CHAPTER 4

DEVELOPMENT OF THEORY

4.1 General

It is proposed to develop the theory of behaviour of reinforced concrete beam columns under biaxial loading by using the functional form of stress strain relation in the following. The assumptions and notations used in the development of the theory are listed in two paragraphs. Theoretical derivations follow in subsequent paragraphs. Summary of the theoretical equations is given in para 4.6. The neutral axis depth factor is determined by solution of cubic equation (see para 5.4, Chapter 5). Computer results are used to prepare interaction curves. Typical interaction curves showing the influence of some selected practical parameters on the ultimate load factors, the ultimate moment factors and the position of neutral axis are presented in Appendices E and F. An explanation regarding location of neutral axis is given in para 4.5.

4.2 Notation

A_s	area of steel in tension
A_c	area of steel in compression
b	breadth of the rectangular section
C_{cs}	total compressive force taken by the compression steel
C	total compressive force taken by concrete
D'_x	eccentricity ratio = d'_x/b
D'_y	eccentricity ratio = d'_y/d_y
d'_x	distance of load point from the compression face in x-direction
d'_y	distance of load point from the compression face in y-direction
d_y	effective depth of rectangular section measured normal to width
dh	effective depth from the extreme compression corner to the centre of gravity of tension steel (measured normal to neutral axis)
E_o	tangent modulus of elasticity of concrete i.e. the initial modulus for elasticity of concrete
E_s	modulus of elasticity of steel = mE_o
e	strain in concrete at a distance y from neutral axis
e_{cs}	strain in compression steel

e_{cc}	eccentricity ratio at any stage earlier to ultimate load
e_{csy}	strain in compression steel at yield
e_s	strain in tension steel
e_{sy}	strain in tension steel at yield
e_u	strain in concrete at ultimate load
K_2dh	cover over steel in compression
g/d_y , n	ratio of depth of neutral axis to effective depth (y-direction), case 1
g_o/g, f	neutral axis inclination factor, case 1 (k)
M	applied moment
M,	moment factor at any stage caused by direct compressive load $M/\sigma_u bd_y^2$
M_u,	ultimate moment factor $M_u/\sigma_u bd_y^2$
M_u	ultimate applied moment
m	modular ratio
mp	dimensionless ratio $E_s A_s/E_o bd_y$
N	dimensionless stress ratio $t_y/m\sigma_u$
n	ratio of depth of neutral axis to effective depth (y-direction), case 1
n_1	g_o/d_y

n_x / b, f	neutral axis inclination factor case 2
n_y / d_y, h	neutral axis depth factor for case 2
n_u	ultimate neutral axis depth factor
P	applied load
P_u	ultimate applied load
P,	load factor at any stage earlier to ultimate load
$P_{u,}$	load factor at ultimate stage $\frac{P_u}{\sigma_u b d_y}$
p	dimensionless ratio of area of steel in tension to the effective cross sectional area $\frac{A_s}{bd_y}$
R	d_y/ b
R_o	radius of curvature of the member measured normal to neutral axis
σ	stress in concrete at a distance y from neutral axis.
σ_u	crushing strength of concrete prism of the size 7.5 cm x 7.5 cm x 15 cm
t_{cs}	stress in compression steel
t_s	stress in tension steel
t_y	yield stress for steel (Nm σ_u)

T	total force taken by tensile steel
Z	dimensionless ratio of area of steel in compression to area of steel in tension (A_c / A_s)
α	rotation factor for elastic case ($E_o\, d_y / \sigma_u\, R_o$)
α_u	rotation factor at ultimate ($2/n_u$)
α_1	inclination of load line with the vertical axis of section
θ	angle of inclination of neutral axis with horizontal

4.3 Assupmtions

The following assumptions are implicit throughout the analysis :

1. A rectangular section is considered for this theory.

2. The strain across the cross section normal to neutral axis is linear upto the ultimate moment.

3. A functional form of stress strain relationship of second degree polynominal upto the point of maximum stress(σ_u) is considered and beyond the point of ultimate load, this stress strain relationship is neglected.

4. The tangent modulus E_0 is considered.

5. For the case of ultimate load, the ratio of maximum strain in concrete to the strain at ultimate in concrete is assumed to be unity.

6. The shape of the stress block has been assumed to be the same as the shape of the stress strain curve upto the ultimate load.

7. Tensile strength of concrete is neglected

8. The bond between the steel and the concrete is assumed to be perfect.

9. The crushing strength of concrete section is assumed to be truly represented by the crushing strength σ_u obtained from compression tests on 7.5 cm x 7.5 cm x 15 cm prisms.

10. The effects of shear and torsion are neglected.

11. The effective cover over the compression steel and tension steel is assumed to be K_2 times the effective depth of the beam measured normal to neutral axis.

12. The time dependent strain is assumed to be zero because of short period of loading.

13. The failure and rupture of concrete takes place locally within the length of the member and this occurs only at one point.

14. It is assumed that the direct compressive load acts at right angles of the plane of the reinforced concrete section and has eccentricities on the axes passing through the centre of gravity of the section. Thus a case of compression accompanied by biaxial bending is assumed.

15. Strain in concrete reaches to its ultimate value at the extreme corner on compression side.

16. Centre of gravities of compression and tension steel lie on a plane normal to the neutral axis (16).

17. For load acting on a diagonal axis, the neutral axis is parallel to the other diagonal (24).

4.4 The Development of Theoretical Equations

The equations for load factor, moment factor and neutral axis depth factor are developed for the following cases.

Case 1. Rectangular section with neutral axis intersecting opposite sides :

(A) Tension failure accompanied by the yield of tension steel and no yield of compression steel.

(B) Compression failure case accompanied by the yield of compression steel and no yield of tension steel.

(C) Balanced design case in which crushing of concrete is accompanied by yield of both compression steel and tension steel.

Case 2. Rectangular section with neutral axis intersecting adjacent sides :

(A) Tension failure case accompanied by the yield of tension steel and no yield of compression steel.

(B) Compression failure case accompanied by the yield of compression steel and no yield of tension steel.

(C) Balanced design case in which crushing of concrete is accompanied by yield of both compression and tension steel

The equations developed are summarised in para 4.6 as equations 1 to 30.

Functional form of relationship assumed (12, 14) is

$$\sigma = Ae + Be^2 \qquad (4.0)$$

The stress strain curve is shown in Fig. 2.3.

At origin $\frac{d\sigma}{de} = E_o$ and

at the ultimate stress $\frac{d\sigma}{de} = 0$,

$$\therefore \quad \sigma = E_o e - \frac{E_o^2 e^2}{4 \sigma_u}$$

Again

$$e = \frac{y}{R_o}$$, since the strain distribution

is linear across a line normal to neutral axis (see Figs. 4.1 and 4.2), y is measured normal to neutral axis and R_o is a radius of beam column curvature.

$$= \frac{E_o y}{R_o} - \frac{E_o^2 y^2}{4 \sigma_u R_o^2} \qquad (4.1)$$

Case 1 (A)

Equations for load factor, moment factor and the neutral axis depth factor for tension failure accompanied by the yield of tension steel and no yield of compression steel are developed as under :

At any stage of loading earlier to ultimate, equation for equilibrium can be written as

$$P = C + C_{cs} - T \qquad (4.2)$$

where

$$C = \int_0^{n d h} \sigma \, b_1 \, \delta y \qquad (4.3)$$

Equation 4.1 gives the value of which is stress in concrete at a distance y from neutral axis.

$$C_{cs} = A_c \; t_{cs}$$

$$= A_c \; e_{cs} \; E_s$$

the strain in compression steel e_{cs} would be

$$\frac{(n - K_2)\, dh}{R_o}$$

$$\therefore \quad C_{cs} = A_c\, E_s \left[\frac{(n - K_2)\, dh}{R_o} \right] \qquad (4.4)$$

$$T = A_s \; t_s$$

$$= A_s\, (N\, m\, \sigma_u) \qquad \left[\begin{array}{l} \text{since} \\ t_s = t_y \end{array} \right] \qquad (4.5)$$

Substituting C, C_{cs} and T in equation 4.2,

$$P = \int_0^{h_2} \sqrt{4\, g_o^2 + b^2} \left[\frac{E_o y}{R_o} - \frac{E_o^2\, y^2}{4\, \sigma_u R_o^2} \right] d_y$$

$$+ \int_{h_2}^{h_1 + h_2} \frac{4\, g_o^2 + b^2}{2 b g_o} \left[\frac{b\, (g + g_o)}{\sqrt{4\, g_o^2 + b^2}} - y \right] \left[\frac{E_o y}{R_o} - \frac{E_o^2\, y^2}{4\, \sigma_u R^2} \right] d_y$$

$$+ \frac{m Z p\; b d_y}{R_o} E_o \left[\frac{b\, (g + g_o)}{\sqrt{4\, g_o^2 + b^2}} - K_2 dh \right]$$

$$- p b d_y\, N\, m\, \sigma_u \qquad (4.6)$$

Substituting

$$h_1 = \frac{2\, g_o\, b}{\sqrt{4\, g_o^2 + b^2}} \qquad (4.7)$$

$$h_2 = \frac{(g - g_o)\, b}{\sqrt{4g_o^2 + b^2}} \tag{4.8}$$

$$E_s = m\, E_o$$

$$= \frac{E_o\, dh}{\sigma_u\, R_o} \tag{4.9}$$

Equation for load factor $P,$ can be developed in the form as shown in Eqn. 1, (para 4.6).

At ultimate condition $P, = P_{u},$

$$n = n_u$$

$$\alpha = \alpha_u$$

where

$$\alpha_u = \frac{E_o\, dh}{\sigma_u R_o}$$

$$= \frac{E_o\, b\, d_y (1 + n_1)}{R_o\, \sigma_u \sqrt{4\, g_o^2 + b^2}}$$

$$= \frac{2\,(1 + k n_u)}{n_u\,(1 + k)} \tag{4.10}$$

here $k = \dfrac{n_1}{n} = \dfrac{g_o}{g} = f$

Substituting

$$P, = P_{u}, = \frac{P_u}{\sigma_u\, b d_y^2}$$

$$n = n_u, \quad \alpha = \alpha_u = \frac{2\,(1 + kn_u)}{n_u\,(1 + k)}$$

in Eqn. 1 and on simplification, an equation for $P_{u,}$ can be developed. The simplified form is as under:

$$P_{u,} = \frac{n_u}{3(1 + k)^2}\,(2 + 3k + k^2) + \frac{2\,mpZ\,(1 + n_1)}{n_u\,(1 + k)}\left[\frac{n_u(1 + k)}{1 + n_1} - K_2\right] - mpN \tag{4.11}$$

(Equation 2 in para 4.6).

Thus we have the load factor $P,$ and $P_{u,}$ valid for any load P and the ultimate load P_u respectively for case 1 (A) summarised as in Equations 1 and 2, of para 4.6.

Again at any stage of loading earlier to ultimate, M can be obtained by taking moments about the neutral axis as

$$M = C \cdot y + C_{cs}\,(n - K_2)\,dh + T\,(1 - n)\,dh \tag{4.12}$$

Substituting C, C_{cs} and T as obtained earlier and simplifying we get Equation 3 for

$$M_, = \frac{M}{\sigma_u b d_y^2}$$

and at ultimate condition $n = n_u$, $\alpha = \alpha_u$, $M_, = M_{u,}$

the relationship for M_u, is obtained as in Equation 4.

Thus we have the moment factor equation 3 and the ultimate moment factor equation 4 valid for any stage of loading and for ultimate load respectively.

Further, at any stage of loading the eccentricity ratio

$$e_{cc} = \left(\frac{M,}{P,} \right) \qquad (4.13)$$

Substituting $\alpha = \alpha_u$ and $n = n_u$, we get equation for the ultimate neutral axis depth factor as shown in equation 5. Thus equation 5, is an equation of neutral axis depth factor at ultimate load for case 1 (A).

Case 1 (B)

Equations for load factor, moment factor and neutral axis depth factor for compression failure case accompanied by the yield of compression steel and no yield of tension steel are developed in similar manner except that

$$C_{cs} = A_c \cdot t_{cs}$$

$$= A_c \cdot t_y \qquad \text{(since the compression steel yields)} \quad (4.14)$$

$$= A_c \cdot Nm \cdot \sigma_u$$

and

$$T = A_s \cdot t_s$$

$$= A_s \frac{(1-n)dh}{R_o} \cdot E_s \qquad (4.15)$$

Again

$$P = C + C_{cs} - T \qquad (4.16)$$

Substituting new values for C_{cs} and T, equation for P, as in equation 6, is developed while substituting $n = n_u$ and $\alpha = \alpha_u$, $P = P_u$, equation for P_u, is developed as shown in equation 7.

Thus we have equations 6 and 7 for P, and P_u, for load at any stage and for ultimate load condition respectively for case 1 (B).

Again at any stage of loading earlier to ultimate,

$$M = C\,y + C_{cs}\,(n - K_2)\,dh + T\,(1 - n)\,dh \qquad (4.17)$$

Substituting the values of C, C_{cs} and T equations for moment factor (M,) at any stage of loading can be developed (see equation 8), while substituting $n = n_u$, $\alpha = \alpha_u$ and $M_u = M_u$, equation for M_u, as shown in equation 9, is developed.

We have therefore equations 8 and 9 for M, and M_u, for load at any stage and for ultimate load respectively for case 1 (B).

Equation 10 is an equation for ultimate neutral axis depth factor.

Case 1 (C)

Equations for load factor, moment factor and neutral axis depth factor for balanced design case in which crushing of concrete is accompanied by yield of both compression and tension steel are developed on similar lines as given below

$C_{cs} = A_c \, t_y = A_c \, Nm \, \sigma_u$ since the compression steel yields and

$$T = A_t \, t_y = mp \, N \, \sigma_u \, bd_y \qquad (4.18)$$

Value of C will be as given in equation 4.3.

Substituting values of C, C_{cs} and T in equation 4.2 relation for P, can be developed (Equation 11) while substituting $P_u = P$, $n_u = n$ and $\alpha_u = \alpha$ equation for P_u, can be developed for ultimate load condition (equation 12).

Thus we have the load factor equation for load at any stage and ultimate load factor equation (Equations 11 and 12).

Again at any stage of loading earlier to ultimate

$$M = C\,y + C_{cs}\,(n - K_2)\,dh + T\,(1 - n)\,dh \qquad (4.19)$$

Substituting C, C_{cs} and T an equation for M, at any stage of loading is obtained. (Equation 13).

At ultimate $n = n_u$, $\alpha = \alpha_u$ and $M = M_u$, we get a relation of M_u, (Equation 14).

Thus we have the moment factor equation 13 for load at any stage and the ultimate moment factor equation 14 for the ultimate load condition.

Further at any stage of loading earlier to ultimate the eccentricity ratio e_{cc} would be

$$e_{cc} = \left(\frac{M,}{P,} \right) \qquad (4.20)$$

Substituting $n = n_u$ and $\alpha = \alpha_u$ and on simplification a neutral axis depth ratio equation 15 is developed.

For pure bending, the eccentricity ratio should be ∞. All the above equations are valid for pure bending cases also.

<u>Case 2 (A)</u>

Equations for load factor, moment factor and neutral axis depth factor for tension failure accompanied by the yield of tension steel and no yield of compression steel are now developed for the case in which neutral axis bisects the adjacent sides (see Fig.4.2).

At any stage of loading earlier to ultimate, equation for equilibrium can be written as

$$P = C + C_{cs} - T \qquad (4.21)$$

$$M = C\,y + C_{cs}\,(n - K_2)\,dh + T\,(1 - n)\,dh \qquad (4.22)$$

where

$$C = \int_0^{n\,dh} \sigma\, b_1 \cdot \delta y \tag{4.23}$$

$$C_{cs} = A_c \cdot e_{cs} \cdot E_s = A_c \cdot E_s \cdot \frac{(n - K_2)dh}{R_o} \tag{4.24}$$

$$T = A_s \cdot t_s$$

$$= A_s \cdot N\,m \cdot \sigma_u \tag{4.25}$$

Substituting the values of C, C_{cs} and T in equation 4.21, we get

$$P = \int_0^{n\,dh} \frac{n_x^2 + n_y^2}{n_x \cdot n_y} \left[\frac{n_x \cdot n_y}{\sqrt{n_x^2 + n_y^2}} - y \right] \left[\frac{E_o y}{R_o} - \frac{E_o^2\, y^2}{4\,\sigma_u R_o^2} \right] dy$$

$$+ \left[\frac{A_c \cdot E_o (n - K_2)\, dh}{R_o} \right] - A_s \cdot N\,m \cdot \sigma_u \tag{4.26}$$

Substituting

$$f = \frac{n_x}{b}, \quad h = \frac{n_y}{d_y} \quad \text{and} \quad P_1 = \frac{P}{\sigma_u\, b d_y},$$

$$\alpha = \frac{E_o dh}{\sigma_u R_o} = \frac{E_o\, b d_y}{\sigma_u R_o} \left(f + \frac{h}{2}\right) \frac{1}{\sqrt{n_x^2 + n_y^2}}$$

$$\alpha_u = \frac{2\left(f + \frac{h}{2}\right)}{fh},$$

equation for load factor P_1 is obtained, (Equation 16). Again $n_u = h = n$, $\alpha = \alpha_u$ and $P_u = P$ give condition for developing equation for P_u, in Equation 17.

Thus we have the load factor P, and the ultimate load factor P_{u}, in Equations 16 and 17.

Again at any stage of loading, earlier to ultimate M is obtained as in equation 4.22.

Substituting C, C_{cs} and T (4.23, 4.24, 4.25), an equation for M, at any stage of loading is obtained as given by equation 18.

At ultimate, $n = n_u$, $\alpha = \alpha_u$ and $M, = M_{u}$, .
Substituting these values in equation 18, a relation of M_{u}, is obtained (Equation 19)

Thus we have the moment factor equation, for load at any stage (Equation 18) and the ultimate moment factor equation 19.

Further at any stage of loading earlier to ultimate the eccentricity ratio e_{cc} is obtained as in 4.13 and substituting $\alpha = \alpha_u$, $n_u = n = h$, we get equation for ultimate neutral axis depth factor as shown in Equation 20.

Case 2 (B)

Equations for load factor, moment factor and neutral axis depth factor for compression failure case accompanied by the yield of compression steel and no yield of tension steel are developed on similar lines except that now the values of C_{cs} and T shall be as under

$$C_{cs} = A_c \cdot t_s = A_c \cdot t_y$$

$$T = A_s \cdot t_s$$

$$= A_s \cdot e_s \cdot E_s$$

$$= A_s \cdot \frac{(1 - n)dh}{R_o} E_s$$

Equations for P, P_u, M, and M_u, and neutral axis depth factor are developed and summarised in equations 21, 22, 23, 24 and 25.

Case 2 (C)

Equations for load factor, moment factor and neutral axis depth factor for balanced design case in which crushing of concrete is accompanied by yield of both compression and tension steel are developed on similar lines except that in this case as in 1 (C),

$$C_{cs} = A_c \cdot t_y = A_c \cdot N m \cdot \sigma_u \quad \text{and}$$

$$T = A_t \cdot t_y = mp \; N \sigma_u \, bd_y$$

Substituting these values in equations of equilibrium for direct load and bending moment, equations for

P, P_u, M, and M_u, and for neutral axis depth factor are developed and summarised in equations 26, 27, 28, 29 and 30.

<u>Critical Values of mp</u>

Critical values of mp for the demarcation between tension failure case and compression failure case.

Case 1 : Neutral axis bisecting opposite sides.

The critical values of mp is obtained as follows

$$e_u = \frac{e_{sy} \cdot n_u dh}{dh\,(1 - n_u)} \qquad \text{(See Figs. 4.1 and 4.2)} \qquad (4.27)$$

also we have

$$\sigma_u = \frac{E_o \cdot e_u}{2} \qquad (4.28)$$

$$e_u = \frac{2\,\sigma_u}{E_o} \qquad (4.29)$$

$$e_{sy} = \frac{t_y}{E_s}$$

$$= \frac{N\,\sigma_u}{E_o} \qquad (4.30)$$

Substituting e_u, and e_{sy} in equation 4.27 we get

$$(n_u)\ \text{critical} = \frac{2}{2 + N\,(1 + k)}$$

$$= \text{NUCR} \qquad (4.31)$$

Now substituting NUCR in equation 15 of the case of balanced design a relation of mp critical can be obtained as given in equation 31.

Case 2 : Neutral axis intersecting adjacent sides

Equations 4.27, 4.29 and 4.30 give the relation as

$$\frac{2}{n_u} = \frac{N}{1 - n_u} \qquad (4.32)$$

Now converting equation 4.32 in terms of f and h we get

$$\bar{h} = \frac{2f}{(n + 2).f - 1} \qquad (4.33)$$

$$= NUC$$

Substituting value of $\bar{h}$ in equation 30 and simplifying we get an equation 32 for mp critical for case 2.

Critical Values of mp for demarcation between the balanced design and tension failure case accompanied by the yield of tension steel and yield of compression steel are developed as under.

Case 1. Neutral axis intersecting opposite sides.

$$\frac{e_u}{n_u dh} = \frac{e_{csy}}{dh\,(n_u - K_2)} \qquad (4.34)$$

(see Figs. 4.1 and 4.2)

Since compression steel has also to yield

$$e_{csy} = \frac{t_y}{E_s}$$

$$= N \frac{\sigma_u}{E_o} \qquad (4.35)$$

Also $$e_u = \frac{2\,\sigma_u}{E_o} \qquad (4.36)$$

Substituting the values of e_{csy} and e_u in equation 4.34, we get

$$\overline{n_u} = \frac{2\,K_2}{2 + 2k - 2\,K_2 k - N - kN} \qquad (4.37)$$

where $\overline{n_u}$ is critical value of neutral axis.

Substituting this critical value in the equation of the case of balanced design i.e. equation 15, a relation for $\overline{mp}$ is obtained as in equation 33.

Case 2 : Neutral axis intersecting adjacent sides.

Equations 4.34, 4.35 and 4.36 give relation as

$$\frac{2}{n_u} = \frac{N}{n_u dh - K_2 dh} \qquad (4.38)$$

Converting equation 4.38 in terms of f and h we get

$$\bar{h} = \frac{2\,K_2\,f}{f(2 - N) - K_2} \qquad (4.39)$$

where $\bar{h}$ is critical value of neutral axis depth factor.

Substitution the value of $\bar{h}$ in equation 30, we get an equation for $\overline{mp}$ (Equation 34).

4.5 Location of Neutral Axis

For square column $\theta = \alpha_1$ is found reasonable approximation, (24). For rectangular sections, the relation between θ and α_1 is obtained by the assumption (which does not lead to large error) that for a load acting on diagonal axis, the neutral axis is almost parallel to the other diagonal, i.e.

$$\tan \theta = \frac{d_y}{b} = R \tag{4.40}$$

while

$$\tan \alpha_1 = \frac{b}{d_y} = \frac{1}{R} \tag{4.41}$$

Thus with three unknowns, i.e. θ, α_1, and R, a relation can be developed. This relation in square section is a straight line. While for rectangular section the relation is a circular curve.

(a) Case 1

$$\tan \theta = \frac{2g_0}{b}$$

$$= 2 \cdot fn \cdot R \tag{4.42}$$

The relation 4.42 is used for selection of neutral axis inclination factor f in appropriate connection of n values.

(b) Case 2

$$\tan\theta = \frac{n_y}{n_x}$$

$$= \frac{h \cdot R}{f} \qquad (4.43)$$

The relation 4.43 is used for selection of neutral axis inclination factor f in appropriate connection of h values.

4.6 Summary of Equations

A summary of equations for load factors, moment factors and ultimate neutral axis depth factors for cases 1 (A), 1 (B), 1 (C) and cases 2 (A), 2 (B) 2 (C) is given here as Equations 1 to 30. Critical values of mp for demarcation between tension failure case and compression failure case as well as demarcation between the balanced design and tension failure are summarised and listed as equations 31 to 34. Steps to derive these equations are explained in para 4.4.

CASE 1

Rectangular Section with Neutral axis Intersecting Opposite Sides :-

A : Tension Failure accompanied by the Yield of Tension Steel and no Yield of Compression Steel. (11)

LOAD FACTOR (P_9) AND ULTIMATE LOAD FACTOR (Pu_9) EQUATIONS :-

$$P_9 = \frac{\alpha n^2 (3 + k^2)}{6(1 + n_1)} - \frac{\alpha^2 n^3 (1 + k^3)}{12(1 + n_1)^2} + mpz\alpha \left[\frac{n(1+k)}{1+n_1} - k_2 \right] - mpN \qquad 1.$$

$$Pu_9 = \frac{n_u}{3(1+k)^2} \left[2 + 3k + k^2 \right] + \frac{2\,mpz(1+n_1)}{n_u(1+k)} \left[\frac{n_u(1+k)}{(1+n_1)} - k_2 \right] - mpN \qquad 2.$$

MOMENT FACTOR (M_1) AND ULTIMATE MOMENT FACTOR (Mu_1) EQUATIONS :-

$$M_1 = \frac{\alpha n^3 (1 + k^2)}{3\,Tc\,(1 + n_1)} + \frac{\alpha^2 n^4 (-5 - 10k - k^4)}{80\,Tc\,(1 + n_1)^2}$$

$$+ \, mpz\,\alpha(1 + n_1)\left[\frac{n + n_1}{1 + n_1} - k_2\right]^2 \div Tc$$

$$+ \frac{mpN(1 - n)}{Tc} \qquad \underline{3.}$$

$$Mu_1 = \frac{2\,nu^2 (1 + k^2)}{3\,Tc\,(1 + k)} + \frac{nu^2(-5 - 10k - k^4)}{20\,Tc\,(1 + k)^2}$$

$$+ \frac{2\,mpz\,(1 + n_1)^2}{nu\,(1 + k)\,Tc}\left[\frac{nu\,(1 + k)}{(1 + n_1)} - k_2\right]^2$$

$$+ \, mpN(1 - nu) \div Tc \qquad \underline{4.}$$

ULTIMATE NEUTRAL AXIS DEPTH FACTOR (nu)

$$A\,nu^3 + B\,nu^2 + C\,nu + D = 0 \qquad \underline{5.}$$

where . . . . p.t.o.

(CONT D.) (EQUA. NO. 5)

$$A = \frac{-15 - 90k - 40k^2 + 20k^3 - 3k^4}{60}$$

$$- \frac{2kD'x(2 + 3k + k^2)}{3}$$

$$B = mpN\left[k(1 + k)^2 (1 + 2D'x)\right]$$

$$+ 2mpz\Big[-(1 + k)^2 k_2 k - (1 + k)^2 \cdot 2kD'x$$

$$+ (1 + k)k_2^2 k^2 + 2(1 + k)k_2 k^2 D'x\Big]$$

$$- \frac{(2 + 3k + k^2)D'y}{3}$$

$$C = mpN\left[(1 + k)^2 (1 + D'y)\right]$$

$$+ 2mpz\Big|-(1 + k)^2 k_2 - (1 + k)^2 D'y$$

$$+ (1 + k)kK_2(2k_2 + 2D'x + D'y)$$

$$D = 2mpz\left[(1 + k)k_2(k_2 + D'y)\right]$$

CASE 1 (CONTD.)

B : Compression Failure accompanied by the Yield of Compression Steel and no Yield of Tension Steel. (21)

LOAD FACTOR (P_3) AND ULTIMATE LOAD FACTOR (Pu_3) EQUATIONS :-

$$P_3 = \frac{\alpha n^2 (3 + k^2)}{6(1 + n_1)} - \frac{\alpha^2 n^3 (1 + k^3)}{12(1 + n_1)^2} + mpzN - \alpha(1 - n)mp \qquad 6.$$

$$Pu_3 = \frac{n_u}{3(1 + k)^2}\left[2 + 3k + k^2\right] - \frac{2mp(1 - n_u)}{n_u(1 + k)} + mpzN \qquad 7.$$

MOMENT FACTOR (M_3) AND ULTIMATE MOMENT FACTOR (Mu_3) EQUATIONS :-

(please turn over)

(CONTD.) (EQUA. NO. 8)

$$M_{,} = \frac{\alpha n^3 (1 + k^2)}{3 Tc (1 + n_1)} + \frac{\alpha^2 n^4 (-5 - 10k - k^4)}{80 Tc (1 + n_1)^2}$$

$$+ m p z N (1 + n_1) \left[\frac{n + n_1}{1 + n_1} - k_2 \right] \div Tc$$

$$+ \frac{m p \alpha (1 - n)^2}{Tc (1 + n_1)} \qquad \underline{8.}$$

$$Mu_{,} = \frac{2 nu^2 (1 + k^2)}{3 Tc (1 + k)} + \frac{nu^2 (-5 - 10k - k^4)}{20 Tc (1 + k)^2}$$

$$+ \frac{m p z N (1 + n_1)}{Tc} \left[\frac{nu (1 + k)}{(1 + n_1)} - k_2 \right]$$

$$+ \frac{2 mp (1 - nu)^2}{Tc (1 + k) nu} \qquad \underline{9.}$$

ULTIMATE NEUTRAL AXIS DEPTH FACTOR (nu)

$$A nu^3 + B nu^2 + C nu + D = 0 \qquad \underline{10.}$$

where ......

(please turn over)

(CONTD.)

$$A = \frac{-15 - 90k - 40k^2 + 20k^3 - 3k^4}{60} - \frac{2kD'x(2 + 3k + k^2)}{3}$$

$$B = 2mp\left[-(1+k)\,x\,(1 + 2D'x)\right] - mpZN(1+k)^2 k\,(kz + 2D'x) - \frac{(2 + 3k + k^2)D'y}{3}$$

$$C = 2mp\left[(1+k)(-1 + k + 2k^2 D'x - D'y)\right] - mpZN(1+k)^2(kz + D'y)$$

$$D = 2mp(1+k)(1 + D'Y)$$

(EQUA. NO. 10)

CASE 1 (CONTD.)

C : Balanced Design : Crushing Of Concrete accompanied by Yield of both Compression and Tension Steel. (31)

LOAD FACTOR (P_3) AND ULTIMATE LOAD FACTOR (Pu_3) EQUATIONS :-

$$P_3 = \frac{\alpha n^2 (3 + k^2)}{6(1 + n_1)} - \frac{\alpha^2 n^3 (1 + k^3)}{12(1 + n_1)^2} + mpzN - mpN \qquad \underline{11.}$$

$$Pu_3 = \frac{n_u}{3(1 + k)^2} \left[2 + 3k + k^2 \right] + mpzN - mpN \qquad \underline{12.}$$

MOMENT FACTOR (M_3) AND ULTIMATE MOMENT FACTOR (Mu_3) EQUATIONS :-

(please turn over)

(contd.)

$$X_c = \frac{\alpha n^3 (1 + k^2)}{3\,Tc\,(1 + n_1)} + \frac{\alpha^2 n^4 (-5 - 10k - k^2)}{80\,Tc\,(1 + n_1)^2}$$

$$+ m p \pi n (1 + n_1) \left[\frac{n + n_1}{1 + n_1} - k_2 \right] \div Tc$$

$$+ \frac{n_1 p n (1 - n)}{Tc} \qquad \underline{13.}$$

$M_u =$ [illegible]

[illegible] $\qquad \underline{14.}$

ULTIMATE NEUTRAL AXIS DEPTH FACTOR (nu)

$$E\,nu^2 + C\,nu + D = 0 \qquad \underline{15.}$$

where ....

(please turn over)

(CONTD.)

$$B = \frac{-15-90K-40K^2+20K^3-3K^4}{60} - \frac{2+3K+K^2}{3}\cdot D'_x \cdot 2K$$

$$C = mpN\left\{(1+K)^2 K(1-ZK_2)+(1-Z)(1+K)^2 D'_x \cdot 2K\right\}$$

$$-D'_y \quad \frac{2+3K+K^2}{3}$$

$$= mpN(1+K)^2 K\left\{(1-ZK_2)+2(1-Z)\cdot D'_x\right\}$$

$$-D'_y \quad \frac{2+3K+K^2}{3}$$

$$D = pNm(1+K)^2\left\{(1-ZK_2)+(1-Z)D'_y\right\}$$

CASE 2

Rectangular Section with Neutral axis Intersecting Adjacent Sides :-

A : Tension Failure accompanied by the Yield of Tension Steel and no Yield of Compression Steel. (A 2)

LOAD FACTOR (P_9) AND ULTIMATE LOAD FACTOR (Pu_9) EQUATIONS :-

$$P_9 = \frac{\sigma_c f^2 h^2}{6(f+h/2)} - \frac{\sigma_c^2 f^3 h^3}{48(f+h/2)^2} + mpzK\left[\frac{fh}{f+h/2} - K_2\right] - mpK \qquad (16.)$$

$$Pu_9 = \frac{fh}{4} + \frac{2mpz(f+h/2)}{fh}\left[\frac{fh}{f+h/2} - K_2\right] - mpK \qquad (17.)$$

MOMENT FACTOR (M_o) AND ULTIMATE MOMENT FACTOR (Mu_o) EQUATIONS :-

$$M_o = \frac{\sigma_1 f^2 h^2}{6\,Tr(f + h/2)} - \frac{\sigma_1^2 f^3 h^3}{36\,Tr(f + h/2)^2} + \frac{mpz\,\sigma_1 (f + h/2)}{Tr \cdot f}\left[\frac{fh}{f + h/2} - x_2\right]^2 + \frac{mpz(1 - x_2 + h/2f)}{Tr} \qquad \underline{18.}$$

$$Mu_o = \frac{T f h^2}{66\,Tr} + \frac{2\,mpz\,(f + h/2)^2}{Tr \cdot h \cdot u}\left[\frac{fh}{f + h/2} - x_2\right]^2 + \frac{mpz(1 - x_2 + h/2f)}{Tr} \qquad \underline{19.}$$

ULTIMATE NEUTRAL AXIS DEPTH FACTOR (h_u)

$$A h^3 + B h^2 + C h + D = 0 \qquad \underline{20.}$$

where . . . .

(please turn over)

(CONTD.) (EQUA. NO.20)

$$A = \frac{2}{15} f + D'x/4$$

$$B = \frac{mpz}{f} (1 - k2/2f)(k2 - 2D'z)$$

$$- mpN/f (1/2 + D'x) + \frac{f}{4} D'y$$

$$C = \frac{2mpzk2}{f} \left[f - k2 - \frac{D'Y}{2} - D'X \right]$$

$$+ mp \left[-N (1 + D'Y) + 2Z D'Y \right]$$

$$D = -2mpzk2 (k2 + D'Y)$$

CASE 2 (CONTD.)

B :- Compression Failure accompanied by the Yield of Compression Steel and no Yield of Tension Steel. (22)

LOAD FACTOR (P_3) AND ULTIMATE LOAD FACTOR (Pu_3) EQUATIONS :-

$$P_3 = \frac{\alpha_1 f^2 h^2}{6(f+h/2)} - \frac{\alpha_1^2 f^3 h^3}{48(f+h/2)^2} + mpzN - mp\alpha_1 \left[(f+h/2) - fh \right] \div (f+h/2)$$

21.

$$Pu_3 = \frac{fh}{4} + mpzN - \frac{2mp}{fh}\left[(f+h/2) - fh \right]$$

22.

MOMENT FACTOR (M_2) AND ULTIMATE MOMENT FACTOR (Mu_2) EQUATIONS :-

$$M_2 = \frac{\alpha_1 f^2 h^3}{12\, TR\,(f + h/2)} - \frac{\alpha_1^2 f^3 h^4}{30\, TR\,(f + h/2)^2} + \frac{mpzx\,(f+h/2)}{TR \cdot f}\left[\frac{fh}{f + h/2} - k2\right] + \frac{mp\tau_1}{TR \cdot f\,(f + h/2)}\left[(f + h/2) - \frac{h}{f}\right]^2$$

. . . . 23.

$$Mu_2 = \frac{7 f h^2}{60\, TR} + \frac{mpzx\,(f + h/2)}{TR \cdot f}\left[\frac{fh}{f + h/2} - k2\right] + \frac{2mp}{f^2 h}\left[(f + h/2) - h/f\right]^2 \div TR$$

24.

ULTIMATE NEUTRAL AXIS DEPTH FACTOR (h)

$$Ah^3 + Bh^2 + ch + D = 0 \qquad 25.$$

where . . . . (please turn over)

(CONTD.) (EQUA. 25)

$$A = \frac{2}{15} f + \frac{D'x}{4}$$

$$B = -\Big[mp/2f^4 (f^2 - 4f + 4) - \frac{mpzN}{f} \left(\frac{Kz}{2} + D'x \right)$$

$$- D'y\, f/4 + mp(1/f - 2)$$

$$+(mp/f - 2mp)\, D'x/f \Big]$$

$$C = -\Big[\frac{2mp}{f^2} (f - 2) - mpzN(Kz + D'y)$$

$$+ 2mp(1 + D'x/f) + mp(1/f - 2)\, D'y \Big]$$

$$D = -2mp(1 + D'y)$$

CASE 2 (CONTD.)

C : Balanced Design : Crushing of Concrete accompanied by Yield of both Compression and Tension Steel (32)

LOAD FACTOR (P_3) AND ULTIMATE LOAD FACTOR (Pu_3) EQUATIONS :-

$$P_3 = \frac{\alpha_1 f^2 h^2}{6(f + h/2)} - \frac{\alpha_1^2 f^3 h^3}{48(f + h/2)^2} + mp_2N - mpN \qquad \text{26.}$$

$$Pu_3 = \frac{fh}{4} + mp_2N - mpN \qquad \text{27.}$$

MOMENT FACTOR (M_3) AND ULTIMATE MOMENT FACTOR (Mu_3) EQUATIONS :-

(please turn over)

(CONTD.)

$$M_1 = \frac{\alpha_1 f^2 h^3}{12\, TR (f + h/2)} - \frac{\alpha_1^2 f^3 h^4}{80\, TR (f + h/2)^2}$$

$$+ \frac{mpZN(f + h/2)}{TR \cdot f} \left[\frac{fh}{f + h/2} - k_2 \right]$$

$$+ \frac{mpN(1 - h + h/2f)}{TR} \quad \ldots\ldots \quad \underline{28.}$$

$$Mu_2 = \frac{F f h^2}{60\, TR} + \frac{mpZN(f + h/2)}{TR \cdot f} \left[\frac{fh}{f + h/2} - k_2 \right]$$

$$+ \frac{mpN(1 - h + h/2f)}{TR} \quad \ldots\ldots \quad \underline{29.}$$

ULTIMATE NEUTRAL AXIS DEPTH FACTOR (h)

$$A h^3 + B h^2 + C h + D = 0 \quad \ldots\ldots \quad \underline{30.}$$

where

$A = 0$

$$B = \frac{2}{15} f + \frac{D^2 x}{4}$$

(please turn over)

(CONTD.) (EQUA. 30)

$$C = \frac{f}{4} D'_Y - \frac{mpN}{2f}\left[1 - zk2 + 2D'_X(1-z)\right]$$

$$D = mpN\,(zk2 - 1 - D'_Y(1-z))$$

CRITICAL VALUES OF mp FOR THE DEMARKATION BETWEEN TENSION FAILURE CASE AND COMPRESSION FAILURE CASE

CASE 1 NEUTRAL AXIS INTERSECTING OPPOSITE SIDES

$$mpcRo = \frac{-(NUCR)^2 A1 + NUCR \cdot A2}{N(1+K)^2\,(NUCR \cdot M1 + M2)} \qquad 31.$$

Where ....

$$NUCR = \frac{2}{2 + N(1+K)} \; ; \quad A_1 = A \text{ OF EQ. 5.}$$

$$A2 = \frac{2 + 3K + K^2}{3} D'_Y \; ; \quad M1 = K(1 - zk2 + 2(1-z)D'_X)$$

$$M2 = (1 - zk2 + D'_Y(1-z))$$

CASE 2 — NEUTRAL AXIS INTERSECTING ADJACENT SIDES

$$mpcra = \frac{(NUC)^2 A_3 + NUC \cdot A_4}{N\,(NUC \cdot M_4 + M_5)} \qquad \underline{32.}$$

Where ....

$$NUC = \frac{2f}{(N+2)f - 1} \;;\; A_3 = A \text{ OF EQ. 20.}$$

$$A_4 = \frac{f}{4} D'y$$

$$M_4 = \frac{1}{2f}\left(1 - Zk_2 + 2(1 - Z) D'x\right)$$

$$M_5 = 1 - Zk_2 + (1 - Z) D'y$$

CRITICAL VALUES OF mp FOR THE DEMARCATION BETWEEN THE BALANCED DESIGN AND TENSION FAILURE CASE ACCOMPANIED BY THE YIELD OF TENSION STEEL NO YIELD OF COMPRESSION STEEL ...

CASE 1 — NEUTRAL AXIS INTERSECTING OPPOSITE SIDES

$$mpbao = \frac{-(NUBAO)^2 A_1 + NUBAO \cdot A_2}{N\,(1+K)^2\,(NUBAO \cdot M_1 + M_2)} \qquad \underline{33.}$$

where .....

$$NUBA0 = \frac{2K2}{2 + 2K - 2K2K - N - KN}$$

CASE 2 NEUTRAL AXIS INTERSECTING ADJACENT SIDES

$$mpbaa = \frac{(NUB)^2 A3 + NUB \cdot A4}{N(NUB \cdot M4 + M5)} \qquad 34.$$

where

$$NUB = \frac{2 + K2}{(2-N)f - K2}$$

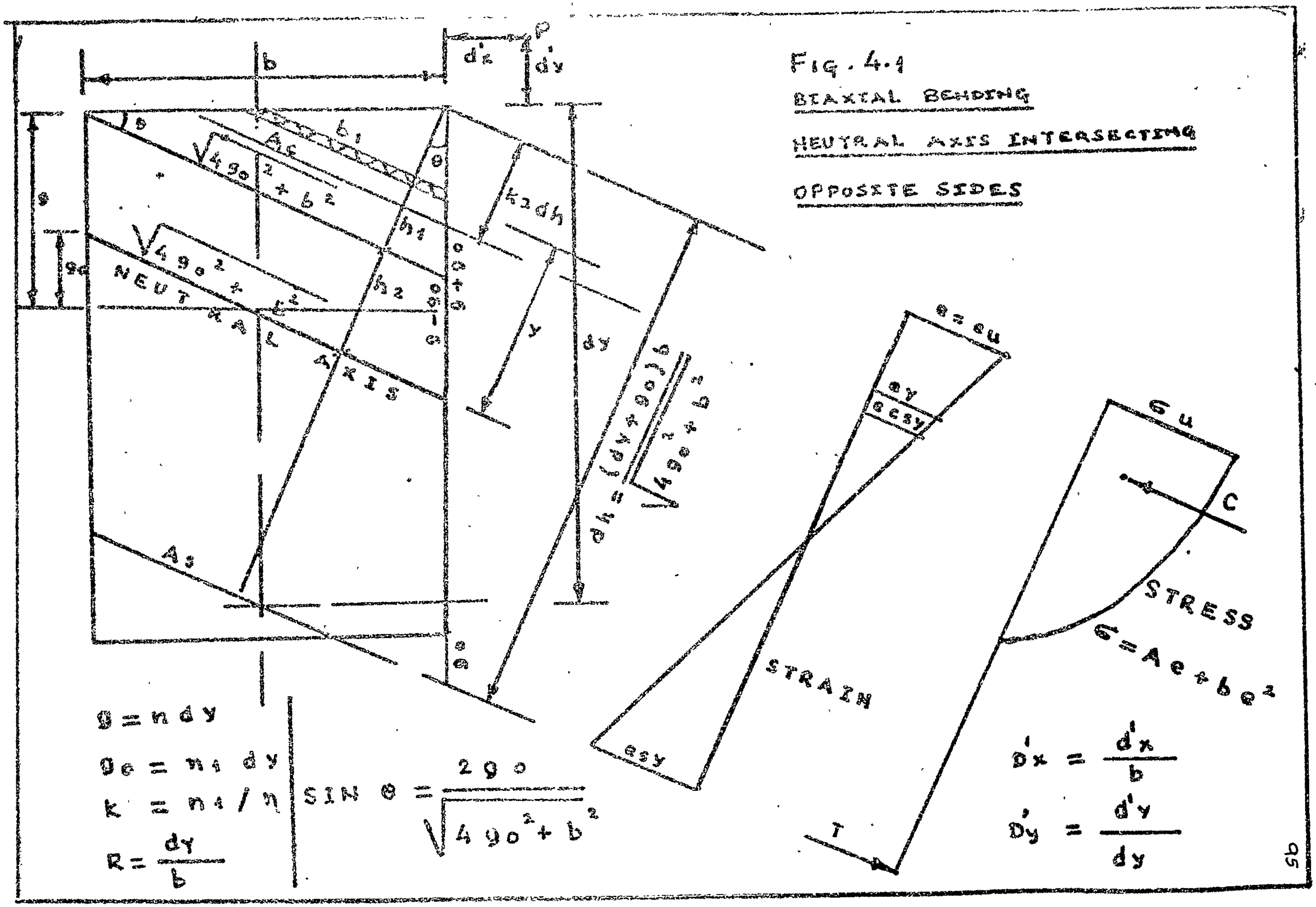


Fig. 4.1
BIAXIAL BENDING
NEUTRAL AXIS INTERSECTING
OPPOSITE SIDES

BIAXIAL BENDING NEUTRAL AXIS INTERSECTING OPPOSITE SIDES

CASE 1

$$h_1 = \frac{2 g_o b}{\sqrt{4 g_o^2 + b^2}}$$

$$h_2 = \frac{(g - g_o) b}{\sqrt{4 g_o^2 + b^2}} \qquad h_1 + h_2 = \frac{b(g + g_o)}{\sqrt{4 g_o^2 + b^2}}$$

$$b_1 = \frac{4 g_o^2 + b^2}{2 b g_o} \left[\frac{b (g + g_o)}{\sqrt{4 g_o^2 + b^2}} - y \right]$$

$$Tc = \sqrt{1 + 4 K^2 n^2 R^2}$$

$$\alpha = \frac{E_o b \, d_Y}{R \, \sigma_U \sqrt{4 g_o^2 + b^2}} (1 + n_1)$$

$$\alpha_u = \frac{2 (1 + n_1)}{n_U (1 + k)} = \frac{2 (1 + k n_U)}{n_U (1 + k)}$$

$$\underline{\text{Eccentricity Ratio}} = \frac{M_y}{P_y} = \frac{e_h}{d_y}$$

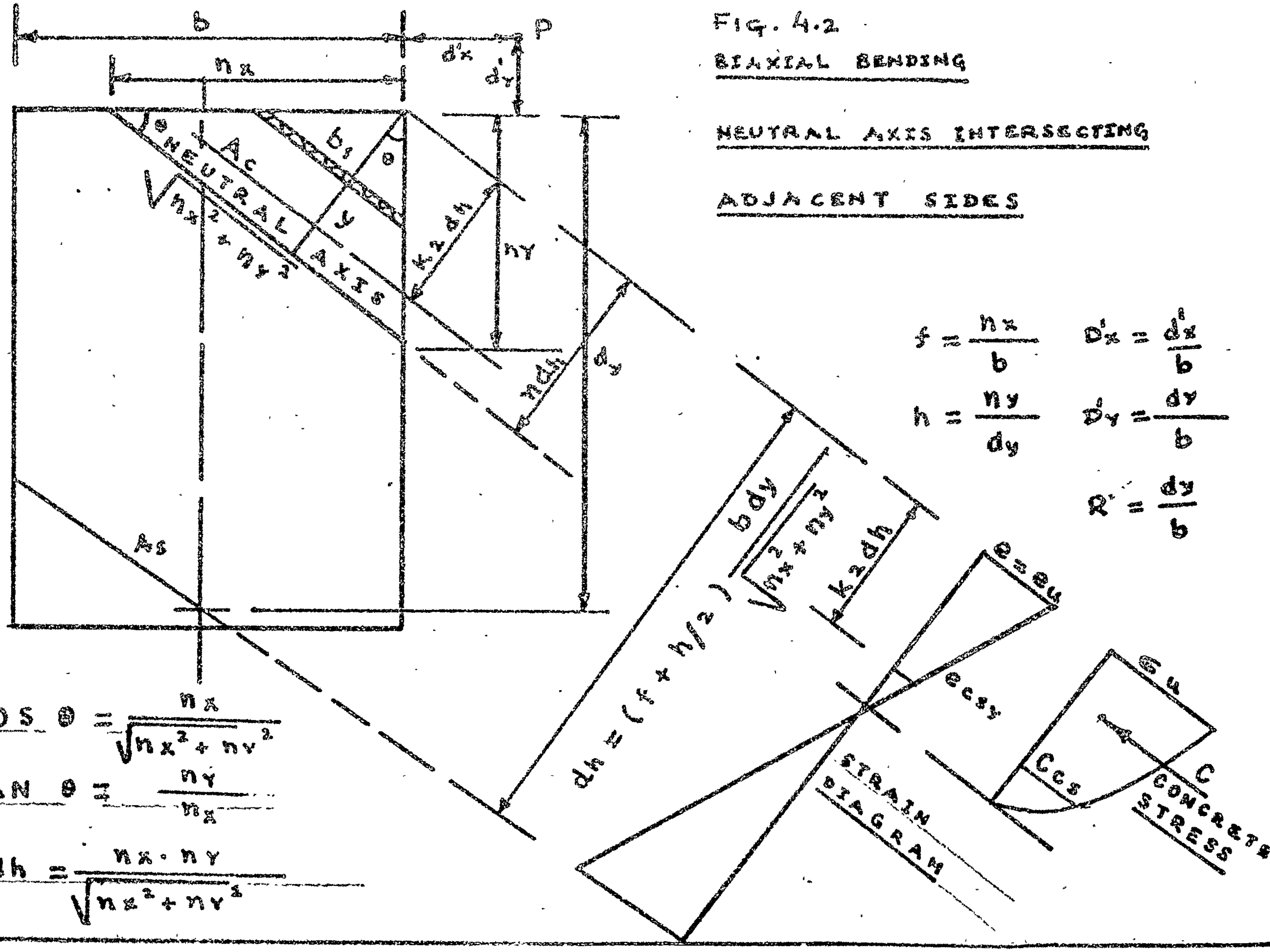
$$= \left[D'y + D'x \cdot 2 n_1 + n (1 + k) \right] \frac{b}{\sqrt{4 g_o^2 + b^2}}$$

FIG. 4.2

BIAXIAL BENDING

NEUTRAL AXIS INTERSECTING

ADJACENT SIDES



$f = \frac{nx}{b}$ $D'x = \frac{d'x}{b}$

$h = \frac{ny}{dy}$ $D'y = \frac{dy}{b}$

$R' = \frac{dy}{b}$

$\underline{\text{COS } \theta} = \frac{nx}{\sqrt{nx^2 + ny^2}}$

$\underline{\text{TAN } \theta} = \frac{nY}{nx}$

$\underline{ndh} = \frac{nx \cdot nY}{\sqrt{nx^2 + nY^2}}$

BIAXIAL BENDING NEUTRAL AXIS INTERSECTING ADJACENT SIDES

CASE 2

$$b_1 = \frac{n_x^2 + n_y^2}{n_x n_y}\left[\frac{n_x n_y}{\sqrt{n_x^2 + n_y^2}} - y\right]$$

$$\alpha = \frac{E_o}{\sigma_u R}\left[b\, d_Y \,(f + h/2)\frac{1}{\sqrt{n_x^2 + n_y^2}}\right]$$

$$\alpha_u = \frac{2\,(f + h/2)}{f\,h}$$

$$TR = \sqrt{1 + h^2 R^2 / f^2}$$

Eccentricity Ratio $= \dfrac{M_y}{P_y} = \dfrac{e_h}{d_y}$

$$= \frac{D'_y + D'_x \cdot h_1 + h'}{\sqrt{1 + h^2 \cdot R^2 / f^2}}$$