Chapter 7

Conclusion and future outlook

In chapter 2, the BEGOE(k) ensemble of interacting spinless bosons is introduced for a finite system of bosons ($m = 10$) which are to be distributed in N number of single particle states. For these m interacting bosons, the the rank of interaction (denoted by k which is always $\leq m$) plays an important role for some of the spectral properties which is discussed in the chapter. Going ahead with the dimensionality of such a system, the basic construction of Hamiltonian is shown. Hence obtaining eigenvalues from the Hamiltonian matrix, various spectral properties which include the study of nature of eigenvalue density for different k values, the transition point where it makes a transition. Further, the nature of nearest neighbor spacing distribution (NNSD) is also observed. To conclude with we say that the state density for the spinless Bosons has a Gaussian to Semicircle transition as the body rank k goes from 2 to m ; with the transition point coming out to be $k = m/2$. But it is observed here that the NNSDs are found to be of GOE type irrespective of the rank of interaction (k).

Moving further towards bosons with spin, chapter 3 introduces a one plus two-body bosonic ensemble with fictitious spin BEGOE(1+2)-F. The $m = 10$ bosons having fictitious spin ($F = 1/2$) are distributed in $\Omega = 4$

number of single particle states that are doubly degenerate. Using mean field defined by random single particle energies, for fixed-(m, F) BEGOE(1 + 2)-F, density of states is found to be close to Gaussian for BEGOE(2)-F. In addition to it, for BEGOE(2)-F, one also observes the the average-fluctuation separation and calculated are eigenvalue centroids and spectral variances. Then analyzed are the spectral fluctuation properties, like Nearest Neighbour Spacing Distribution (NNSD) and Dyson-Mehta (3) statistic, as a function of λ- the two-body interaction strength. When we observe the behavior of the NNSDs, we see that for very small value of λ they are found to be Poisson in nature. But as the strength of λ increases, one can observe a transition to GOE indicating chaos. On the other hand, similar situation is also seen for theΔ_3 statistics. The two distributions (short and long range) are been analyzed for three different spins $F = 0$, 2 and 5. Moreover, spin dependence of the transition point λ_C is also obtained. It is seen that the value of the critical two-body strength parameter (λ_C) decreases as F-spin increases. Lastly, $1/f$noise behaviour of the BEGOE(1 + 2)-F ensemble is also verified using periodogram analysis and we come to know that the spectra exhibit $1/f$ noise.

Chapter 4 discusses about the BEGOE(1+2)-$S1$ Gaussian orthogonal ensemble of random matrices generated by random two-body interactions in presence of a mean-field for spin one boson systems. The significance of this model is well understood by its applicability for spinor BEC and to analyze generic structures generated by the 3rd version of the interacting boson model (called IBM-3) of atomic nuclei. In this chapter, the basic construction of Hamiltonian of this ensemble is explained and presented are analytical formulation with numerical results. The system chosen here is (m, Ω)=(10,4) where Ω are single particle states that are triply degenerate due to the spin one degree of bosons. Again, the reason for choosing a finite system is the computational constraints but the results are fair

enough to justify the properties of the ensemble; it has been verified for 200 members. First of all we discuss about the nature of eigenvalue density and conclude that in strong interaction limit ($\lambda = 0.2$) it is close to Gaussian. This is done for three spin values $S = 0$, 1, 2 and also for the lowest two moments (γ_1 and γ_2) of the two point function. Next analyzed is the Nearest Neighbor Spacing Distribution (NNSD) which shows GOE behavior again for a very strong two body interaction. Also, a transition from Poisson to GOE is observed as the value of λ increases gradually. It is possible to deal with much larger spaces if we use direct construction of H matrix in a good S basis. This is being attempted and using this in future a more detailed study with sufficiently large size examples will be reported. Also it is seen that the spectral widths are nearly constant for lower spins ($S < S_{max}/2$) and increase thereafter. The results for self-correlations in energy centroids ($[\sum_{11}(m, S : m, S)]^{1/2}$) and in spectral variances ($[\sum_{22}(m, S : m, S)]^{1/2}$) as a function of spin S/S_max are also presented. It is seen that the centroid fluctuations are large for $S = 0$ with $m \gg \Omega$ and decreases with increase in S value. For small m, the variation of $[\sum_{11}]^{1/2}$ with spin S is weak. For fixed m, $[\sum_{11}]^{1/2}$ decreases with increase in Ω. The value of $[\sum_{22}]^{1/2}$ are always smaller than $[\sum_{11}]^{1/2}$. For large m, with Ω being very small, the widths are quite large but they decrease quickly with increasing Ω. Thus, the width of the fluctuations in spectral widths is found to be much smaller, unlike the width of the fluctuations in energy centroids. Other than this, the preliminary aspects of one of the embedding algebras $SU(\Omega) \otimes SU(3)$ and also two pairing algebras in the space defining BEGOE(1+2)-$S1$ are discussed in this chapter. More detailed study of the effects of random interactions in presence of the two pairing interactions will be useful for gaining new insights into IBM-3 model of atomic nuclei [65, 66] and this will be discussed in future. In the same manner, the results of expectation values calculated for pairing

terms are equally important to study the IBM-3 model. It is seen that the expectation values are largest near the ground states and then decrease as we move towards the center of the spectrum. Extension of BEGOE(2)-$S1$ to BEGUE(2)-$S1$ and to the more restricted BEGUE(2)-$SU(3)$ with H preserving $SU(3)$ symmetry for spin one boson systems are possible [122] for preliminary results for BEGUE(2)-$SU(3)$. Finally, applications of BEGOE(1+2)-$S1$ ensemble to spin one BEC would be possible in future.

Chapter 5 brings up the analysis of a fermionic ensemble EGOE(1+3)-s which focuses on the contribution from the three body interaction. The applications of EGOE(1+2)-s to study the properties of multi-qubit systems give us a further motivation to probe the effect of the higher order interaction. The higher order (here up to 3 body) even though weak cannot be neglected. A method of constructing the ensemble EGOE(1+3)-s for a given (Ω, m, S)=$(8, 8)$ is employed in the good spin basis. From the eigenvalue spectrum the eigenvalue density of the system is calculated for fix value of three body interaction strength ($\lambda = 0.1$), which is strong enough to manifest about the chaotic regime. It is observed that the eigenvalue density (smooth part) is close to Gaussian irrespective of the value of spin, may it be highest or lowest. Also, apart from the smooth part of the density the average-fluctuations are evaluated for $S = 0$ and $S = 1$. It is seen that as the order (ζ) in the Gram-Charlier expansion is increased, the fluctuations exhibit GOE behavior. Moreover, talking about the spacing statistics both - short range (NNSD) and long range (Δ_3) statistics are studied here in the same strong interaction limit ($\lambda = 0.1$). For NNSDs, we observe a consistent Wigner form for every value of spin. Similarly, the Dyson Mehta (Δ_3) statistics also show the same pattern like NNSDs. All the analysis is done using 20 members due to computational limits. Hence these were some preliminary results for EGOE(1+3)-s, the chaos markers for EGOE(1+3)-s and other such properties will be studied in near future.

Lastly, Chapter 6 is a study of transition from the regular domain (Poisson) to the chaotic domain (GOE) but using a recently defined method of ratio of consecutive level spacings $[P(r)dr]$. The chapter initially introduces this method and compares it with the previously done traditional methods to actually locate the perfect regime of a quantum many particle system. Further introduced is an interpolating matrix ensemble with a parameter λ giving Poisson ($\lambda = 0$) to GOE ($\lambda \to \infty$) transition. For such a matrix ensemble the $P(r)dr$ has been studied for different matrix dimensions ($d \leq 1000$). We find that for the curves of $\langle r \rangle$ and $\langle \tilde{r} \rangle$ versus Λ, the transition from Poisson to GOE is faster as the value of dimensionality (d) increases. Results are shown for matrix dimension d going from 300 to 1000 (for 1000 members). This reflects the dependency of matrix dimension and hence cannot be called as Universal. Hence a new transition parameter $\Lambda \sim \lambda^2 d$ is introduced and the related averages $\langle r \rangle$ and $\langle \tilde{r} \rangle$ are plotted against Λ. One finds that the curves achieve a universal form independent of the matrix dimension. To verify the correctness of the new introduced parameter Λ, we make a mapping between the 3×3 and $d \times d$ matrices and the results are satisfactory. Apart from this we also verify the method of $P(r)dr$ to study the spacing statistics for some of the bosonic and fermionic system discussed in previous chapters and the results fairly match. At the end of this chapter we introduce one-dimensional (1D) finite interacting spin-1/2 lattice chains using Heisenberg's XXX and XXZ models. Here the chaos inducing factor is the defect site which is fixed at the middle of the lattice chain, the strength of which matters for the onset of chaos. The first model is an isotropic XXX Heisenberg model wherein we have $L = 14$ lattice sites and the defect site to be at $d = 7$. We see a clear transition from Poisson to GOE as the strength of defect ε_d is increased, also the behavior of the related averages are observed herein. The second model is an an-isotropic XXZ model investigated using $L = 18$ lattice sites, but the results go hand

in hand with the previous model. Hence for both the cases, the Poisson to GOE transition is observed using the given method of distribution of ratio of consecutive level spacings.