Abstract

One of the most prominent methods to thoroughly understand the complex quantum systems which show a stochastic behavior is the famous Random Matrix Theory (RMT). This theory was first developed by Wishart (1928) in the field of mathematical statistics. Because of its unique feature of universality, the theory of random matrix has found ample applications in the field of physics, mathematics, stock market, information technology, etc. In particular, RMT is unanimously accepted worldwide as an important tool to study the concepts of chaos at the very quantum level in many-particle systems. Previously, several groups like Wigner, Dyson, Mehta, Gaudin, Porter, French, Bohigas, etc. emerged with the applications of RMT in the branch of nuclear physics. Wigner studied the neutron excitation spectra of heavy nuclei obtained using neutron resonances and conjectured that for many body systems whose interaction is complex enough, the Hamiltonian representing the system should behave like a large random matrix. This conjecture was further proved to be correct which in turn strengthened the trueness of RMT, and it was manifested that the concepts of RMT can be useful to many branches of science. Well, for such a study one needs to have knowledge about different symmetry sectors in space and time like rotational symmetry and time-reversal symmetry. On the basis of these symmetries classified are the three classical Gaussian ensembles : (i) Gaussian Unitary Ensemble (GUE) (ii) Gaussian Orthogonal Ensemble (GOE) and (iii) Gaussian Symplectic Ensemble (GSE). These classical ensembles show invariance under different transformations and hence they behave differently, a detailed explanation is given ahead in the first chapter.

The interest in RMT increased when Bohigas, Giaonni and Schmit conjectured that RMT should be equally applicable to the spectra of all chaotic systems. Later on it was theorized by Haake and Stockmann that all the generic quantum systems whose classical analogue is chaotic follow random matrix statistics. In this way RMT was successful to find a path way for the study of systems like quantum billiards, metallic grains, Sinai billiards etc. Similarly the theory can also be applied to the system of many particles where interaction is up to m-body in character and they are dominative. But the inter-particle interaction is one-body or two-body in realistic many particle systems. From here came the concept of Embedded Ensembles (EE) which was first introduced by French and co-workers where the two-body ensembles are defined by representing the two-particle Hamiltonian by one of the classical ensembles. Further, the m ($m > 2$) particle H-matrix is generated by using geometry of the direct product structure. But for realistic systems, the Hamiltonian is an adjoint of a mean-field part and the two-body interaction part, hence termed as EE(1+2). Initially, using the Gaussian Orthogonal Ensembles, the fermionic systems were studied called EGOEs. On similar grounds, EGOE for bosonic systems are studied recently, called BEGOEs.

Bosons and fermions are bifurcated in many senses but we shall concentrate mainly on the most important demarcation which is based on the number of particles (m) that are allowed in a single particle state (N). Governed by the Pauli's exclusion principle, the fermions possess a limiting case called "the dilute limit" ($m \to \infty$, $N \to \infty$, $m/N \to 0$) where only one particle is allowed per state. Conversely for bosons in addition to the dilute limit we have "the dense limit" ($m \to \infty$,$N \to \infty$, $m/N \to \infty$) which allows the accumulation of more than one particles in a particular single particle state. For the present investigation we shall focus on the dense limit of bosons using three bosonic ensembles and dilute limit of fermion using one fermionic ensemble, with different spin degrees of freedom for each. The basic construction for the Hamiltonian of these ensembles is shown and discussed are various spectral properties based on the eigenvalue spectrum of the system. The whole work here is

done by computational stimulation using FORTRAN codes. Further discussed is the chapter-wise content of this thesis.

Chapter 1 begins with the introduction of how and when the Random Matrix Theory (RMT) came into existence. Also discussed is about its universality and hence applicability to the complex systems whose behavior is undetermined. In this chapter we shall learn about the different symmetries and then switch over to the classification of the classical Gaussian ensembles (GUE, GOE, GSE). Here we shall also discuss about the rise of Two Body Random matrix Elements (TBRE) by French and group which became an initiative to study the Embedded Ensembles (EE). The chapter includes a review of various investigations done in past and then gives an outline of the different bosonic and fermionic models considered in this thesis.

Chapter 2 brings up a basic model of m spinless bosons with k-body interaction, BEGOE(k). Choosing a finite system of spinless bosons, the Hamiltonian is constructed and thereby generated are the eigenvalues. From these energy eigenvalues, studied is the shape of the eigenvalue density. Also, a transition in the density is studied for different body ranks (k) and the point of transition is computed. Moreover, the short range spacing statistics namely the Nearest Neighbor Spacing Distribution (NSND) is also studied for different body ranks.

In Chapter 3 we talk about a one plus two-body bosonic gaussian orthogonal embedded ensemble where the bosons possess a fictitious spin, BEGOE(1+2)-F. Here, each boson is assigned with a fictitious spin of ($F = 1/2$) so the single particle states Ω are doubly degenerate. Again we choose a finite system of bosons and construct the Hamiltonian of the system. For such an ensemble, NNSDs are constructed and then using the Brody distribution, we obtain the chaos markers for different spins. Also, the long-range statistics (Δ_3) is studied to observe a transition from integrability to chaos. The analysis is done for varying strength (λ) of the two body part. At the end one can also observe the 1/f noise behaviour for the ensemble.

Further in Chapter 4, we progress our work by introducing a one plus two-body

bosonic gaussian orthogonal embedded ensemble, but this time the bosons possessing spin one degree of freedom. We consider finite number of bosons which are to be distributed in Ω number of single particle states that are triply degenerate. The embedding algebra is given as $U(3\Omega) \supset G \supset G1 \otimes SO(3)$ with $SO(3)$. A method for constructing the ensemble for a given (Ω, m, S) has been developed here. The analysis part covers the nature of state density for fixed-(m,S), the behaviour of nearest neighbour spacing distribution and fluctuations in energy centroids and spectral widths. In accordance with the strength of the two-body part (λ), the change in nature of the spectral properties is also probed. Moreover, two types of pairing in BEGOE(1 + 2)-$S1$ space are introduced and some numerical results for ground state structure and pairing structure are presented.

Switching over from bosons to fermions Chapter 5 presents a fermionic ensemble. A way different from the perviously discussed systems, here we deal with the contribution coming from the three body part. In other words, we have a one plus three-body fermionic gaussian orthogonal embedded ensemble where the fermions are assigned with spin $s = 1/2$, hence called EGOE(1+3)-s. The two-body part is neglected here for a while and the strength of the three-body part is determined by the parameter (λ). Again, the construction part of the Hamiltonian is shown and various statistical properties are studied in detail. Starting from the nature of the eigenvalue density, we proceed to check the average-fluctuation separation in state density. Also, the behavior of spacing statistics like NNSD and Dyson Mehta (Δ_3) statistics are investigated for different interaction strengths (λ).

Now in chapter 6, a new method that is developed to study the Poisson to GOE transition using 'ratio of consecutive level spacings' $[P(r)dr]$ is discussed. The $[P(r)dr]$ method which serves as an alternative to the traditional NNSDs has a different concept of taking the ratio (r) of the consecutive energy level spacings. Using an interpolating matrix ensemble with a parameter λ giving Poisson ($\lambda = 0$) to GOE ($\lambda \to \infty$) transition, average of the ratio of consecutive level spacings has been

analyzed for matrix dimension $d \leq 1000$. Here we also introduce a new transition parameter denoted by Λ which is very important for the condition of universality. Along with that some examples of transition curves are presented for the models studied in previous chapters as well as for one-dimensional (1D) finite interacting lattice chain of spins-1/2.

Chapter 7 compiles the summary of every chapter briefly and discusses about the possible future prospects of random matrix theory and the concept of embedded ensembles.