlon (1999) conducted experimental study for the students of grade-VI, where students were two groups randomly selected as control group with 25 students while experimental group of 27 students. A quantitative and qualitative analysis of all the results showed that problem centred learning was indeed effective in the opinion in the involved stakeholders. Makanong (2000) conducted the study was quasi-experimental research using pretest-posttest non-equivalent control group design. Classes comprising 164 of grade-IX students were taken as sample. The constructivist Teaching Based Model was developed. Four types of instrument the Mathematics problem solving process test, the Mathematics achievement test, the interview task problems and the classroom observation protocol were employed to collect data. Ross (2006) examined the effects of Constructive teaching approaches on middle school students. A mixed methods design was utilized by implementing multilevel structural equation modeling and constant comparison within the analysis. Kamp (2007) study was based on a quasi-experimental design with 65 students of seventh-grade and PLATOedu test was used to administered thru pre-test and post-test. A paired t-test and ANOVA were used to analyze the data. Fengfeng (2008) examined the use of educational computer games in a summer Math program to facilitate 4th and 5th graders. The results indicated that students developed more positive attitudes toward math learning through five-week computer math gaming, but there was no significant effect of computer gaming on students' cognitive test performance or meta-cognitive awareness development. Warren (2008) examined an Achievement was measured using two teacher developed tests. This study compared students from 1999-2002 who were taught with traditional instruction and students taught after 2002 who were taught with constructivist instruction. A parametric t-test for unpaired independent samples was used to analyze the data for significant differences. Lopez et al. (2010) examined whether social constructivist activities can improve the mathematical competency of grade five students. The sample included 342 students and eight teachers from two rural and urban schools. Data from semi-structured interviews also revealed that some students lacked basic math skills and were not exposed or guided to solve complex problems. Due to the unexpected result, the aim of this study became to explain why these results came about and what implications for teaching were highlighted by the consideration.

While, summarizing the findings as revealed from the reviews are as: Jain (1979) pointed as along with other factors, attitude towards Mathematics plays vital role in and correlated with learning Mathematics. Rosaly(1992) stated as the achievement in

Mathematics are highly correlated and that urban boys & girls have a more positive attitude than rural boys & girls. Karen (1998) concluded as students also disliked many things, including: negative group experiences, the extent of writing, the amount of work involved, and a desire for more time when working on a project. Çetin et al. (2005) resulted that 23% of the students having an attitude as it is a course that they usually have difficulty in understanding. Mriano (2005) stated based on Pearson correlation that there is a significant correlation between the two dependent variables attitude toward learning mathematics and mathematics achievement. National Research Council (1985) reported as students with more ability represent problems using the abstracted, deeper-level features of the problem. The recognition of important features within a problem is directly related to the "completeness and coherence" of each problem solvers' knowledge organization. Krishnan (1990) concluded that the essential problem in school Mathematics is how to teach problem solving strategies to students. So that they may becomes efficient problem solvers. Yeo (2004) outlined the findings about the factors for difficulties in problem solving as lack of comprehension of the problem posed, lack of strategy knowledge, inability to translate the problem into Mathematical form and inability to use the correct Mathematics. Steen, Brooks & Lyon (2006) revealed that students showed increased motivation and increased time on task when treated with virtual manipulative. Ramani & Patadia (2012) PLM was found to be effective in teaching probability to XI standard science streams students. Yuliang (2013) concluded on usage of multimedia as manipulative have theoretical and practical international implications for K-12 education. Now, about the Constructivism, Ridlon (1999) revealed as the problem centered learning appeared promising and worth further investigation. Makanong (2000) concluded as the qualitative data indicated that constructivist teaching was a promising approach capable getting students more involved in learning Mathematics. Ross (2006) revealed as Constructivist approaches were found to have a positive effect on both types of student learning in middle school Mathematics. Fengfeng (2008) concluded the study findings have highlighted the value of situating learning activities within the game story, making games pleasantly challenging, scaffolding reflections, and designing suitable off-computer activities. Warren (2008) concluded that the adoption of a constructivist methodology does not negatively impact test scores. Lopez et al. (2010) indicated as besides the need for careful examination of social constructivist activities on performance, this study underscores the importance of relevant teaching and learning activities, the important role of teachers during social constructivist activities and the need to identify suitable forms of assessment to measure performance.

Particularly, Johnson (2000) revealed from the research study conducted through review of various literatures about the other attributes like attitudes, problem solving, interactions and responses, manipulative, constructivism, assessment and achievements. For the attitude, this study revealed critically as: One out of every two students thinks that learning mathematics is primarily memorization. Students develop positive attitudes toward mathematics when they perceive mathematics as useful and interesting. And about the attitude of the teachers it is outlined as, the attitude of the mathematics teacher is a critical ingredient in the building of an environment that promotes problem solving and makes students feel comfortable to talk about their mathematics. For the problem solving, this study concluded as A problem needs two attributes if it is to enhance student understanding of mathematics, first is a problem needs the potential to create a learning environment and second is a problem needs the potential to lead student investigations into unknown. Mathematics teachers who help students improve as problem solvers tend to ask frequent questions and use problem resources other than the mathematics textbook. About the interactions and responses it is revealed as, high-confidence students have more interactions about mathematics with their teachers than low-confidence students and these interactions tend to be on a higher cognitive level. When teachers have increase their wait times then the length of the student responses increases, the numbers of student responses increases, and the apparent confidence of students in their responses increases. In terms of manipulative, Mathematics teachers need much more assistance in both how to select an appropriate manipulative for a given mathematical concept and how to help students make the necessary connections between the use of the manipulative and the mathematical concept. Timing is the key. Once students have learned a rote procedure, it is quite difficult for students to acquire a conceptual understanding of that procedure. Thus, teachers need to focus each student's initial instruction on using manipulative to build a solid understanding of the concepts and processes involved. For the Constructivism, the role of teachers and instructional activities in a constructivist classroom is to provide motivating environments. From multiple research efforts on creating a constructivist classroom, concluded that "not only are children capable of developing their own methods for completing school mathematics tasks but that each child has to construct his or her own mathematical knowledge. In order to study about the assessment and achievements it is found as, the use of open-ended problems in a mathematics classroom is a teacher's "best chance" to assess a student's level of understanding or development of meaning in the mathematics. A variety of alternative assessments in mathematics must be used to generate the information a mathematics teacher needs to determine what his/her students are thinking, how his/her students are reasoning, and what the next instructional steps should be.

2.3 PART-III: REVIEWS ON INSTRUCTIONAL STRATEGIES FOR MATHEMATICS EDUCATION

Anderson (1978) opted research study on "Using Errors to Improve the Quality of Instructional Programs". In this research study, Clinchy and Rosenthal's error classification scheme was applied to test results to determine its ability to differentiate the effectiveness of instruction in two elementary schools. Mathematics retention tests matching the instructional objectives of both schools were constructed to measure the understanding of arithmetic concepts and the ability to perform computations and algorithmic operations. Inter-school comparisons were made with respect to the types of errors made: computational, algorithmic, and omission. Fifth grade students in one school made significantly fewer computational errors than in the other school. In the remaining grades, there were no significant differences. Additional practice exercises were recommended to correct this error. Other students made significantly more algorithmic errors and a four-step approach to teaching algorithms was outlined to alleviate this weakness. Significant inter-school differences in omissions errors were noted in grades 5 and 6--the students concerned would also benefit by a more systematic approach to algorithm teaching. In sum, the error classification scheme can be used to evaluate instructional programs and to suggest instructional improvements.

Kothari (1985) investigated the efficacy of different instructional media into the teaching of Mathematics to the pupils of class IX in relation to certain variables. The sample of 120 students was selected from two schools of Anand. The tools used were Junior Index of Motivation, Reasoning ability and Criterion Tests. The study disclosed under observation that pupils were very eager to know about the different instructional media. It was their demand that all the units of Mathematics should be taught through visual projection. In case of instructional media namely Activities and experiment, pupils were very busy in drawing figures. They enjoyed studying through this media as it was activity oriented. Visual projection is comparatively more effective than any other Instructional media like Activities and experiment or even programmed learning material for teaching of Mathematics. The low achieving pupils are comparatively more benefited by programmed learning material than the high achievers and the average achieving pupils.

Mohpatra (1990) looked into the critical appraisal of the secondary school Mathematics curriculum of Orissa. The sample consisted of two hundred and twenty secondary school teachers and five hundred and fifty six students. The tools used were questionnaires for students studying Mathematics and for teachers teaching Mathematics. The Mathematics teachers were conservative in their outlook as far as the objectives of teaching Mathematics were concerned. They emphasized the Fundamental Mathematical operations, familiarity with Mathematical concepts and terms, development of Mathematical skills, objectives like development of discipline, determination and a sense of proportion were given the least importance. The students were, by and large, pragmatic in their approach and considered Mathematics to be a utilitarian subject. The teacher provided high ranking to the traditional topics and resisted the intrusion of new topics.

Kalamaros (1991) tried to study instructional method and decreased student errors on math worksheets. Teachers often express concern about student's poor performance on classroom math worksheets. Performance deficits may be attributed in part to personal internal variables and/ or to external factors such as materials or instructional methods. Unfortunately, student's math ability often is evaluated based on performance on math worksheets, regardless of the many factors that may be impacting the individual. The purpose of this study was to explore the effect that instructional methods have on student performance on math worksheets. A multiple baseline single organism study was completed with 11 third grade subjects. The subjects were referred for participation by their classroom teacher based on the teacher's belief that the student had "difficulty following written directions". Teachers need to be aware of the potential relationship between regarding ability and math performance. When teachers are interested in determining students' skills in math, the effects of reading must be taken into account. Teachers must evaluate the types of errors students made and take the time to show students explicitly how to correct those errors. Without this effort, errors are likely to be repeated. Teachers should always consider the impact that attitudes and beliefs about math ability have on student performance. Controlling for these influences increases the likelihood that students will demonstrate their true math skills.

Mishra (1991) had reported that with appropriate teaching strategies even arithmetically disabled children can learn addition and subtraction. However, such improved techniques have to be developed by painstaking research.

Dandapani (1992) identified the process variables and the characteristics of mathematics teachers which contribute to the effective teaching of mathematics. The sample consisted of six hundred and eighty nine teachers of high schools and higher secondary schools of Tanjore district in Tamilnadu. The tools used to collect data developed by the investigator include a Teacher's Perception Scale of Effective Teaching of Mathematics and Characteristics of Effective Mathematics teacher Description form. It was found that female teachers had a significantly higher perception than the male teachers. Teacher's perception had been found to vary with their years of experience. The perception of teacher did not differ because of their qualification, place of work, type of management, type of school and number of periods of teaching mathematics.

Singh (1992) compared the results of computer assisted instruction (CAI) with conventional method of instruction in teaching Mathematics for certain selected units of the mathematical curriculum. The study was conducted in four higher secondary schools having facilities of three to five BBC micro computers. The students belonged to different socio-economic groups. Three units of the Mathematics syllabus for class IX namely, simultaneous equations in algebra, statistical data and their graphical representation in statistics and triangles and their congruence in geometry were chosen for the study. The tools used in the study included rating scale by the researcher, Genus Intelligence Test, the attitude scale towards Mathematics and educational software. The statistical technique used included mean, and t-test for data analysis: The major findings were: The group taught through CAT in all the schools showed a substantial progress. The CAI method of teaching Mathematics had proved to be more effective. Both boys and girls gained from the computer treatment. A significant favourable change in the attitude of the pupils of the experimental groups over the control groups was observed.

Kumar (2008) investigates on the formation of concepts in Mathematics among the pupils of std. VI, VII, VIII & its relation to correlated approach in teaching learning process. The sample consisted of 948 students from the three randomly selected schools in greater Mumbai affiliated to the Maharashtra state Board of Secondary & Higher Secondary Education, Pune. The design of the study was experimental post test only Control group design. Tools used were Achievement Test and data were analyzed using Mean, SD, and t-test. It was found that the students who were taught using correlated approach were highly benefited. Knowledge when perceived as whole enables the pupils to link the previous knowledge with the present knowledge and form a better configuration of knowledge.

Anthony & Walshaw (2009) studied about the Characteristics of Effective Teaching of Mathematics: A View from the West have stated that in New Zealand a collaborative knowledge building strategy—The Iterative Best Evidence Synthesis Program—has been implemented at policy level. Drawing on findings from the mathematics Best Evidence Synthesis Iteration, and more recent research studies, this paper offers ten principles of effective pedagogical approaches that facilitate learning for diverse learners. In examining the links between pedagogical practices and a range of social and academic student outcomes we draw on the histories, cultures, language, and practices for the New Zealand context and comparable international contexts. The ten principles of effective pedagogy of mathematics are (1) An Ethic of Care: Caring Classroom Communities that are focused on Mathematics goals help develop students' Mathematical identities and proficiencies. (2) Arranging for learning: Effective teachers provide students with opportunities to make sense of ideas both independently and collaboratively. (3) Discourse in the classroom. (4) Mathematical language: The use of Mathematical language is shaped when the teacher models appropriate terms and communicates their meaning in a way that students understand. (5) Mathematical tasks (6) Making Connections: Effective teachers support students to create connections, between different ways of solving problems, between mathematical topics, and between mathematics and everyday experiences. (7) Tools and representations: Effective teachers carefully select tools and representations to provide support for students' thinking. (8) Teacher learning and knowledge. (9) Building on students thinking. (10) Mathematical Communication.

Aguele et al. (2010) conducted a study on Effectiveness of Selected Teaching Strategies in the Remediation of Process Errors Committed by Senior Secondary School Students in Mathematics. The purpose of the study was to determine the effectiveness of selected teaching strategies in the remediation of process errors committed by students in mathematics in senior secondary schools. The study employed the quasi-experimental design. Sample for the study consisted of two hundred and seven (207) students drawn from six senior secondary schools randomly selected from the three hundred and sixty senior secondary schools in Edo State. The Diagnostic Test on Mathematics (DIATOM) was used to collect data for the study. Data collected were analyzed using analysis of covariance (ANCOVA) and z-test for two population proportions. Results of data analysis revealed that the direct instruction was a more effective strategy for the remediation of process errors committed by students in mathematics. Sex and school location were shown not to have had any significant influence on the effectiveness of either strategy. The study recommended that enough practice activities should be given to students during class sessions to assist them develop mastery of content taught.

2.3.1 SUMMARY OF REVIEWS IN PART- III

The reviews revealed about various instructional strategies of mathematics are as: Anderson (1978) in this research study Clinchy and Rosenthal's error classification scheme was applied to test results to determine its ability to differentiate the effectiveness of instruction and two elementary schools were taken as sample. Inter-school comparisons were made with respect to the types of errors made: computational, algorithmic, and omission. Fifth grade students in one school made significantly fewer computational errors than in the other school. In the remaining grades, there were no significant differences. Additional practice exercises were recommended to correct this error. Kothari (1985) investigated the efficacy of different instructional media into the teaching of Mathematics to the pupils of class IX with sample of 120 students selected from two schools. Junior Index of Motivation, Reasoning ability and Criterion Tests used as tools. Visual projection is comparatively more effective than any other Instructional media like Activities and experiment or even programmed learning material for teaching of Mathematics. Mohpatra (1990) The sample consisted of two hundred and twenty secondary school teachers and five hundred and fifty six students. The tools used were questionnaires for students studying Mathematics and for teachers teaching Mathematics. Dandapani (1992) study conducted for 629 teachers of high schools and higher secondary schools. The tools used to collect data developed by the investigator include a Teacher's Perception Scale of Effective Teaching of Mathematics and Characteristics of Effective Mathematics teacher Description form. It was found that female teachers had a significantly higher perception than the male teachers. Singh (1992) in this study, four higher secondary schools having facilities of three to five BBC micro computers taken as sample. Three units of the Mathematics for class IX namely, simultaneous equations in algebra, statistical data and their graphical representation in statistics and triangles and their congruence in geometry were chosen for the study of computer assisted instruction (CAI) with conventional method. Tools like rating scale by the researcher, Genus Intelligence Test, the attitude scale towards Mathematics and educational software were used and mean as well t-test were used for data analysis. Kumar (2008) experimented with post test only Control group design. Tools used were Achievement Test and data were analyzed using Mean, SD, T-test. The sample consisted of 948 students from the grade VI, VII, VIII of three randomly selected schools. Aguele et al. (2010) the study employed the quasi-experimental design. Sample for the study consisted of two hundred and seven (207) students drawn from six senior secondary schools randomly selected. The Diagnostic Test on Mathematics (DIATOM) was used to collect data and data were analyzed using analysis of covariance (ANCOVA) and z-test for two population proportions. Sex and school location were shown not to have had any significant influence on the effectiveness of teaching strategy.

Anderson (1978) concluded as the error classification scheme can be used to evaluate instructional programs and to suggest instructional improvements. Kothari (1985) revealed as the low achieving pupils are comparatively more benefited by programmed learning material than the high achievers and the average achieving pupils. Mohpatra (1990) study resulted as the teacher provided high ranking to the traditional topics and resisted the intrusion of new topics. Dandapani (1992) Teacher's perception had been found to vary with their years of experience and did not differ because of their qualification, place of work, type of management, type of school and number of periods of teaching mathematics. Singh (1992) revealed majorly as the CAI method of teaching Mathematics had proved to be more effective. Both boys and girls gained from the computer treatment. Kumar (2008) stated as the knowledge when perceived as whole enables the pupils to link the previous knowledge with the present knowledge and form a better configuration of knowledge. Aguele et al. (2010) revealed as the study recommended that enough practice activities should be given to students during class sessions to assist them develop mastery of content taught.

2.4 PART-IV: REVIEWS ON CONCEPTUAL UNDERSTANDING OR IN-DEPTH LEARNING IN MATHEMATICS

In the context of Mathematics, 'in-depth learning' and 'rote learning' have variously been defined and explained. According to Jenkins (2010), in-depth learning manifest itself in Mathematical thinking which is characterized in terms of how students make of sense of Mathematics, the strategies they apply to solve problem, the conceptual representation they create, the argument they make and the conceptual understanding they demonstrate. Skemp (1976) has presented his views on types of learning: "relational" and "instrumental", the ideas explained in the context of instrumental and relational learning are relevant to the practice of teaching Mathematics in any context. Relational learning is explains both as what to do and why (knowing with reason), where as instrumental learning is described as "rules without reasons". The National Research Council (2001) set forth in its document *Adding It Up: Helping Children Learn Mathematics* a list of five strands, which includes *conceptual Understanding*: comprehension of mathematical concepts, operations, and relations.

Williams (1980), in a study of the understanding of Mathematical proofs by eleventh grade students discovered that: (1) Less than 30 percent of the students demonstrated any understanding of the role of proof in mathematics. (2) Over 50 per cent of the students stated there was no need to prove a statement that was "intuitively obvious". (3) Almost 80 percent of the students did not understand the important roles of hypotheses and definitions in a proof. (4) Less than 20 percent of the students understood the strategy of an indirect proof. (5) Almost 80 percent of the students did understand the use of a counter example. (6) Over 70 per cent of the students were unable to distinguish between inductive and deductive reasoning, which included being unaware that inductive evidence does not prove anything. (7) No gender differences in the understanding of Mathematical proofs were evident.

Johnson (2000) summarised the reviews on **Mathematical reasoning** in terms to enhance or improve **Comprehension** as: Conceptual understanding of the four basic operations incorporates connections among representations—concrete, pictorial, symbolic, and real-world. The goal of many research and implementation efforts in mathematics education has been to promote learning with understanding. But achieving this goal has been like searching for the Holy Grail. There is a persistent belief in the merits of the goal, but designing school learning environments that successfully promote understanding has been difficult" (Hiebert and Carpenter, 1992). Few high school students are able to comprehend a mathematical proof as a mathematician would, namely as a "logically rigorous deduction of conclusions from hypotheses" (Dreyfus, 1990). Part of the problem is that students also do not appreciate the importance of proof in mathematics (Schoenfeld, 1994). Students of all ages (including adults) have trouble understanding the implications of a conditional statement (e.g. If-then). This trouble is due to a focus on seeking information that verifies or confirms the statement when the focus should be on seeking information that falsifies the statement (Wason and Johnson-Laird, 1972). Teachers can improve their students' ability to construct and evaluate mathematical proofs if they (1) transfer to students the responsibility of determining the truth value of mathematical statements and (2) integrate their teaching of proof constructions into other mathematical content rather than treat it as a separate unit (Balacheff, 1988). Students' understanding of logical statements is significantly correlated with the frequency of mathematics teachers' use of conditional reasoning (e.g. use of "if-then" statements) in their own verbal responses (Gregory and Osborne, 1975).

Johnson (2000) summarised the reviews on **Mathematical connection or relations** in terms to promote **Understanding or Comprehension as**: Students need to build meaningful connections between their informal knowledge about mathematics and their use of number symbols, or they may end up building two distinct mathematics systems that are unconnected—one system for the classroom and one system for the real world (Carraher et al., 1987). Students need to discuss and reflect on connections between mathematical ideas, but this “does not imply that a teacher must have specific connections in mind; the connections can be generated by students”. Hodgson (1995) demonstrated that the ability on the part of the student to establish connections within mathematical ideas could help students solve other mathematical problems. However, the mere establishment of connections does not imply that they will be used while solving new problems. Thus, teachers must give attention to both developing connections and the potential uses of these connections. A mathematical connection that is explicitly taught by a teacher may actually not result in being meaningful or promoting understanding but rather be one more “piece of isolated knowledge” from the students’ point of view (Hiebert and Carpenter, 1992).

Johnson (2000) summarised the reviews on **Application of Mathematical concepts** as: The skills and concepts learned in school mathematics differ significantly from the tasks actually confronted in the real world by either mathematicians or users of mathematics (Lampert, 1990). Students often can list real-world applications of mathematical concepts such as percents, but few are able to explain why these concepts are actually used in those applications (Lembke and Reys, 1994). Vocational educators claim that the continual lack of context in mathematics courses is one of the primary barriers to students’ learning of mathematics (Bailey, 1997; Hoachlander, 1997). Yet, no consistent research evidence exists to support their claim that students learn mathematical skills and concepts better in contextual environments (Bjork and Druckman, 1994).

Lai & Murray (2005) conducted their research on ‘Teaching with Procedural Variation: A Chinese Way of Promoting Deep Understanding of Mathematics’ and reported as:. In mathematics education, there has been tension between deep learning and repetitive learning. Western educators often emphasize the need for students to construct a conceptual understanding of mathematical symbols and rules before they practise the rules (Li, 2006). On the other hand, Chinese learners tend to be oriented towards rote learning and memorization (Marton, Watkins & Tang, 1997). One aspect of the criticism is that rote learning is known to lead to poor learning outcomes

(Watkins & Biggs, 2001). However, Chinese students consistently outperform their Western counterparts in many international comparative studies on mathematics achievement such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1997; Mullis, Martin, & Foy, 2008) and PISA (OECD, 2004; OECD, 2010). This study aims to contribute to an understanding of the "paradox of the Chinese learners" (Marton, Dall' Alba & Lai, 1993) by exploring the procedural variation and its place in the development of mathematical understanding.

Even & Tirosh (2008) studied about the learning of mathematics and mentioned that prior knowledge of primary concepts provides a foundation upon which learning of subsequent concepts is based. Evolution of mathematical thinking and mathematical reasoning thus becomes a process which can be stimulated or in one way the other be influenced by the external factors or conditions, which, in many researchers' view, could be controlled, to a great extent, by the teacher.

Ali (2011) conducted research study based on literature reviews on the aspects of Understanding in Mathematics and summarised the reviews in a report as: Research provides evidence that children who learn subject matter knowledge with thorough understanding demonstrate an enhanced ability to think flexibly when dealing with novel problems (Newton, 2000; Sierpinska, 1994). In-depth learning in mathematics facilitates further learning; it enables critical abilities such as reasoning and analytical skills and helps develop learners' creative faculty of mind (Newton, 2000; Perkins, 1993). Moreover, the prior knowledge students bring to the learning situation is considered to be a vital factor in facilitating in-depth learning (e.g. Gollub et al., 1993; Perkins, 1993; Mayhill & Brackley, 2004). Both National Curriculum (2006) and Education Policy (2009) stress upon a marked shift in teacher's role from transmitter of information to creator of learning environment in classroom which supports students in developing rational understanding of the mathematical concepts. Leder (1991) concluded that teachers' poor grounding in mathematics could be blamed on students' difficulties in understanding mathematics. Further, she suggested that a shortage of well qualified mathematics teachers at all levels of the educational system continues to be a matter of concern. Joseph & Yoe (2010) and McLaren (2010) both the research studies stated that, teachers' central role in promoting deeper learning requires them to understand and practice some of the basic principles of the conceptual learning in mathematics. These principles include teaching general knowledge or generic concepts in the subject and helping students in overcoming the difficulties they face while mathematical concepts. Teachers can use a wide variety of activities and techniques such as discussion, stories, songs, role play, visual illustrations, patterns seeking, using examples from real life, use of analogy and explanations, to help build prerequisite knowledge and strengthen connections between what students already know about a concept what they need to know more about it.

Rahman et al. (2013) conducted study to report on strategies implemented with objectives of (i) enhancing students' understanding; (ii) supporting self-regulated learning; and (iii) improving teaching practice for Engineering Mathematics 3. To support the realization of the KES approach (Knowledge – Experiential – Self-regulated), the team uses a framework which they had previously developed (Roselainy et al, 2012a) to encourage students to adopt self-regulated learning behaviour in an active learning environment. We also ensured that the teaching, learning and assessment activities were constructively aligned (Biggs & Tang, 2010). An action research methodology was implemented to improve teaching practice and thus data collected was used to modify subsequent teaching and learning activities. The total number of students was 65 made up of 33 students from the second year Electrical Engineering (SMJE) and 32 students from the Mechanical Engineering (SMJM) Programmes. Data were through various methods such as, (a) observations of students, (b) students' work, (c) students' reflections on their learning and finally, (d) students' answers on how they make sense of the mathematical knowledge learnt, and finally (e) performance in examinations. The strategies were successful in encouraging and supporting students to embrace and take charge of their own learning. Students' results were also better than their previous achievements in Engineering Mathematics 2 (Differential Equations). The study concluded with that the students have to be supported in an appropriately designed learning environment for successful independent learning.

2.4.1 SUMMARY OF REVIEWS IN PART- IV

Here in this part, the reviews related with the conceptual understanding or in-depth learning in Mathematics are constituted. Williams (1980) conducted study on the understanding of Mathematical proofs by eleventh grade students and almost 80 percent of the students did not understand the important roles of hypotheses and definitions in a proof. Lai & Murray (2005) stated as in mathematics education, there has been tension between deep learning and repetitive learning. Ali (2011) conducted research study based on literature reviews on the aspects of Understanding in Mathematics. Rahman et al. (2013) in this study an action research methodology was implemented to improve teaching practice. As 33 students from the second year of Electrical Engineering (SMJE) and 32 students of Mechanical Engineering (SMJM) Programmes means total 65 students were selected for the sample. Data were collected through observations of students, students' work, students' reflections on their learning, students' answers on how they make sense of the mathematical knowledge learnt and finally, performance in examinations.

Williams (1980) concluded as that no gender differences in the understanding of Mathematical proofs were evident. Even & Tirosh (2008) revealed as the prior knowledge of primary concepts provides a foundation upon which learning of subsequent concepts is based. Ali (2011) stated as the Teachers can use a wide variety of activities and techniques such as discussion, stories, songs, role play, visual illustrations, patterns seeking, using examples from real life, use of analogy and explanations, to help build prerequisite knowledge and strengthen connections between what students already know about a concept what they need to know more about it. Rahman et al. (2013) concluded with that the students have to be supported in an appropriately designed learning environment for successful independent learning.

Johnson (2000) revealed critically as Conceptual understanding of the four basic operations incorporates connections among representations—concrete, pictorial, symbolic, and real-world. Students need to build meaningful connections between their informal knowledge about mathematics and their use of number symbols and need to discuss and reflect on connections between mathematical ideas. Students often can list real-world applications of mathematical concepts such as percents, but few are able to explain why these concepts are actually used in those applications. The ability on the part of the student to establish connections within mathematical ideas could help students solve other mathematical problems.

2.5 PART- V: <u>REVIEWS ON S.O.L.O. TAXONOMY</u>

From the literature on the SOLO taxonomy, it was suggested that SOLO is a hierarchical model that is suitable for measuring learning outcome of different subjects. Here, reviews included those where SOLO model had been used in assessing students' cognitive attainment in Science, Counseling, Practice subject, and several Mathematics areas and skills like statistics, algebra, probability, geometry, error analysis and problem solving.

Bennett (1987) conducted research entitled with "The quality of problem solving in mathematics: an application of the SOLO Taxonomy to think aloud solutions of mathematical word problems". The research study summarised as the quality of problem solving in mathematics was considered for a group of 10 students, approximately 15 years of age, attending a public secondary school in South Australia. The SOLO (Structure of the Observed Learning Outcome) Taxonomy developed by Biggs and Collis (1982) and presented by them as a means of expressing the quality of learning outcomes is used in this study to determine the quality of think aloud responses to mathematical word problems. This application required the development of a framework for analysis and scoring based on the component dimensions of the published taxonomy. In this way the study adds to the available literature on the SOLO Taxonomy in general and more particularly to its use in mathematics and problem solving. The developments for application here were fruitful and confirm the taxonomy as a useful research instrument in this form. They also refine the SOLO Taxonomy as a means of determining performance quality in ways which could be of value to teachers in classrooms.

Chick et al. (1988) presented that how the extended SOLO Taxonomy can be applied in analyzing error in a variety of mathematics topics. In order to illustrate the SOLO levels, task analysis maps were devised by Chick (1988) to describe the types of errors being made by students in solving the mathematics problems. It was used to accomplish whether students had been successful in attaining the level of solutions desired. According to Chick, a correct solution would not be obtained if the solver was operating at unsatisfactory levels in any one of the necessary concepts areas. Thus, it was interesting to determine the areas in which the difficulties lie based on the pictorial nature of the task analysis and response maps. This method is useful for teachers in analyzing students' mistakes, showing the potential for facilitating the comparison of students' solution with 'ideal' or correct solution, identifying the conceptual difficulties and misconceptions and examining the way of students in using the information given in the problem.

Gates (1994) conducted research study on the 'Transfer of Abstract Thinking in Mathematics'. Mathematics teachers' influence on student learning of mathematics could interfere and limit the learning of higher order Mathematics. To assess the affects of this influence, 1st year University mathematics students were selected as a study sample. These students' Mathematical understanding was explored using 'Mathematical items' designed specifically for this study. The students' responses were assessed and evaluated using the SOLO taxonomy. The findings tentatively suggest that prior learning affects the depth and clarity of University students' understanding of Mathematics. This study portrays a growing concern of our children's lack of appreciation (or poor attitude) and poor understanding of Mathematics. To address this concern there is a need for teachers who are confident in their own Mathematical knowledge and who themselves have a grasp of mathematical concepts and ideas. The training and education of secondary and primary mathematics teachers is an important link in the Mathematics education of our children and the area of interest for this research. The research began with the need to identify the levels of mathematical learning and understanding that the pre-service secondary Mathematics teachers have been, in the past, assumed to have acquired during their pre-tertiary schooling and education. Piaget's theory of 'cognitive development' was used to examine and assess the levels of Mathematical understanding of the pre-service Mathematics teachers. The trial sample was 54 first year University mathematics students (23 enrolled in Dip.Eng. & 31 B.Eng.) and three experienced Mathematics teachers. These figures tentatively suggest that prior Mathematical knowledge has an influence in Mathematical understanding of higher order levels.

Boulton et. al. (1996) also used the SOLO Taxonomy to analyze student learning. In their study, the content of written statements from 40 teachers enrolled in a graduate course was categorized by structural organization according to the SOLO model. Their findings indicated that 80% of student responses fit the multistructural level indicating that students need help in structuring the content of their learning to reach a relational or abstract level. They advocated that students be provided opportunities to distinguish between models written at different SOLO levels and that students write and rewrite material individually and in groups until a relational level is met. Highly regarded in the present study is the ability to distinguish between deep and surface learning.

Peter & Yeen (1996) had investigated on ‘Rasch Analysis of Math SOLO Taxonomy Levels Using Hierarchical Items in Testlets’. This study attempted to estimate Structure of Learning Outcome (SOLO) levels in mathematics using the Partial Credit and Rating Scale models. A 30-item test comprising 10 testlets of 3 items each was designed and administered to 674 lower secondary school students. The items were arranged in a hierarchical manner, each testing SOLO levels in this order: Unistructural, Multistructural, Relationship and Extended Abstract. The item response matrix was fitted into the Partial Credit and Rating Scale models. Results showed that: (1) the observed testlet response patterns fitted those expected; (2) the dataset fitted the psychometric models reasonably well; and (3) the proportion of examinees getting SOLO items correct decreased in order from level 1 to level 3 along the math proficiency continuum between -2.0 and +2.0. The results of the study have implications for a criterion-based approach in interpreting test results based on SOLO testlets. Results also showed the viability of testlet item bank development, test construction, and computerized testing using testlets.

Jones et al. (1997) had applied SOLO model to formulate a framework for assessing 15 middle school students’ thinking in conditional probability and independence. The structured interview protocol (14 tasks) was used to collect the students’ responses. A double-coding procedure then was used to identify the level based on the descriptors in the framework. The framework then was validated through data obtained from eight grade three children who served as case studies. The results suggested that although the framework produced a coherent picture of students’ probability thinking, there was ‘static’ in the system which generated inconsistencies within levels. The levels of thinking appeared to be in agreement with level of cognitive development based on SOLO model and provided a theoretical foundation for designers of curriculum and assessment program in elementary school probability.

Levins (1997) attempted to show that how SOLO model was appropriate to categorize the students' written responses into the cognitive classification framework for similar or different ages, which these existed at different degrees of consistency of particular ideas about certain scientific concepts. In this study, three questions were given to 190 students from years 7-12. Two cycles reflected an increase in the students' power of abstraction from the first to the second cycle. The first cycle was one in which the students acquired basic skills and concepts. For instance, they suggested some features such as steam, water, heat and gas which were the descriptions concerning evaporation. In other words, they reacted to reality as they saw it. In the second cycle, the students were able to conceptualize the ideas they possessed concerning their ideas of evaporation. Thus, this study provided an analysis of the growth of the evaporation concept within the theoretical framework of the SOLO model. This model enclosed the growth of basic understanding about the concepts must have in place before the transition to the more demanding abstract ideas.

Chick (1998), in an attempt to understand the nature of cognitive processes at the highest level of formal thinking, used the SOLO Taxonomy to examine the stages of mathematical cognition of a mathematics researcher by analyzing the data she collected as a graduate student. Chick stated that while both undergraduate and graduate students operated in a formal mode, there was a difference between the two levels of formal functioning, formal-1 and formal-2. One significant difference Chick observed was between creating (formal-2) and understanding knowledge (formal-1). A second distinction between the two levels was observed in responses. Chick found it difficult to assess formal-2 cognition in student responses to prompted questions. While both formal-1 and formal-2 modes can produce relational responses on the SOLO Taxonomy, satisfactory performance at the formal-2 level, a criterion qualifying the individual as a "researcher," was evidenced most often with the ability to produce relational responses. Chick concluded that outcomes indicative of formal-2 cognition can be evaluated using the SOLO Taxonomy and that the levels of the taxonomy reflect the worthiness of the results, just as the SOLO Taxonomy in concrete-symbolic and early formal modes has been applied successfully.

Burnett (1999) presented that SOLO model provides an existing way to view the quality of learning outcomes of counseling within a learning framework. The clients' written responses were analyzed and classified by using this model. In the exploratory study, the clients were asked to write a letter to a friend describing as much detail as possible what they had learned and how they gained or benefited from counseling. The result obtained from this study revealed that majority of the clients fall within the multistructural category based on the classification of SOLO level. It means that most of the clients were not integrating the knowledge learned in the counseling context. The aspects learned were still treated as unrelated. This approach may have implication for the process of counseling because counselors can use this technique to enhance their clients' learning.

Lake (1999) outlined about an adaptation of SOLO model that provides student and teacher with a pedagogically sound template which can be used to develop critical numerical skill especially interpretation of graph and table in learning of biology. In this context, SOLO model is visualized as a spiral learning structure repeating itself with increasing levels of abstraction; each level built on the skill that was acquired at the previous one. Thus, it was useful to be designed to classify the problem solving processes by stages (unistructural, multistructural, relational and extended abstract) and adapted to provide a useful four-step template of generalized questions that leaded students from the basic skills to critical analysis.

Reading (1999) used SOLO model to classify 180 students' responses (academic years 7 to 12) concerning the statistical understanding in data tabulation and representation. They were allowed to respond as much information as he or she felt was necessary. Three major grouping of the levels were identified based on the depth to which the responses indicated the ability of the student to understand the representation of the data. The first group dealt with only the requirement of the question. The second group was concerned with attempting to understand the data. The third group of responses indicated a readiness to describe the information contained in the data in a more acceptable form. Also, there was an increase in level with academic year. Younger students were more likely to use judgments in their descriptions while older students were more likely to use statistics.

Pegg (2001) used SOLO model to describe the development of algebraic knowledge and thinking ability. In this study, SOLO model comprised a recurring cycle of three levels referred to as unistructural, multistrustural and relational. Two algebraic examples were presented in which one of the item had been applied to a number of secondary classes in 2001 and another one had been used initially by Biggs (1982). Based on the students' responses, SOLO model was used to classify algebraic solving ability in UMR (unistructural, multistructural, relational) cycle in the concrete symbolic second cycle mode which is closely to formal one mode. In this study, SOLO model was used in reverse and combining it with the idea of superitem. That is, within any superitem, a correct response to a question would indicate the cognitive ability at the certain level reflected in the SOLO structure of that question. The criteria used to write the four level questions are as follow (Collis, Romberg, & Jurnak, 1986): i) Unistructural – one obvious piece of information was used. The information was obtainable directly from the stem (the story or problem situation). ii) Multistructural – all the necessary pieces of information was used in a sequence but do not integrate them. The information given may use as a recipe where a set of instructions are followed to solve the problem. iii) Relational – the given information is insufficient to solve the problem immediately. Alternatively, the information must be carefully inter-related to produce a satisfactory solution or to form a structure. iv) Extended Abstract - a response that use of an abstract general principle or hypothesis derived from or suggested by the information in the stem to a new and more abstract situation. There are four levels of structure response that had been applied to construct four levels questions in a superitem about algebraic equation. The correct achievement of the first question (first level: unistructural) in the superitem would indicate that the algebraic solving ability at least of the unistructural level.

Chan et al. (2002) made an attempt on applying SOLO model in a practice subject. The scripts of long essay papers and short classroom discussion responses from postgraduate students who had taken an advanced practice subject in mental health were analyzed using SOLO model. In the finding, it was found that SOLO was suitable for measuring the work in content variation of practice subjects and it could be applied to the students who were from different levels of cognitive learning outcome. They discovered that the classification of levels that sub-levels were added to SOLO model that would reduce the ambiguity and increase agreement between rater (inter-rater reliability). For instance, presturctural, unistructural, multisturctural moderate level, multistructural high level, relational moderate level, relational high level and extended abstract level.

Mooney (2002) attempted to build an integrated picture of students' thinking that incorporates four key statistical processes. The SOLO model framework had been developed in characterizing the development of middle school students' thinking across four processes, namely describing data, organizing and reducing data, representing data and analyzing and interpreting data. The profile showed a strong internal consistency in students' statistical thinking across the four processes. Also, found that 5/12 of the students achieved the same level of thinking for any three out of the four statistical processes. Based on this model, it can be concluded that the four statistical processes closely linked although examined separately.

Vallecillos & Moreno (2002) described that SOLO model can be used as a framework to characterize and assess the learning of elementary statistical inference amongst 49 secondary students aged 14-15 years old and 17-18 years old. Two different parts with 12 items each were constructed in three different contexts, namely concrete, narrative and numeric. Four constructs had been established namely: i) population and samples and the relationship; ii) inferential process; iii) sample sizes; and iv) sampling types and biases. The researchers described three levels of observed learning outcomes (unistructural, multistructural and relational) for each construct. The findings demonstrated that the higher age students performed better than lower age students especially in two of the constructs, namely: i) populations and samples and their relationship; and ii) sample size.

Hattie & Brown (2004) conducted research on ‘Cognitive processes in asTTle: The SOLO Taxonomy’. This research report provides an explanation of the SOLO taxonomy used to ascertain cognitive processing in asTTle assessment questions and tasks. asTTle (Assessment Tools for Teaching and Learning) is funded by the Ministry of Education to Auckland Uniservices Ltd. at the University of Auckland to research and develop an assessment application for Reading, Writing, Mathematics for the students of Years 4-10 (Levels 2-6) for New Zealand schools. This taxonomy has been used in asTTle to categorise student performance on every task in Reading and Mathematics. This report explains the SOLO taxonomy and its psychological basis, and provides examples of using SOLO in assessment and education in general. The key issue addressed in this report is how to devise an assessment model that values a balance of surface and deep processing. The key for the asTTle development is the use of a defensible taxonomy of processing -- the SOLO taxonomy. Like most taxonomies, SOLO describes the processes involved in asking and answering a question on a scale of increasing difficulty or complexity.

Lian & Idris (2006) studied about **'**Assessing Algebraic Solving Ability Of Form Four Students'. Mathematics researchers generally agree that algebra is a tool for problem solving. However research works on assessing students' algebraic solving ability is sparse in literature. The purpose of this study was to use the SOLO model as a theoretical framework for assessing Form Four students' algebraic solving abilities in using linear equation. The content domains incorporated in this framework were linear pattern (pictorial), direct variations, concepts of function and arithmetic sequence. This study was divided into two phases. In the first phase, students were given a pencil-and-paper test. The test comprised of eight superitems of four items each. Results were analyzed using a Partial Credit model. In the second phase, clinical interviews were conducted to seek the clarification of the students' algebraic solving processes. Results of the study indicated that 62% of the students have less than 50% probability of success at relational level. The majority of the students in this study could be classified into uni-structural and multi-structural. Generally, most of the students encountered difficulties in generalizing their arithmetic thinking through the use of algebraic symbols. The qualitative data analysis found that the high ability students seemed to be more able to seek the recurring linear pattern and identify the linear relationship between variables. They were able to coordinate all the information given in the question to form the algebraic expression and linear equations. Also, the students with low ability were shown more ability on the method of drawing and counting. They lacked understanding of algebraic concepts to express the relationship between the variables. The results of this study provided evidence on the significance of SOLO model in assessing algebraic solving ability in the upper secondary school level.

Aoyama (2007) used SOLO model to investigate the hierarchy of interpretations of graphs among 175 students from different education levels in Japan (junior high school to graduate students). All participants completed a questionnaire including three or four items each based on a different theme, and having three to five questions asking about the interpretation of a graph and its context. Rasch model was used to clarify the construction of hierarchy. Five different levels of interpretations of graphs were identified, namely idiosyncratic, basic graph reading, rational/literal, critical and hypothesizing and modelling. Generally the students' performance progressed with age, but the performance of junior college students was slightly lower than those of high school students. The hierarchy of interpretations of graphs was found to be very useful in preparing further guidelines for teaching statistical literacy. In this context, SOLO model was applied to measure the learning outcome of students who were in different academic year. It can therefore be seen that age influenced the thinking ability in statistics. The degree of abstraction that was utilized by the learner in handling the element of the tasks closely related to the stages of development.

Serow (2007) integrated SOLO model and Rasch model to gain insight into students' understanding of class inclusion concepts in geometry. The partial credit analysis was used to provide data concerning the distances between response categories and clusters of response categories. The qualitative study involved in-depth interviews with 24 students of higher mathematical ability, purposely selected, within Years 8–12 (ages 13–18 years) in two secondary schools. In the tasks concerning relationships among figures, and those concerning relationships among properties, a hierarchical framework emerged that is evident in the SOLO categorizations and is reinforced by the application of the Rasch analysis. Each of the items followed the SOLO sequence of levels within cycles without exception. The step difficulty between a U2(CS) response and an M2(CS) response concerning relationships among figures has a mean of 0.64. It was also found to be difficult by the sample of students to respond at R2(CS) compared with M2(CS) concerning relationships among figures (mean 1.18). This was similar to the step difficulties concerning relationships among properties, where M2(CS) to R2(CS) (mean 0.70) was found to have a comparatively high step difficulty.

UNICEF (2007) conducted a research study based on SOLO Taxonomy and drafted a 'Report of the East Asia Learning Achievement Study (EALAS)'. EALAS introduced the use of Structure of Observed Learning Outcomes (SOLO) taxonomy to the region.

This approach defines a student's understanding of a subject at increasing levels of complexity and is applicable to most subject areas. To record and analyze findings, SOLO taxonomy was coupled with Rasch modeling, a data analysis tool for creating multi-item scales. A total of 11 UNICEF country offices and their counterparts joined in the EALAS technical workshops. Nine of these, Mongolia, China, the Democratic People's Republic of Korea, VietNam, Myanmar, Indonesia, the Philippines, Timor-Leste and the Pacific, have successfully carried out the pilot process in at least 20 schools. Each country conducted a pilot study to test instruments and procedures. The scale of the pilot studies varied from country to country, but usually at least 20 schools was selected, with two classes chosen from each school. In most cases this represented a sample of approximately 1,200 students. The schools were purposively selected for the pilot study to reveal expected differences between school types and sizes, districts or provinces, or between urban and rural schools. This kind of purposive sampling allowed for a wide range of responses to the cognitive and other instruments, which was useful for assessing the appropriateness of response categories and for testing administrative procedures in a range of settings. For different countries, the broad subjects or content domains had considerable similarity in mathematics, more variability in language and great variability in life skills, which tended to cover a combination of science, social science and moral or ethics content. The variation in the life skills domain reflects the lack of a well-defined syllabus. The test design matrix, which has the content areas on one axis and the SOLO taxonomy levels on the other was defined to reflect not only the range of subject topics in the curriculum, but also a range of item difficulty to ensure that the test difficulty reflected the ability of the target group. With reference to the Assessment, it is a Dynamic model from which it infers the interactions between assessment, curriculum, teacher development and student learning. SOLO supports this dynamic since it provides guidance to teachers and curriculum development for the structuring of instructional sequences, for assessing levels of student comprehension, and for reviewing the appropriateness of the curriculum and test items. Using SOLO taxonomy and Rasch modeling together offers a number of direct applications. At the class level, teachers and headmasters can know how students are doing in different subjects and what level of achievement they have attained. Schools can determine how well a curriculum is being mastered by individual students and how well teachers are doing comparatively based on student performance.

Kayani et al. (2010) investigated on 'Teachers' Perception Regarding Examination Based On SOLO Taxonomy'. This research study reports the finding of the study carried out in 30 districts of Punjab (Pakistan). The study was designed to seek the teachers' perception about the newly introduced examination system called structure of observed learning outcomes (SOLO) taxonomy. Sample of the study consisted of twelve schools (eight from rural areas and four from urban area) from each district and total 360 teachers. A questionnaire was developed for teachers based on five point Likert scale which was used for collection of data. Findings of the study revealed that examination system based on SOLO taxonomy increased reliability and validity of the grade V examination system and it enhanced creative thinking and reading, writing and comprehension skills of students. It was suggested that a comprehensive training program emphasizing construction of test SOLO type test items, scoring and interpretation be arranged for teachers. The paper concluded that, the first and foremost aim of teaching is to make students creative and independent learners. New examination system based on SOLO Taxonomy had a lot of strengths. It increased reliability and validity of the grade-V examination system. It developed creative thinking among students and improved students' reading, writing and comprehension skills, and it discouraged habits of selective study among students. It also discouraged cramming and rote memorization in paper preparation. Marking system of papers was easy, as well as standard. The study recommended as, (i) the concept of SOLO Taxonomy based examination is new in Pakistan, therefore, a comprehensive training program may be arranged for teachers. (ii) In order to examine effectiveness of SOLO examination system more research may be conducted.

Bhattacharyya et al. (2012) conducted experimental study on 'Impact of SOLO Taxonomy in Computer Aided Instruction to Qualitative Outcome of Learning for Secondary School Children'. The Qualitative Outcome of Learning (QOL) measures the level of attainment in learning efforts through Structure of the Observed Learning Outcome (SOLO) taxonomy. In this work, they attempted to evaluate the QOL of learners in geometry for secondary school children studying under the Central Board of Secondary Education curriculum. The learners are challenged to test their understanding in lessons related to specific topics, rather than just knowing the content. The contents of each topic is based on SOLO taxonomy and is set to test the understanding level of learners with increasing complexity in that topic. Different learning tools are employed - an intelligent tutoring system Shikshak, CD based technology and printed textual material of the same content through which learners appear to promote active cognitive learning. The focus of this study was on the change in the QOL following the use of these three different learning tools employing SOLO taxonomy.

Lynn Rider (n.d.), the aim of this study was to investigate the potential benefits of a multi-Representational curriculum on students' understanding of and connections among graphical, tabular, and symbolic Representations of Algebraic concepts. The participants of the study were 313 college students enrolled in developmental college Algebra at two southern universities. This study utilized a quasi-experimental design in which instructors at one university (control) taught the course from a traditional Algebraic perspective while instructors at the other university (treatment) taught the course from a functional approach simultaneously introducing Representations and Multiple Representations. The effect of a multi-Representational curriculum on student success and Representational Preference was assessed with a pre-tests and post-tests of five problems, each with three Representations graphic, tabular, and symbolic. The problems were chosen because of their prevalence in most developmental college Algebra curricula. Although both curricula were successful in increasing student achievement, students from the multi-Representational curriculum scored significantly higher and were significantly more adept in using Representational methods other than Algebraic to solve the problems. Qualitative interviews were also conducted with eight participants from each school to examine the connections that students were making and their ability to move flexibly among the graphical, tabular, and Algebraic Representations. The interviews were analyzed using Biggs and Collis's SOLO Taxonomy. This research showed that a multi-Representational curriculum could be effective in expanding students' web of connected knowledge of Algebraic and functional concepts. The SOLO Taxonomy and rubric defined in this research gives teachers an effective way of measuring student learning.

Moila (n.d.) conducted a research study to investigate the use of educational technology in Mathematics teaching and learning. In order to achieve this goal, a school by the name of Phusela Secondary was visited for the whole week to conduct the study. A lot of the literature reviewed was on developed countries as there is limited literature on developing countries concerning the use of educational technology in mathematics teaching and learning and rural schools. Learners achievements compared to the Solo taxonomy measured effective usage of educational technology. The investigation followed a mixed method approach that was more evaluative and as one case was investigated it was a case study. Participants were sampled mathematics learners who were willing to participate in the study and willing mathematics educators of Phusela Secondary School. It was found from the study that there are no plans on the use of educational technology tools in mathematics teaching and learning, inadequate educators′ training on the use of educational technologies in teaching and learning and lack of relevant educational technology tools for rural schools. These were the major reasons for the school not to use the educational technology tools in mathematics teaching and learning. However these tools were sometimes used for other purposes other than mathematics teaching and learning. Recommendations were made on how Phusela Secondary School can improve its usage of educational technology tools in mathematics teaching and learning effectively for the development of higher order thinking skills. Recommendations for further study in as far this study was concerned were made.

2.5.1 SUMMARY OF REVIEWS IN PART - V

In this Part – V, about twenty five reviews collected with reference to SOLO Taxonomy where said taxonomy had been applied either in teaching-learning process or assessment or any other manner in any of the discipline have been included.

SOLO Taxonomy was used in various discipline as Lake (1999) and Levins (1997) experimented it with Science; Burnett (1999) used it in Counselling; Chan et al (2002) in Practice subject; Jones et al (1997) and Chick (1998) in Thinking; Hattie & Brown (2004) in Reading, Writing as well in Mathematics; Kayani et al (2010) in Examination System; Bhattacharyya et al (2012) used in CAI. While Bennett (1987), Chick et al (1988), Gates (1994), Peter & Yeen (1996), Reading (1999), Pegg (2001), Mooney (2002), Vallecillos & Moreno (2002), Lian & Idris (2006), Aoyama (2007), Serow (2007), Moila (n.d.) and Lynn Rider (n.d.) used in various areas and aspects of Mathematics. In Mathematics, the studies conducted in various area like Bennett (1987) in Problem solving; Chick et al (1988) in Error Analysis; Gates (1994) in

Understanding. While Peter & Yeen (1996); Reading (1999), Mooney (2002), Vallecillos & Moreno (2002), Aoyama (2007) used in Statistics; Pegg (2001), Lian & Idris (2006), Lynn Rider (n.d.) in Algebra; Serow (2007) in Geometry; Moila (n.d.) employed this taxonomy in Educational Technology.

As looking to the methodologies, Bennett (1987) conducted a study on a group of 10 students, approximately 15 years of age, attending a public secondary school in South Australia. Chick et al. (1988) devised SOLO levels wise task analysis and response maps to study the types of errors being made by students in solving the mathematics problems. In the study of Gates (1994), the trial sample was consisting of 54 first year University mathematics students (23 enrolled in Dip.Eng. & 31 B.Eng.) and three experienced Mathematics teachers. Mathematical understanding was explored using 'Mathematical items' designed specifically for this study and the students' responses were assessed and evaluated using the SOLO taxonomy. Boulton et. al. (1996) in their study, the content of written statements from 40 teachers enrolled in a graduate course was categorized by structural organization according to the SOLO model. Their findings indicated that 80% of student responses fit the multistructural level indicating that students need help in structuring the content of their learning to reach a relational or abstract level. Peter & Yeen (1996), this study attempted to estimate SOLO levels in mathematics using the Partial Credit and Rating Scale models. A 30-item test comprising 10 testlets of 3 items each was designed and administered to 674 lower secondary school students. Jones et al. (1997), the structured interview protocol (14 tasks) was used to collect the 15 middle school students' responses. Levins (1997), in this study, three questions were given to 190 students from years 7-12 and attempted to show that how SOLO model was appropriate to categorize the students' written responses into the cognitive classification framework. Chick (1998) investigated about the responses of both undergraduate and graduate students operated in a formal mode to observe the significant difference between creating and understanding knowledge. Burnett (1999) analyzed the clients' written responses and classified by using SOLO model in terms to view the quality of learning outcomes of counseling. And found as majority of the clients fall within the multistructural category based on the classification of SOLO level. It means that most of the clients were not integrating the knowledge learned in the counseling context. Lake (1999) adapted SOLO model and stated as that provides student and teacher with a pedagogically sound template which can be used to develop critical numerical skill especially interpretation of graph and table in learning of biology. Reading (1999) used SOLO model to classify 180 students' responses (academic years 7 to 12) concerning the statistical understanding in data tabulation and representation. Pegg (2001), based on the students' responses, SOLO model was used to classify algebraic solving ability in UMR (unistructural, multistructural, relational). Chan et al. (2002), the scripts of long essay papers and short classroom discussion responses from postgraduate students who had taken an advanced practice subject in mental health were analyzed using SOLO model. Mooney (2002) attempted to build an integrated picture of middle school students' thinking that incorporates four key statistical processes using the SOLO model framework. Vallecillos & Moreno (2002) experimented the learning and assess of 49 secondary students aged 14-15 years and 17-18 years for Statistical concepts through the SOLO model based framework. Hattie & Brown (2004) developed an assessment application for Reading, Writing of Mathematics for the students of Years 4-10 (Levels 2-6) to study the cognitive processing with respect to the SOLO levels. Lian & Idris (2006) used SOLO model as a theoretical framework for assessing Form Four students' algebraic solving abilities in using linear equation. Students were given a pencil-and-paper test and results were analyzed using a Partial Credit model. Results of the study indicated that 62% of the students have less than 50% probability of success at relational level and majority of the students could be classified into uni-structural and multi-structural. Aoyama (2007) SOLO model was used to investigate the hierarchy of interpretations of graphs among 175 students from different education levels in Japan (junior high school to graduate students) using Questionnaire. Rasch model was used to clarify the construction of hierarchy. Serow (2007) used SOLO model, Rasch model and the partial credit analysis to gain insight into students' understanding of class inclusion concepts in geometry. The qualitative study involved in-depth interviews with 24 students of higher mathematical ability, purposely selected, within Years or grades 8–12 (ages 13–18 years) in two secondary schools. UNICEF (2007) SOLO taxonomy was coupled with Rasch modeling to assess the learning achievement to observe the understanding of a subject at increasing levels of complexity and were carried in terms of Pilot study to test instruments and procedures in 20 schools of 11 countries of UNICEF constituted to the sample about 1200 students. Kayani et al. (2010) study the teachers' perception through questionnaire about the newly introduced SOLO taxonomy based examination system. Sample of the study consisted of twelve schools (eight from rural areas and four from urban area) from each of the 30 district and total 360 teachers. Bhattacharyya et al. (2012), the Qualitative Outcome of Learning (QOL) measures the level of attainment in learning efforts through SOLO Taxonomy in geometry for secondary school children studying under the Central Board of Secondary Education curriculum and the learners were challenged to test their understanding in lessons.

Mostly all the reviews of the aforesaid studies concluded with the positive favour about the SOLO Taxonomy. Studies concluded and recommended as: Bennett (1987) concluded as a study was fruitful and confirm the taxonomy as a useful research instrument in this form and as a means of determining performance quality in ways which could be of value to teachers in classrooms. Chick et al. (1988) found it useful for teachers in analyzing students' mistakes, showing the potential for facilitating the comparison of students' solution with 'ideal' or correct solution, identifying the conceptual difficulties and misconceptions and examining the way of students in using the information given in the problem. Gates (1994) portrays a growing concern of children's lack of appreciation (or poor attitude) and poor understanding of Mathematics. These figures tentatively suggest that prior Mathematical knowledge has an influence in Mathematical understanding of higher order levels. Boulton et. al. (1996) stated as its highly regarded in the present study is the ability to distinguish between deep and surface learning. Peter & Yeen (1996), the results of the study have implications for a criterion-based approach in interpreting test results based on SOLO testlets. Jones et al. (1997) reported as the levels of thinking appeared to be in agreement with level of cognitive development based on SOLO model and provided a theoretical foundation for designers of curriculum and assessment program in elementary school probability. Levins (1997) stated as this SOLO model enclosed the growth of basic understanding about the concepts must have in place before the transition to the more demanding abstract ideas. Chick (1998) concluded that outcomes indicative of creative cognition can be evaluated using the SOLO Taxonomy and that the levels of the taxonomy reflect the worthiness of the results. Burnett (1999) agreed as this approach may have implication for the process of counseling because counselors can use this technique to enhance their clients' learning. Lake (1999) reported as SOLO model was useful to be designed to classify the problem solving processes by stages (unistructural, multistructural, relational and extended abstract) and adapted to provide a useful four-step template of generalized questions that leaded students from the basic skills to critical analysis. Reading (1999), results were determined based on the SOLO levels and concluded for the study as younger students were more likely to use judgments in their descriptions while older students were more likely to use statistics. Pegg (2001), attainment of SOLO levels wise questions responded correctly help to determine the solving ability of the learners. Chan et al. (2002), they discovered that the classification of levels that sub-levels were added to SOLO model that would reduce the ambiguity and increase agreement between rater (inter-rater reliability). For instance, presturctural, unistructural, multisturctural moderate level, multistructural high level, relational moderate level, relational high level and extended abstract level. Mooney (2002) found that 5/12 of the students achieved the same SOLO level of thinking for any three out of the four statistical processes. Vallecillos & Moreno (2002) SOLO level helped to demonstrate that higher age students performed better than lower age students in constructing various statistical concepts. Hattie & Brown (2004), the key issue addressed in this report is how to devise an assessment model that values a balance of surface and deep processing. Like most taxonomies, SOLO describes the processes involved in asking and answering a question on a scale of increasing difficulty or complexity. Lian & Idris (2006) The results of this study provided evidence on the significance of SOLO model in assessing algebraic solving ability in the upper secondary school level. Aoyama (2007) SOLO level helped to conclude that the degree of abstraction that was utilized by the learner in handling the element of the tasks closely related to the stages of development. Serow (2007) results were determined using SOLO level showing positive favor to get idea about level of difficulties. UNICEF (2007) pointed as the SOLO taxonomy helpful though schools can determine how well a curriculum is being mastered by individual students and how well teachers are doing comparatively based on student performance. Kayani et al. (2010) state that new examination system based on SOLO Taxonomy had a lot of strengths. It’s a new concept so comprehensive training program may be arranged for teachers. In order to examine effectiveness of SOLO examination system more research may be conducted. Bhattacharyya et al. (2012), the focus of this study was on the change in the QOL following the use of different learning tools employing SOLO taxonomy.

2.6 RESEARCH TRENDS AND RESEARCH GAPS: MATHEMATICS

From all the reviews and summaries mentioned above, it is learnt and gained insights about the research trends were there for the various concerns of the Mathematics. Also, it is learnt about the research gaps which are presented below. But before that and in the same link, some overview has been given here about the status of research in Mathematics in Indian context.

As an important agenda for Mathematics education in India is Research in Mathematics Education. **Arunachalam (2001)** had analysed towards the same in a research study on *Mathematics research in India* -as reflected by papers indexed in *Mathsci* (1988-1998), is quantified and mapped. India has a very long tradition of excellence in Mathematics and Astronomy and in modern times India has produced much work in both pure and applied Mathematics. According to Basil Gordon, Russia and the USA are the top contributors to the literature of Mathematics, followed by England, France, Germany and then India and China. Comments derived from the review studies are as: None of the universities have come up to the international levels. "We have failed in having institutions where high level research and teaching go together", says Seshadri. Echoes Varadarajan: "... the basic Mathematics curriculum has not been changed except perhaps in a cosmetic fashion and that there is a virtual famine of well-motivated and qualified teachers."

According to **Banerjee (2012)** in a research paper on “Innovations And Initiatives In Mathematics Education In India” abstracted with some suggestions as**:** In the absence of strong empirical evidence and sound theoretical background, policy formulation becomes a difficult task. This holds true for the **NCF (2005)** as well where studies are required to critically examine the translation of guidelines given in the document to the textbooks and in the classroom. A few small scale studies, carried out in the primary and middle school grade levels, do indicate that a lot needs to be still achieved to fulfil the visions of the document. This may also indicate the need to critically examine the underlying assumptions in the design of the framework and the textbooks and the organization of content across grades. One needs to address the question of children’s learning of Mathematics as a discipline (with certain concepts, ideas, language, symbols, ways of reasoning and arguing, dealing with abstractions and generalizations) till the middle school level, which may serve as the terminal point of education for many children in this country. One also needs to ponder whether changing the framework and revising the textbooks would automatically lead to the desired overall change. Teacher preparation continues to be the weakest link in our education system. The departments and colleges have not been able to come up with a good model of training teachers at both the pre-service and in-service levels. Simultaneously, efforts have to be made to develop capacities among teacher educators and administrators in the system. Similarly, assessment is another area which has not radically changed since the NCF (2005) came into being. This is also one area which needs serious rethinking and research.

So, in Indian context it is required to think to change the thoughts for conducting the researches and especially in Mathematics and Mathematics Education. It needs to emphasize more on researches at micro level in Mathematics Education. From the years, concerns are there to improve the interest and achievement for the Mathematics at school level as well as at higher level, but still there are gaps or lacking to identify the actual responsible factors for the disinterests or fears for Mathematics or for not acquiring expected grip or mastery on Mathematics. Some of the thoughts as learnt and derived from above all the reviews, findings and conclusions are pointed below in terms to look further to conduct researches in depth in terms to gain actual or concrete solutions.

- ► In Indian context, major of the researches were based on the cognitive aspects for the Mathematics education and very less focus had been given on Psychological or emotional aspects except attitude and motivation but that are also very less in manner. Also, found very less or no emphasis on the psychomotor aspects of learning Mathematics at higher level.
- ► Majority of the findings are with respect to the designing Achievements tests and Achievements in terms of gaining marks or grades are considered as the parameters or benchmarks to measure the learning of the subject Mathematics majorly with respect to the knowledge only.
- ► Assessment or Evaluation in the research study of mathematics is generally referred to as Achievement test and that is generally meant to the pen-paper

tests. Very less or no aspects of practical or situational based assessment have been found in the reviews.

- Majority of the responsible factors are termed for the poor learning in Mathematics are presented in terms of gender, socio-economic status, urban or rural criterions, attitude, readiness, infrastructures, ICT, teachers and teaching methods etc.
- More of the focus was on learning of the Mathematics and very less focus was on Understanding the Mathematics.
- Very less emphasis on various concepts and conceptual understanding of the Mathematics
- Majority of the studies were conducted in Mathematics were at primary, upper primary and secondary school level and very less studies were found for higher secondary or higher education level.
- Not much study found or researcher has not come across about researches conducted for the learning difficulties in the areas of Mathematics like calculus, function, derivatives or integration, limit etc.

2.7 IMPLICATIONS FOR THE PRESENT STUDY

The review of the researches collected in the context of Mathematics Education, Mathematics teaching-learning and SOLO taxonomy, were divided into five parts in order to evidently relate it with the present problem study. A brief implication stated below as:

In the first part, all the reviews revealed with various difficulties or misconceptions or misunderstandings or errors identified very critically within the areas or concepts or topics of the Mathematics, which taking places during the mathematics learning among the learners of any levels. At the first sight these difficulties or misunderstandings may look very common or casual mistakes that are generally ignored or neglected or undetected during the classroom teaching-learning due to many reasons but many times these misconceptions in learning remains and affect the prolong or lifelong studies of the Mathematics of the learners. These are the gaps where researchers yet really need to work to overcome or bridge the gaps with some new approaches for instructional strategies or methodologies. The insights gained through these research studies in the first part, were useful for the present study in order to design or develop the appropriate instructional strategy in terms to make efforts to address such misconceptions in Mathematics learning at some extent.

The reviews in the second part were collected only for some of the important attributes concerned with the Mathematics which are most relevant with the present study. These attributes considered are based on skills (problem solving, interactions, constructivism, manipulative); psychological aspect (attitude); and the evaluation (assessment and achievement). All the reviews of this part-II revealed about the importance of the learners' 'response/s' and factors related with it. Labinowicz (1985) and Yackel et al. (1990) concluded as student's activity and responses are always rational and meaningful to themselves. One of the teacher's responsibilities is to determine or interpret the student's "rationality" and meaning. For the present study also, more emphasis was on the responses or interactions as that were means of 'learning outcomes'. For the present study, the said responses were measured with respect to the levels of the said SOLO taxonomy.

The reviews in the third part are related with various instructional strategies or the teaching of Mathematics. Different ways, methodologies, various medium and medias were used to design and implement the instructional strategies or variations in teaching of Mathematics were practised in terms to improve efficacy. These ideas were useful for the present study in terms to gain novel ideas as well as to get appropriate directions to develop and implement the instructional strategies along with various or additional components of pedagogy.

In the fourth part, research reviews revealed with the insight for the importance of the 'Understanding or Comprehension' and the 'In-depth learning'. Many researches have been identified with the thoughts that maximum of the Mathematics learning is 'mechanical learning', 'rote-memorisation or learning', 'instrumental learning', 'mug-up learning' or 'surface learning' which needs to convert or transform into the 'relational learning', 'constructivist learning', 'higher order learning' and the 'deep learning'. Kenny & Silver (1997) found that one out of every two students thinks that learning mathematics is primarily memorization. These studies revealed as need to give more focus on 'understanding' and more on 'conceptual understanding' to improve learning in Mathematics. Insights from these reviews were useful to deal with the aspects of the ‘progressive understanding’, ‘deep understanding’ and the ‘conceptual understanding’ which was the most focused component of the present study.

In the fifth part, research reviews are on SOLO taxonomy. Mostly the SOLO taxonomy was used to design the testlets for the assessment of the learning and theses reviews revealed the positive aspects about the taxonomy in a fair manner. As the founder of the SOLO taxonomy advocated that five levels are structured to increase the complexity in understanding through the constructive alignment which improve the learning and understanding as well as the teaching at some extent and also it helps to measure the learning. Chick et al. (1988) presented that how the extended SOLO Taxonomy can be applied in analyzing error in a variety of mathematics topics. In order to illustrate the SOLO levels, task analysis maps were devised by Chick (1998) to describe the types of errors being made by students in solving the mathematics problems. Boulton et al. (1996) also used the SOLO Taxonomy to analyze student learning. The findings from this study indicated that 80% of student responses fit in the multi-structural level indicating that students need help in structuring the content of their learning to reach to a relational or abstract level. Thus these studies were helpful to understand the level wise attainment of the said SOLO taxonomy in the context of researches rather than theoretical and how many ways it could be employed. The findings as well suggestions were helped to learn to deal with the identified aspects appropriately in terms to minimize the misconceptions using the said taxonomy and to maximise the efficacy of the teaching-learning of Mathematics. And it helped while developing the SOLO Taxonomy based instructional strategy for the present study.

From the above overview on review of studies, the researcher found the gap or problem of ‘the need of systematic or stepwise development of understanding in Mathematics’. Thus to proceed further in the same direction and it’s also understood by the researcher that the levels of SOLO Taxonomy advocate the constructive approach and helps to organise the responses as well as the understanding of the learners. Also, the researcher didn’t come across any SOLO Taxonomy based academic practices or researches in India. With these considerations, the researcher was proposed to conduct the research on the implications of the SOLO taxonomy for the Mathematics teaching-learning. Hence, the researcher had use the SOLO taxonomy in designing and developing the instructional strategy as well as in the assessment criterions of this experimental research study.

●●●●●●●●●●