CHAPTER 2

REVIEW OF ULTIMATE LOAD THEORIES

2.1 Stress Strain Relationship.

(a) General.

Response of concrete to compression has been studied since long and experimental data is rich. A shape of the stress strain curve is influenced primarily by concrete strength, type of the cement and the aggregates, proportions of materials, size and shape of the test specimen, curing, age and rate of loading. A consolidated expression which can incorporate all these factors can be an ideal relationship. It is, however, difficult to accommodate all these considerations simultaneously. Hence different expressions in different situations are inevitable.

Stress-strain relationship in the form of triangle, rectangle, trapezium, hyperbola, ellipse, cubical parabola etc. are tried by investigators. Simplified relationships are used by investigators for achieving simplification in analytical approach. Empirical expressions are also used frequently.

(b) Summary of Stress - Strain Equations :

Stress - strain equations developed and used by some of the investigators (6, 14) are collected and presented here as under :

1. Saenz
$$f = E\epsilon\left[1 + \left(\frac{3E_0}{E} - 2\right)\left(\frac{\epsilon}{\epsilon_o}\right) + \left(1 - \frac{2E_0}{E}\right)\left(\frac{\epsilon}{\epsilon_o}\right)^2\right]$$

2. Saenz
$$f = \frac{E\epsilon}{1 + \left(\frac{E}{E_0} - 2\right)\left(\frac{\epsilon}{\epsilon_o}\right) + \left(\frac{\epsilon}{\epsilon_o}\right)^2}$$

(For general use)

3. European Concrete Committee ;
$$f = E\epsilon\left(1 - \frac{\epsilon}{2\epsilon_o}\right)$$

4. Torroja;
$$f = (0.43\ E\ \epsilon_o)\left[1 - \left(1 - \frac{\epsilon}{\epsilon_o}\right)^{7/3}\right]$$

5. Desayi and Krishnan :
$$f = \frac{E}{1 + \left(\frac{\epsilon}{\epsilon_o}\right)^2}$$

6. Tulin and Gerstle :

$$f = \frac{E\epsilon}{1 + \left(\frac{\epsilon}{\epsilon_o}\right)^n}$$

7. Baumann: $f = E\epsilon\left[1 - \left(1 - \frac{E_0}{E}\right)\left(\frac{\epsilon}{\epsilon_o}\right)\right]$

8. Popovics :

$$f = \frac{(n - 1)\ E\epsilon}{(n - 1) + \left(\frac{\epsilon}{\epsilon_o}\right)^n}$$

9. Shah and Winter :

$$f = E\epsilon\left(\frac{E_0}{E}\right)\left(\frac{E\epsilon - 2}{E\epsilon_o - 2}\right)^m$$

10. Smith and Young :

$$f = E\epsilon\ e^{-\frac{\epsilon}{\epsilon_o}}\ ; \quad \left(\frac{E}{E_0} = e\right)$$

11. Bach : $f = C_1\epsilon^n$

12. Sturman et al. :

$$f = C_2\epsilon\left(1 + C_3\ \epsilon^{n - 1}\right)$$

13. Liu, Nilson, Slate :

$$f = \frac{E\epsilon}{1 - \gamma\alpha}$$

14. Thattey : $\sigma = E_o \epsilon - \dfrac{E_o \epsilon^2}{2 \epsilon_u}$

where C_1, C_2, m, n, ... = experimental parameters

E = initial modulus of elasticity of concrete

E_o = secant modulus of elasticity of concrete at f_o ultimate stress ($E_o = f_o/\epsilon_o$)

f, σ = axial stress in concrete

f_o = ultimate stress

= unit strain in concrete caused by stress f

ϵ_o, ϵ_u = unit strain in concrete at ultimate stress f_o.

A brief discussion on the relationships is presented in the following. The relation given by Bach is a simple power function. It assumes a great initial modulus of elasticity. Saenz satisfies several boundary conditions for the ascending branch of the curve. A cubic term can be added to the relation to make it to represent a descending portion of the curve. Some of the relations are special cases to match specific conditions, like a fixed value attached to E/E_o. It is theoretically inappropriate to fix up any constant value for E/E_0 which is a ratio of modulus of elasticity to the initial one. This ratio is about .4 for normal concrete of 70 kg/cm^2 to about 1.3 for concrete of high strength like that of 700 kg/cm^2. A relation given by Desayi and Krishnan (Fig. 2.1) contains f which is stress at strain ϵ and ϵ_o which is strain corresponding to the maximum stress f_0. E is a constant assumed equal to $2f_0/\epsilon_o$. Saenz indicated that the relationship of Desayi and Krishnan is but a special case of the more general relation given by the former (Saenz). For low and high strength concrete, the expression gives poor correlation with experimental data. Saenz (10) subsequently proposed a general equation for all the grades of concrete. He gives

$$\epsilon_o = 10^{-5} \sqrt[4]{f_0}\,(31.5 - \sqrt[4]{f_0})$$

and
$$E = \frac{10^5 \sqrt{f_0}}{1 + 0.006 \sqrt{f_0}}$$

Tulin and Gerstle's (6) relationship is a generalised form of Desayi and Krishnan (1) equation.

Sturman et al. has suggested a value of $n = 1.9$ for eccentrically loaded specimens. Shah and Winter have given expanded form of equation given by Smith and Young (4).

(c) Unit Strain ϵ_o at Ultimate Stress (f_o) :

Some researchers like Ros, Emperger, Brandtzaeg, Jager, Sanez and the Hungarian Code have given formulae for estimation of ϵ_o i.e. unit-strain at ultimate stress f_o and E_o for concrete under compression. The fundamental agreement among them is that ϵ_o and E_o (Secant modulus of Elasticity) at the ultimate stress increase with the increase in strength of concrete. Measured values, however, show scattering results in regard to the performance of most of the formulae. The fact that the unit weight of concrete, which by itself is an important aspect is not given its due consideration in formulation of expressions is a reason for scattering results. However, useful information regarding peak values is obtained by such formulations.

Terzaghi formulates the strain relationship as $\epsilon = f/E + C_5 f^n$ while Ros has given the relation $\epsilon = f/E + C_6 f / C_7 - f$.

Hognestad et al. have formulated the relation to cover the complete stress-strain curve of concrete as

$$f = \epsilon \left(\frac{df_a}{d\epsilon} \right) + f_a$$

where the term $\frac{dfa}{d\epsilon}$ can be approximated by finite differences.

Sinha et al. presents a curve for unloading stress pattern as

$$f = \left(\frac{A_5}{X} \right) (\epsilon - X)^2 + A_6$$

where the parameter X is a function of the number of load repetitions, applied stress and the strength of concrete.

Several empirical formulae (4) to relate f_o and ϵ_o are presented below. Additional relationships can be obtained from these for other values of f_o and E_o. The relationships between stress and strain listed in article 2.12 can be conveniently used by application of the values of f_o and ϵ_o, presented here.

	ϵ_o, percent (f in psi)
Ros	$0.0546 + 2.56 \times 10^{-5} f_o$
Emperger	$3.7 \times 10^{-3} \sqrt{f_o}$
Emperger	$2.2 \times 10^{-3} \sqrt{f_o}$
Brandtzaeg	$f_o/(6800 + 2.6 f_o)$
Jager	$3.7 \times 10^{-3} \sqrt{f_o}$
Hungarian Code	$f_o/(7900 + 3.95 f_o)$

where, in addition to previous symbols, the following symbols are used

A_5, A_6, C_5, C_6, ρ... = experimental parameters

f_a = average compressive stress in concrete compression zone during flexure.

(d) Selection of Stress - Strain Curve of Concrete

Sandoor Popovics (6) has shown that the curvatures of various equations differ considerably. Selection of a suitable stress strain curve becomes easy in such situations. Lower the cement content, the more curved the diagram.

He has also observed that inspite of all variations, the design of reinforced concrete members for simple flexure is not considerably influenced by the diagram of concrete.

Descending branch of stress-strain curve is incorporated in some design procedures because maximum concrete strain is greater than the value of ultimate strain. In view of less marginal effect, the descending curve can justifiably be omitted.

Attempts have been made to assess the strain behaviour of concrete by models. Model approach has its own weakness though it is simple.

If modern methods of analysis are to be used, biaxial stress strain formulations is necessary. Such formulation incorporates poisson's effect and micro-crack confinement effects. Liu and Nilson (7) have proposed biaxial stress-strain law which compares well with experimental results. The matrix constitutive equations are suitable for plane stress finite element investigations. Nilson in his thesis (10) uses a constitutive matrix equation for eccentrically loaded members. A nonlinear response in a biaxially loaded member is confirmed but unfortunately no useful information has yet been obtained. It is reasonable to suppose that the stress intensity in one direction also affects the tangent modulus in the direction normal to it. With regard to the interaction between inelastic behaviour in one principal direction and the properties in the normal direction, the present state of knowledge does not permit a final statement.

Farah and Huggina (11) have used a continuous fourth order polynomial fitted by the least square method to stress strain data obtained from tests on concrete cylinders. The stress strain curve used is of the form

$$f_c = f'_c \sum K_i \epsilon^i$$

where K_i is a constant, (i = 1, 2, 3 and 4), and f_c = stress in concrete, f'_c = maximum concrete strength.

The curve shows an excellent agreement with Hognestad's curve. The relationship between the stress and the strain provides information concerning the failure mechanism of concrete.

(e) Functional Form of Stress Strain Curve

Everard and Thattey (12, 13, 14) have used stress strain relationship in a functional form of second degree polynomial upto the point of maximum stress, as under

$$f = \sigma = Ae + Be^2,$$

where e is strain at a point at stress f in concrete. At origin

$$\frac{d\sigma}{de} = E_o$$

and at ultimate load condition

$$\frac{d\sigma}{de} = 0$$

which reduces the functional form into

$$= E_o e - \frac{E_o^2 e^2}{4 \sigma_u}$$ (See Fig. 2.3).

Simplified forms like triangle, trapezium, rectangle, etc., neglect the actual behaviour of concrete under loading. Desayi (1), Torroza (6) and other formulations lead to elaborate expressions, Baumann's equation does not possess flaxibility for derivations. Some functional and other formulations have large exponential forms which may be deleted in view of the fact that refinement to that extent is not necessary. Liu and Nilson (7, 10) formulations are out of question as they are best suited for finite element analysis. Terzaghi and Ros equations (4) are complicated in mathematical forms. Hognestad et al. (4) have suggested a numerical approach of finite differences which is not within the scope of the work.

Biaxial stress strain relationship is fundamentally nonlinear. A functional form is suitable. The functional form of the relation as given by Huggins and Farah (11) is ideally suitable. However, the fourth degree relation may be abandoned and hence Everad and Thattey's formulation is accepted for the present work. They used the relation for analysis of beam-columns subjected to uniaxial bending and thrust. The theoretical results have shown good agreement with experimental work.

2.2 Ultimate Load Theories

(a) General

Ultimate load and moment capacities of structural members are estimated by ultimate load theory. Elastic theory is practically replaced by a plastic theory. It is established that an elastic or straight line theory does not give reliable predictions. The factor of safety is uncertain, while on the other hand, an ultimate approach permits lower load factors in case of certainty of loading, and moment thrust behaviour is predicted more accurately. It is also concluded that 30 to 40% saving is achieved by the use of the ultimate load approach (31), for both axial and eccentrically loaded members. A case study of a six storey framed structure carried out shows that the savings can be as much as 42% in steel if ultimate design procedures are adopted. Since the use of ultimate load design method is by IS : 456 - 1964 and other international codes. There is no reason why the saving should not be economically realised (54). Moreover, the ultimate load design procedures have the merit of rationlity and simplicity. The ultimate strength of a section is affected by stress redistribution due to shrinkage, creap and cracking.

(b) Background of Ultimate Load Theories

Study of reinforced concrete columns, loaded axially and eccentrically, is reported extensively in literature. Survey of classical and recent approaches suggest that the analytical as well as the empirical procedures are developed, for biaxially loaded compression members. Some of the theories are briefly reviewed.

(c) Thattey's Theory for Uniaxial Eccentricity (14, 61)

Starting from a functional form of stress-strain curve of concrete suggested by K. A. Everard, a theory for members subjected to uniaxial eccentricity is developed by Thattey. Relationship among the several important variables like the neutral axis depth factor, ultimate load, ultimate moment for several parameters is obtained. Typical interaction curves are presented.

The equations for load factor, moment factor, and neutral axis depth factor are developed for three cases as under.

(i) Tension failure case accompanied by the yield of tension steel and no yield of compression steel.

(ii) Compression failure case accompanied by the yield of compression steel and no yield of tension steel.

(iii) Balanced design case in which crushing of concrete is accompanied by yield of both compression steel and tension steel.

The functional form of second degree stress strain curve is used. (See Fig. 2.3)

The load factor P_u, and the moment factor M_u, are in the form as under at the ultimate condition.

$$P_u = A + B \cdot mp$$

$$M_u = C + D \cdot mp$$

where A, B, C and D depend on the position of neutral axis, ratio of compression to tension steel, amount of reinforcement, and the stress ratio. Relation between ultimate axial force and flexural rigidity at origin and the concept of local curvature are also developed.

The theory is compared with other works and the results of P_u, and M_u, are tabulated (61) as under :

Theory	P_u,	M_u,
Mensch	0.510	0.346
Dyson	0.785	0.453
Whitney	0.458	0.333
Cox	0.470	0.360
Jensen	0.578	0.411
Baker	0.480	0.365
Jain		
e_u = 0.003	0.534	0.366
e_u = 0.006	0.663	0.405
Thattey		
e_u = 0.003	0.460	0.341
e_u = 0.006	0.547	0.379

(d) Ultimate Strength Design Charts for Columns Controlled by Tension (Uniaxial Eccentricity) (60).

Tung Au has developed basic equations for design of eccentrically loaded concrete columns (uniaxial) based on ACI - ASCE Committee's Report on ultimate strength design. The charts can be effectively used for columns controlled by tension, and for rectangular sections with symmetrical reinforcement for square and circular sections with spiral reinforcement subjected to combined bending and axial loads and controlled by tension.

The charts give relation between $\frac{P_u}{f'_c\, bt}$ and mp for values of

$$\frac{P_u e'}{f'_c\, bt^2}.$$

(e) ACI Design Charts (45, 62)

In practice the design and analysis of column sections can be carried out quickly using design charts and tables. A compressive series of charts and tables have been published by the ACI for columns with uniaxial eccentricity.

The design charts are sets of interaction diagrams that plot the ultimate load and moment in dimensionless form. Known column size, material strength and ultimate load and moment, a chart can be used for getting a value of mp, which specifies the area of steel.

These charts have $\frac{P_u}{f'_c\, bh}$ and $\frac{P_u e}{f'_c\, bh^2}$ on axes with parameters of mp and eccentricity ratio to be fixed on curves.

The ACI design charts and tables were determined from first principles, using the conditions of equilibrium and compatibility of strains. The steel does not yield at ultimate loads. The steel is assumed as placed uniformly distributed in a thin tabular shape.

(f) Approximate Methods

Approximate methods for biaxial bending fall under three headings. (a) Method of superposition, (b) Method of equivalent uniaxial eccentricity and (c) Approximations of the shape of interaction surface.

(a) Method of Superposition.

Inclined bending is reduced to bending about the major axis of the section thus allowing the use of procedures for uniaxial bending. In this method the reinforcement required for each of the loading cases (P_u, M_{uy}) and (P_u, M_{ux}) is determined separately and added algebraically. This method has no theoretical basis and should not be used since it may lead to large errors on the unsafe side because the full strength of concrete is taken into account twice in the design. This method is used in the code of Venezuela, (34) with some modification. Sometimes the load P_u is replaced by two statically equivalent forces P_{ux} and P_{uy} acting for each axis separately. The reinforcement required for each of the loading cases P_{ux} with the concrete strength of $\frac{f'_c P_{ux}}{P_u}$ and P_{uy} with the concrete strength

$$\frac{f'_c P_{ux}}{P_u}$$

are determined separately and added together. Although this approach has no theoretical support, Moran comments that the solutions obtained in the considered cases seem to be satisfactory (34).

(b) Method of Equivalent Uniaxial Eccentricity

In this method an equivalent uniaxial eccentricity is obtained by an interaction line for a given rectangular section with biaxial bending. The ultimate load for any point of application (e_y, e_x) on such line is the same as the ultimate load for a point of application with uniaxial eccentricity e_o. If the shape of the interaction line is known, it would be possible toddesign for the load P_u acting at equivalent uniaxial eccentricity e_o.

Moran quotes the equation adopted by the Spanish Code 1968, for the determination of equivalent uniaxial eccentricity.

$$e_o = e_x + \left(\frac{1-\beta}{\beta}\right) e_y$$

where β is a factor depending on the level of axial load and the steel content tabulated in the code (See Fig. 2.4).

(c) Approximations in the shape of the interaction surface.

Various suggestions have been made for the shape of the interaction surface from which, knowing the uniaxial strengths, the biaxial bending strengths may be calculated.

The following expression is given in the Russian Code,

$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_o}$$

where P_u is the ultimate load under biaxial bending,

P_{ux} is the ultimate load under eccentricity e_x only.

P_{uy} is the ultimate load under eccentricity e_y only.

P_o is the ultimate load when there is no eccentricity.

This expression has disadvantage of being more suitable for analysis than for design. Bresler (34) found that test results do not deviate more than 9.4%.

It is also suggested that the family of interaction lines corresponding to the various levels of constant load P_u can be approximated by the equation

$$\left(\frac{M_{ux}}{M_{uxo}} \right)^m + \left(\frac{M_{uy}}{M_{uyo}} \right)^n = 1$$

where M_{ux} is $P_u \cdot e_y$ and M_{uy} is $P_u \cdot e_x$ and M_{uxo} and M_{uyo} are the uniaxial flexural strengths about the x and y axes for the constant load. The constants m and n depend on column properties and are determined experimentally.

Parma et al. (36) modified the above equation and stated it as follows :-

$$\left[\frac{M_{ux}}{M_{uxo}} \right]^{\frac{\log 0.5}{\log \beta}} + \left[\frac{M_{uy}}{M_{uyo}} \right]^{\frac{\log 0.5}{\log \beta}} = 1$$

where β is the parameter of the shape of the interaction line. The effect of different values of β on the shape of the interaction line is represented in Fig. 2.5.

Other suggestions for the shape of the interaction surface have been made by Pannell (37) and

Furlong (38). John Meek (25) has also contributed for the approach of interaction surfaces.

(g) Meek's Approach (25)

An approximation to interaction surface of M_{uy} and M_{ux} at constant load P_u is suggested by Meek (25). He has indicated that the curved interaction line can be replaced by two straight lines. Accordingly an interaction curve A B C, where all the three points are known, can be replaced by a straight line A B and a straight line B C.

For rectangular reinforced concrete column, Meek has generated interaction curves for $P_u - M_{ux}$ and $P_u - M_{uy}$ and for a given ratio of $M_{uy}/M_{ux} = \alpha$, various values of P_u are calculated. A horizontal line or curve is generated on the interaction surface. For rectangular section this contour line will be some type of curve perhaps an ellipse. Its shape will depend on dimensions of the column, the amount of reinforcement and its location, and the eccentricity of loading. Such contour lines are developed by Meek.

It is observed that the concept of the interaction surface exemplified by Meek is useful (see Fig. 2.6). Set of curves given by Meek will reduce difficulty of analysis and design of biaxially loaded columns.

(h) Tung Au Theory (16)

The procedures of proportioning rectangular reinforced concrete sections subjected to unsymmetrical bending in two directions based on ultimate strength method are formulated by Tung Au (16). The theory for its derivation assumes the different locations of neutral axis. Charts are provided to simplify the procedures of evaluating the dimensions of the equivalent compressive stress block from the equations. Solution of problem is obtained by first assuming the dimensions of the section and the arrangement of the reinforcement and then by checking the stresses to verify the validity of the assumed section. Such work often requires several repetitions but can be simplified by use of the charts. The equations developed are expressed in the form of dimensionless quantities. The theory assumes equivalent rectangular stress block. Fig. 27 shows section detail and Fig. 2.8 gives a typical chart for designs.

(i) Fleming and Werner's non-Dimensional Design Curves (47)

It is a simplified ultimate strength method of design for columns subjected to bending about both principal axes where a set of non dimensional design curves for one particular geometry is given for commonly encountered values of steel percentage, concrete strength and steel yield stress. The basic assumptions associated with the theory (47) are of standard nature namely rectangular stress block, negligible concrete tensile strength, linear strain distribution at ultimate strength and ultimate concrete strain of 0.003. The development of the theory is done by fixing one end of neutral axes and varying the other end by increments i.e. value of m in Fig. 2.9.

For every position of the other end of neutral axis, ultimate load and bending moment about the principal axis were computed. The first end of the neutral axis was then incrementally changed and all possible trials were executed. The equivalent rectangular stress block is used for the development of the theory.

The loading condition at ultimate was computed from the conditions of equillibrium. The axial load was found from summing forces while its eccentricities from the center of the section were found by summing moments about the extreme compressive concrete fibre.

(j) Huggins and Farah's Polynomial Stress Block (11)

Hinged reinforced concrete columns subjected to biaxial bending are analysed by an integration method which leads to three simultaneous non linear equations by Huggins and Farah (11). The equations are solved by Newton-Raphson method. The method assumes a strain distribution. Loads and moments on the section based on that assumption are compared with applied loads and moments. Initial strains are readjusted and new comparison is made. The procedure is continued until the equilibrium is obtained.

The stress strain curve used in analysis was a continuous fourth order polynominal fitted by the least square method to stress strain data obtained from tests on concrete cylinders. Stress-strain curve is discussed in para 2.1 (d). The ultimate concrete compressive strain is taken as 0.004 and the corresponding stress as 0.85 f'_c.

Stress strain curve in the above mentioned reference work for steel is taken as elastic upto the yield point with a line of constant stress after that stage.

The theory considers strains at three corners as ϵ_1, ϵ_2 and ϵ_3 (See Fig. 2.11). Strain at any point is defined with suitable origin and reference axes.

The method is cumbersome except that iterative process can reduce the difficulty. The procedure is to assume ϵ_1, ϵ_2, ϵ_3 and iterate until the correct strains corresponding to the given P, M_x and M_y are determined. The convergence is rapid. The ultimate loads predicted by the theoretical analysis vary from 2 to 5 percent less than that of tested values.

(k) Ramamurthy and Mallick Approach (24).

A concept of failure surfaces in columns is used for the method (24). The determination of ultimate load of square and rectangular reinforced columns under biaxial eccentric loads is direct.

The neutral axis varies both in direction as well as position with the result that the shape of the compressive zone varies from a triangle to a pentagon according to position of load. Nonlinear stress strain curve is used.

For any assumed position of the neutral axis, the expressions for the ultimate load and its moments about the principal axes reduce to the following dimensionless forms.

$$\lambda = A + B \cdot mp$$
$$m_x = C + D \cdot mp$$
$$m_y = E + F \cdot mp$$

where A, B, C, D and E are constants which depend on the position of the neutral axis and other variables. Fig. 2.12 shows (a) section (b) relation between and (c) stress strain relations and (d) interaction curve in connection with this theory.

(1) Weber, Row and Paulay Charts (34)

Weber has produced a series of charts for square columns bending about a diagonal. It is a most practical design method. Row and Paulay have improved accuracy of this process by using a more accurate concrete compressive stress distribution and have produced design charts, for biaxial bending on axes at various angles with the major axes, thus allowing linear interpolation between a number of points on the interaction lines Weber used conditions of equilibrium and compatibility of strains to derive from first principles, interaction curves of P_u versus $P_u e$ for square columns with the load applied at various eccentricities along the line of a diagonal of the section. Triangular stress block was represented by equivalent rectangular stress block. The charts are used for design of reinforced concrete columns. The charts can also be used for analysis, wherein moment capacities at angle θ can be calculated by linear interpolating between the uniaxial and the diagonal bending moment capacities. Good accuracy is obtained and the maximum error factor was 5.3% for steel area or moment capacity compared with the full theoretical solution using the equivalent rectangular stress block distribution. The charts are useful for square as well as rectangular sections.

Row and Paulay's Charts (Fig. 2.13) use the direction of eccentricity e of the load in a modified manner. They have developed a dimensionless parameter $K = e_x h / e_y b$ where $K = 0$ implies that the loads are on Y-axis and $K = 1$ implies that loads are on the diagonal and $K = \infty$ implies load on the X-axis.

Charts by Row and Paulay for a number of values for K enable the shape of the interaction surface to be obtained accurately. The reinforcement is assumed to be uniformly distributed as a thin tube with 0.25 A_{st} in each face of the section. The charts were calculated from first principles, using the conditions of equilibrium and compatibility of strains and assuming a stress strain curve for the compressed concrete which is parabolic upto a stress of $0.85\ f'_c$ at a strain of 0.002 and then has a constant stress of $0.85\ f'_c$ upto a maximum strain of 0.003 at the extreme compression fibre. The charts do not include the capacity reduction factor. In Weber's charts, a rectangular stress block was used, while in Row and Paulay's charts, parabolic stress pattern is used. Again Row and Paulay's charts cover various eccentricities. Steel requirements can be seen from the charts. It is observed that the steel found by Row and Paulay's chart (34) is greater than steel content determined using Weber's method. The difference is mainly due to the concrete compressive stress distribution assumed. Stress strain curve adopted by Row and Paulay is conservative as the compressive stress is about 8½ % less. Row and Paulay and Weber have used maximum strain of 0.003. However at maximum load the strain at the extreme fibre of a triangular compressed area will be larger. The combined effect of error makes the difference of 19% in steel areas when using the charts.

<u>(m) Medland and Donald Method (26)</u>

Theoretical expressions for compression, uniaxial bending and the flexural rigidity of reinforced concrete sections are developed by Medland and Donald.

The concrete stress strain relationship used by the authors is a polynomial relationship based on Taylor's curve.

The axial load P is calculated by

$$P = F_c + F_s - F_{ct} - F_{s2}$$

where F_c and F_s are load contribution by compression in concrete and steel and F_{ct} and F_{s2} are forces due to concrete in tension and force due to tensile steel. On a similar line, equation for moment is also developed.* Moment axial load interaction diagrams are prepared.

(n) Interaction Exponent (52)

Values of biaxial interaction exponent for concrete columns are determined by Shanmughasundaram where he has used the relation used by Brastle (35) and Pannell (37). The relation between moments about X and Y axes and uniaxial bending capacities are given as under :

$$\left(\frac{M_x}{M_{ox}}\right)^{\gamma} + \left(\frac{M_y}{M_{oy}}\right)^{\gamma} = 1$$

where M_x and M_y are moments about X and Y axes; and M_{ox} and M_{oy} are the corresponding uniaxial bending capacities and γ is the interaction exponent, which is a function of the uniaxial load, section and material properties of the column. Knowing γ for a given moment capacity about one axis, the moment capacity about the other axis is determined from the above relation. The author determined the value of the exponent γ. The author has presented a general approach for computing directly the interaction exponent γ for a rectangular reinforced concrete column at a given axial load. With the application of the theory developed (52), it is indicated, that the design and analysis of columns need not be restricted or approximated to the concrete and steel strengths, concrete cover and the pattern of the reinforcement for which tables and charts are to be used. The method is simpler and more practical compared to the tedious process of obtaining β first (36).

The author has used the stress strain relation given by Desayi and Krishnan.

The values of γ calculated for loads ranging from 62 Kips to 127 Kips are in the range of 1.215 to 0.935. The author used maximum concrete strength f_o as f'_c instead of 0.85 f'_c as the columns were cast horizontally.

2.3 Closure

An analysis of rectangular reinforced concrete columns for axial loads with biaxial bending has been a subject of interest and discussion for several years because it is one of the most important and difficult problems in reinforced concrete theory. Only a limited number of test results have been published. A rational method for calculations for ultimate load of biaxially loaded columns with eccentric loads has been given by Whitney and Cohen (42) and expanded by Mattock, Kriz and Hognestad (51). An approach of interaction curve is well known for eccentricity in one direction and several writers have suggested extension of that idea for biaxially loaded columns. Approximate methods of analysis and design for biaxial bending such as the method of superposition and equivalent uniaxial eccentricities are discussed by Park and Paulay (34). They have also given an excellent treatment of the subject by design charts of Weber, Row and Paulay.

A simplified design of rectangular concrete members by use of rectangular stress block and design charts is presented by Tung Au (16), where a procedure has been formulated for proportioning rectangular reinforced concrete sections subjected to unsymmetrical bending in two directions.

Huggins and Farah (11) have presented an integration method using interative procedure where those writers have used a polynomial form of fourth degree of concrete stress strain relationship. The problem has attracted attention of many outstanding researchers like Whitney (42), Pannel (37) and B.B.Breslar (35) and Furlong R. W. (38).

Ramamurthy and Mallick (24) have used a parabolic stress strain relationship with a maximum strength ordinate of 0.9 times the concrete strength and ultimate strain of = 0.0035.

Fleming and Werner (47) presented a simplified ultimate strength method of design of columns subjected to biaxial bending by using a set of non-dimensional design curves for one particular section geometry for commonly encountered percentage of steel and concrete strength.

Everard and Thattey (12, 13, 14, 61) have used the stress strain relationship of concrete as $f = A e + B e^2$ where f is the stress and e the strain in concrete for analysis of uniaxial bending and thrust. The analysis and experimental work (14) projects a unique theory for uniaxial bending with axial loads, by used of a functional stress strain relationship. The subject has an excellent previous background, as well a good scope for further work.

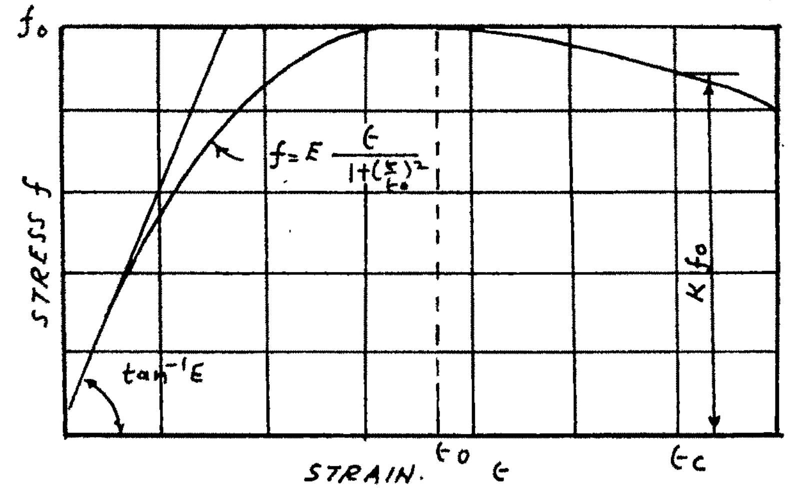


FIG. 2.1 STRESS- STRAIN CURVE (DESAYI & KRISHNAN)

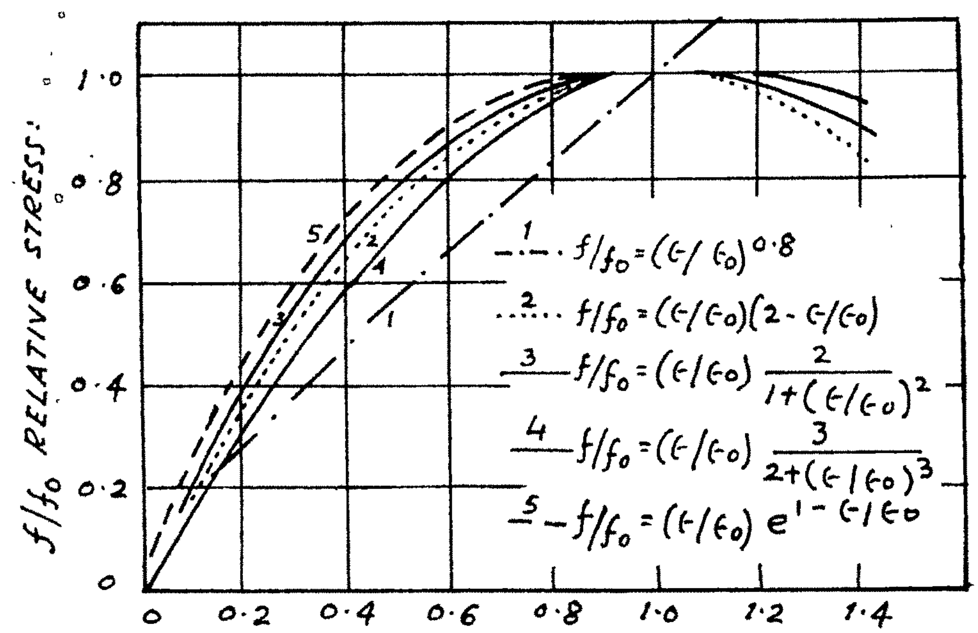
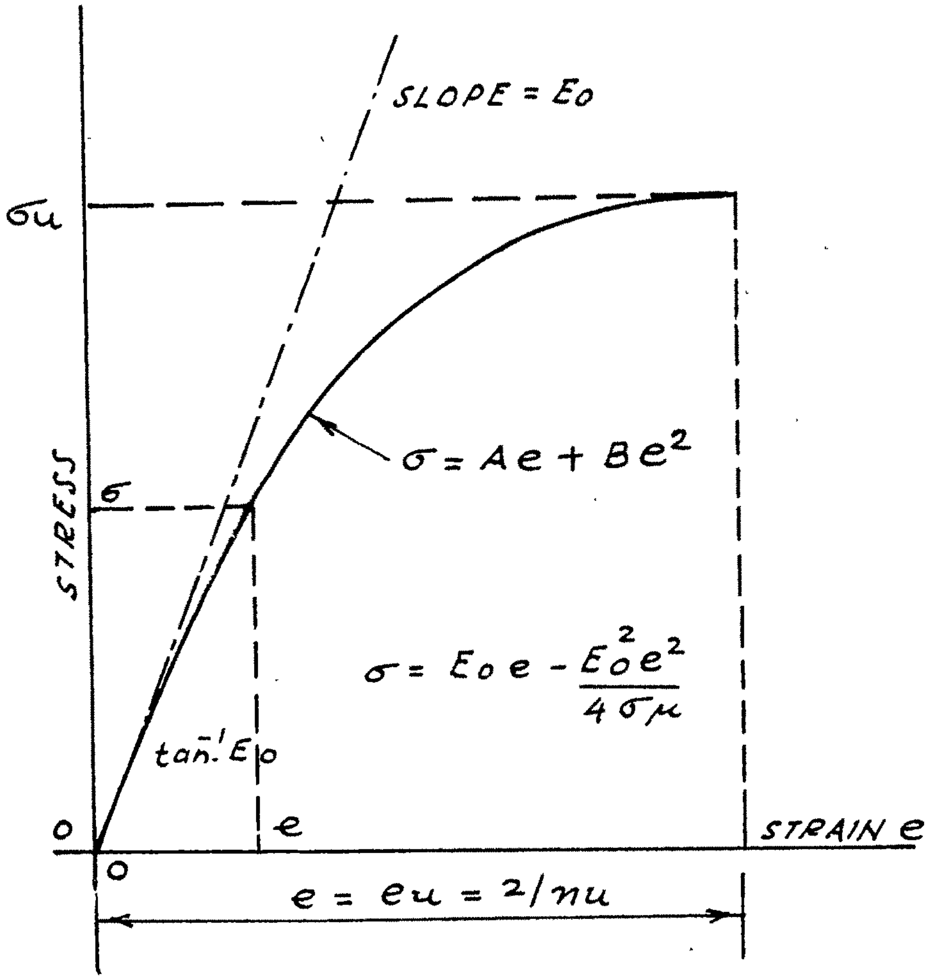
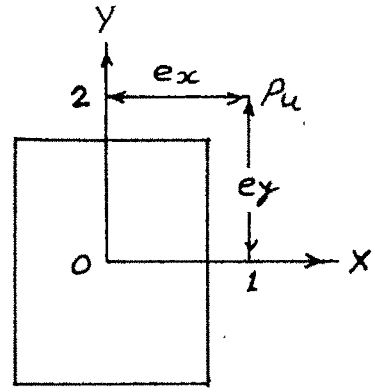


FIG. 2.2 COMPARISON OF FORMULAS FOR STRESS-STRAIN DIA. OF CONCRETE.

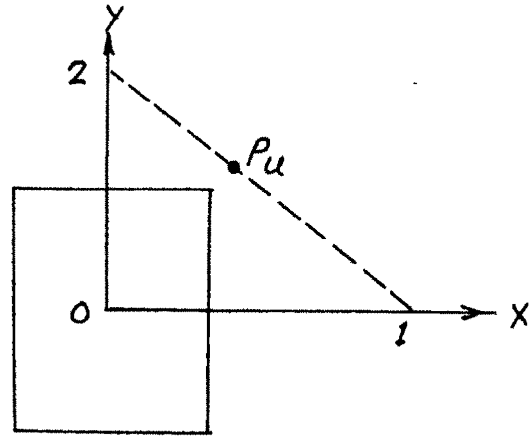


ASSUMED STRESS-STRAIN RELATIONSHIP FOR CONCRETE

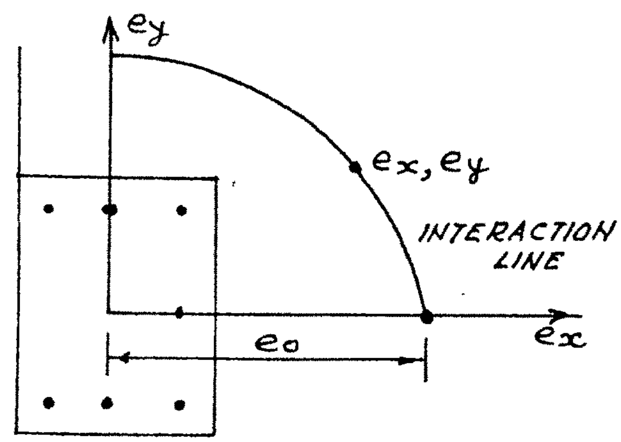
FIG. 2·3 FUNCTIONAL FORM



EQUIVALENT LOADING ECCENTRICITY



CODE OF VENEZUELA



APPROXIMATE DESIGN METHODS FOR BIAXIAL BENDING

FIG. 2·4

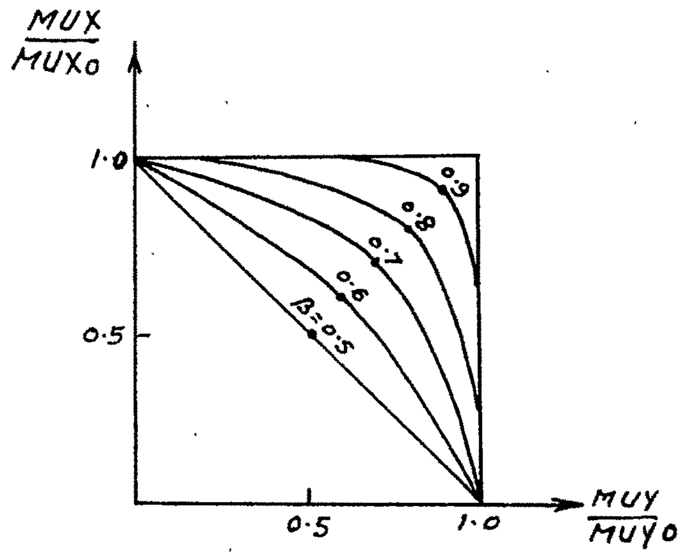


FIG. 2·5 INTERACTION LINES FOR COLUMN WITH BIAXIAL BENDING UNDER CONSTANT PU

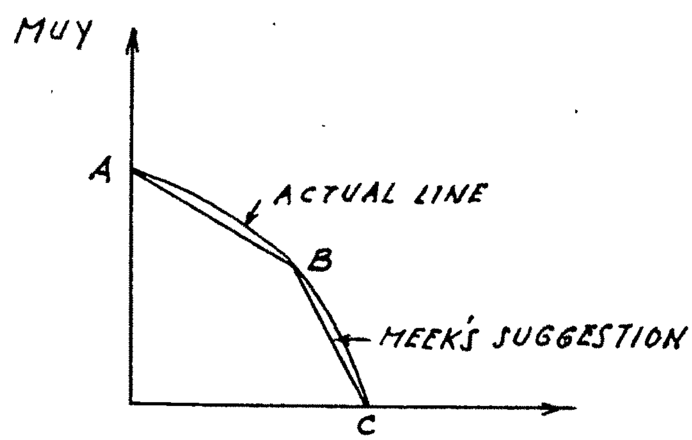


FIG. 2·6 CURVED INTERACTION LINE AND 'TWO STRAIGHT LINES REPLACEMENT'

INTERACTION LINE AT CONSTANT Pu,

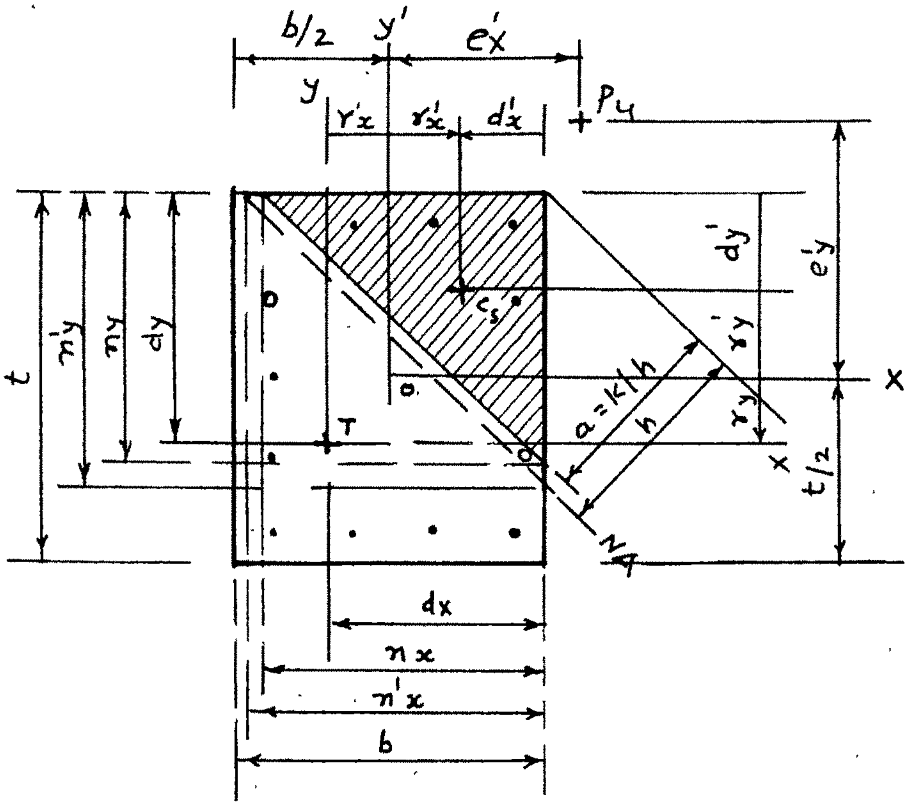
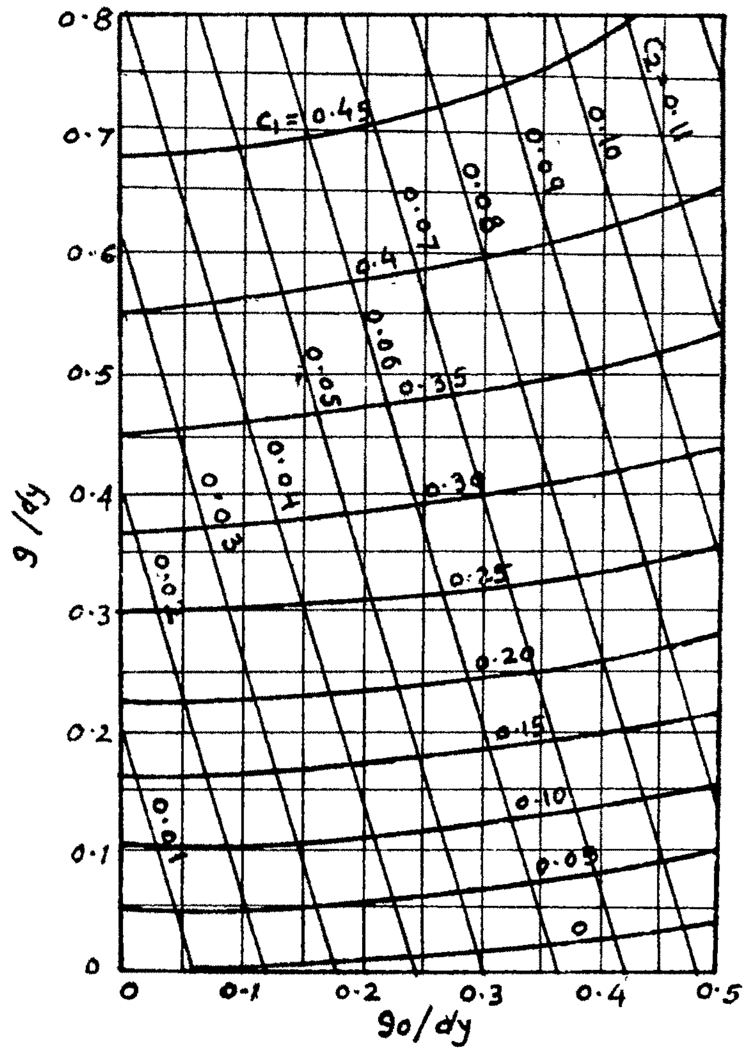


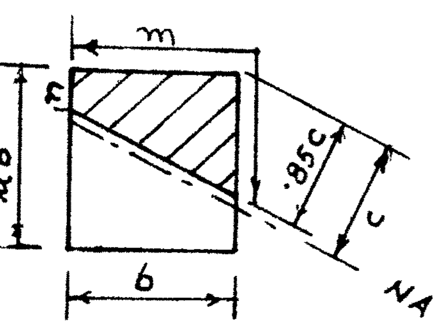
FIG. 2·7 SECTION DETAILS 'TUNG AU THEORY'



TYPICAL DESIGN CHART

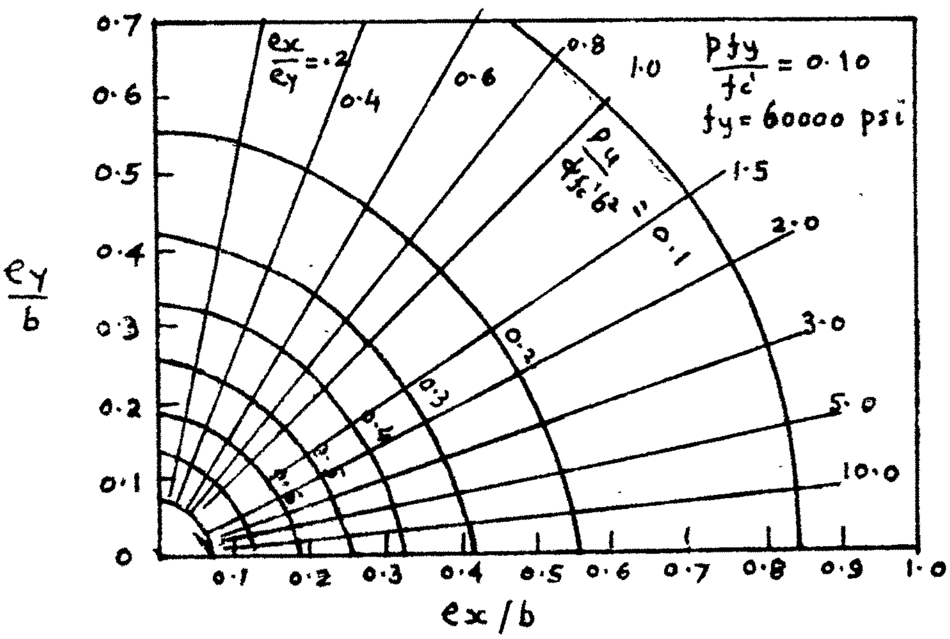
AU TUNG THEORY $r_x/b = \cdot 05$

FIG. 2·8



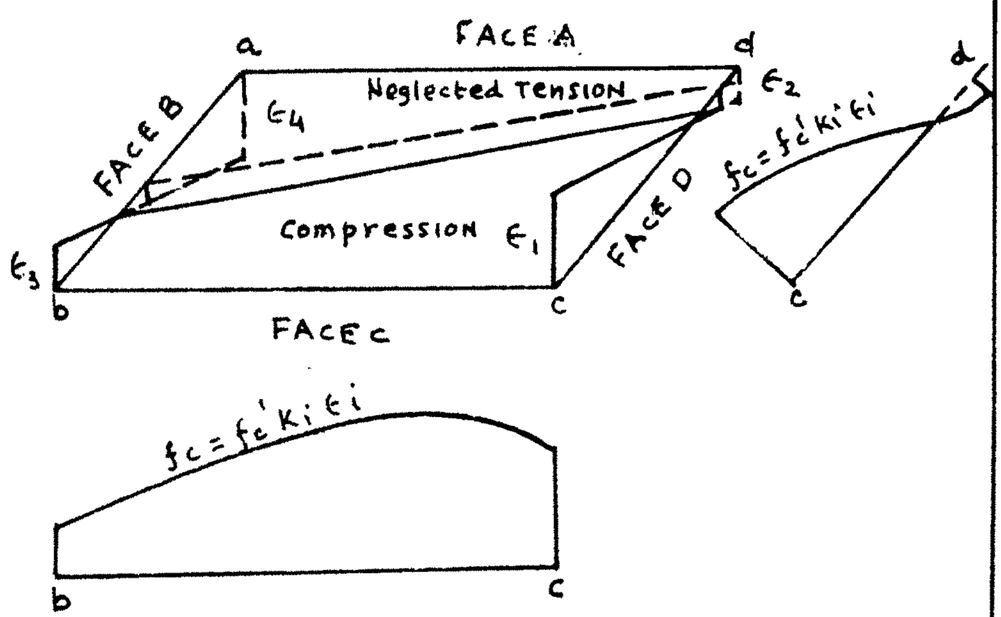
COMPRESSION ZONE FLEMING AND WERNER

FIG. 2·9



DESIGN CURVE
FLEMING AND WERNER

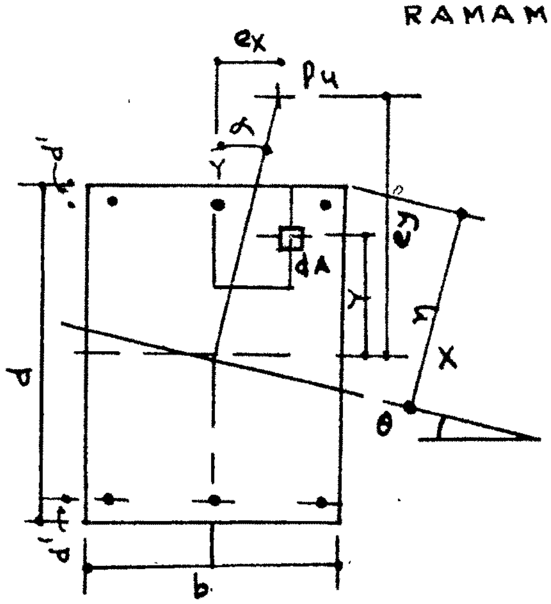
FIG. 2.10



STRESS STRAIN DISTRIBUTION
FARAH AND HUGGINS:

FIG. 2.11

FIG. 2.12

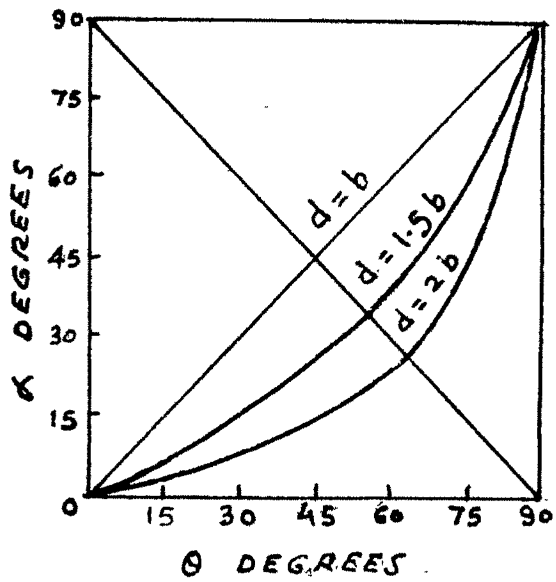


(a) COLUMN SECTION

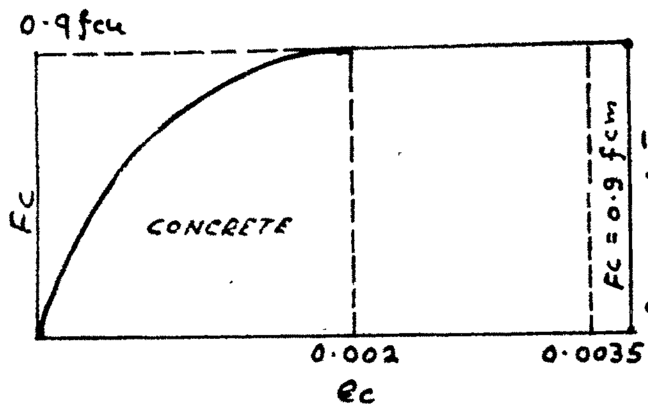
$P_u + T = C$

$P_u\, e_x + \Sigma f\, dA\, x = 0$

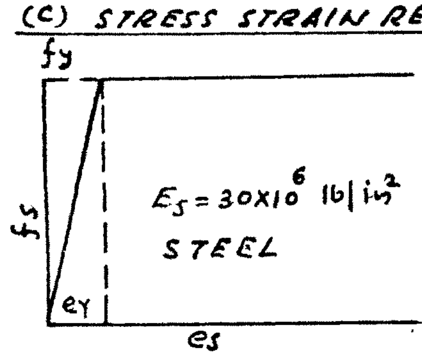
$P_u \cdot e_y + \Sigma f\, dA\, y = 0$

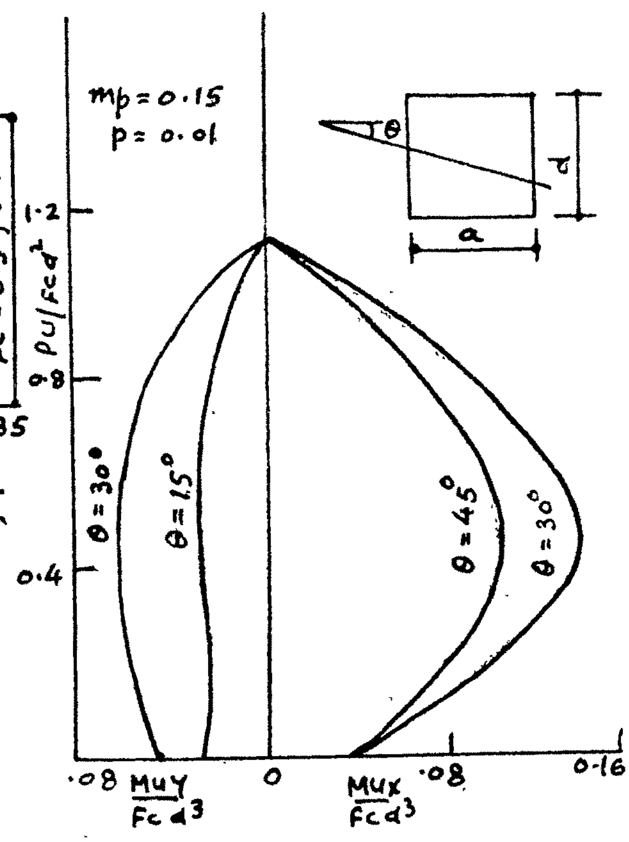


(b) RELATION BETWEEN θ AND α.

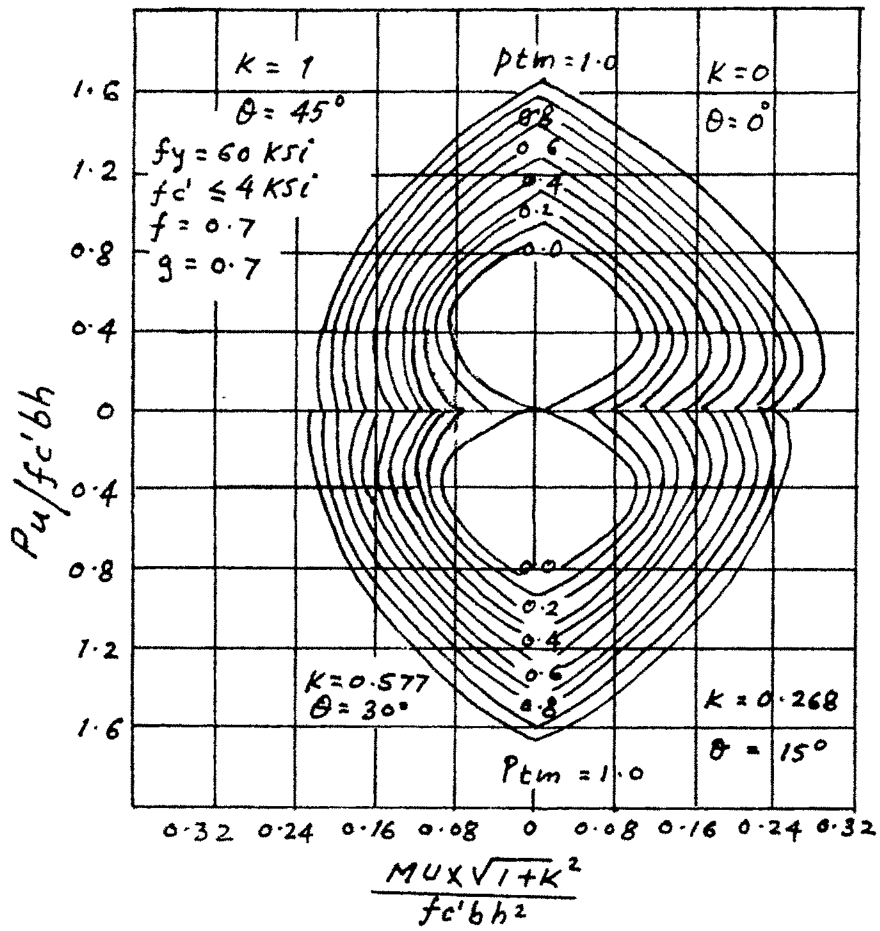


(c) STRESS STRAIN RELATIONS





(d)



DESIGN CHARTS - ROW AND PAULAY

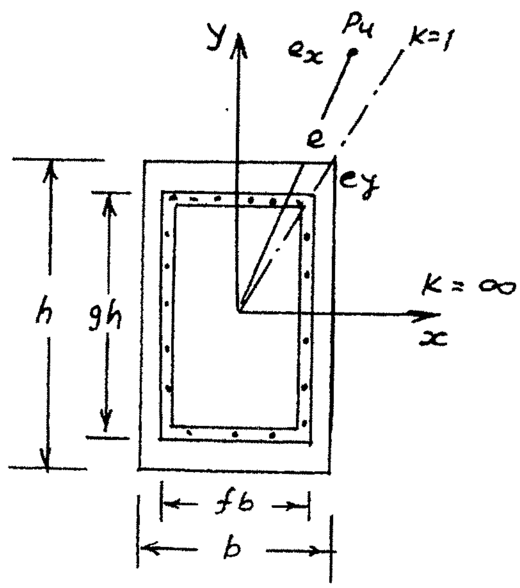


FIG. 2.13

COLUMN SECTION WITH BIAXIAL BENDING