Chapter 2

Carrier Scattering in Semiconductors

2.1 Introduction

As carrier traverse a device, their motion is frequently interrupted by collisions with impurity atoms, phonons, crystal defects, or with other carriers. The importance of phonons and their interactions in bulk materials is well known for solid state physics [1]. The electrical response of any electrical device depends upon the electronic configuration as well as on the interaction of carrier with vibrations of lattice with corresponding temperature. Carrier mobilities and dynamical processes in polar semiconductors, such as gallium arsenide, are in many cases determined by the interaction of longitudinal optical (LO) phonons with charge carriers [2, 3]. An electron in a polar semiconductor will be accelerated in the presence of the external electric field until the electron's energy is large enough for the electron to emit an LO phonon. When the electron's energy reaches the threshold for LO phonon emission, there is a significant probability that it will emit an LO phonon as a result of its interaction with LO phonon [4]. Of course, in the process, the electron will continue to gain energy from the electric field. In the steady state, the processes of the electron energy loss by LO phonon emission and electron energy gain from the electric field will come into the balance and the electron will propagate through the semiconductor with a velocity known as the saturation velocity. The experimental values for this saturated drift velocity generally fall in the range 10^7 cm/s to 10^8 cm/s [5].

For polar semiconductors, the process of the LO phonon emission plays a major role in determining the value of saturation velocity [6, 7]. In non-polar materials such as Si, the deformation potential interaction results in electron energy loss through the emission of phonons. In all such cases, the electron mobility will be influenced strongly by the interaction of electrons with phonons. The saturation velocity of carriers in a semiconductor provides a measure of operating speed of a microelectronic or nanoelectronic device. The minimum time for the carriers to travel through the active region of the device is given approximately by the length of the device divided by the saturation velocity. The practical switching time of such a microelectronic device will be limited by the saturation velocity and thus it is clear that phonons play a major role in the fundamental and practical limits of such microelectronic devices [8].

The importance of carrier-phonon interactions in modern semiconductor devices is also recognized by the dynamics of carrier capture in the active quantum well region of a polar semiconductor quantum-well LASER [9, 10]. For the LASER to operate, an electron must loose enough energy to be 'captured' by the quasi bound state. This energy is of the order of 100 meV or more for many quantum well semiconductor LASERs. The energy loss rate of the carrier (thermalization rate) in polar semiconductor quantum well is determined by both, the rate at which the carrier's energy is lost by optical phonon emission and the rate at which the carrier gains energy from optical phonon absorption. The rate at which the carriers gain energy from optical phonon absorption can be significant in quantum wells due to the accumulation of emitted phonons by energetic carriers. Since the phonon density in many dimensionally confined semiconductor devices are typically well above those of the equilibrium phonon population, there is a probability that these non-equilibrium or 'hot'-phonons will be reabsorbed [11]. Clearly the net energy loss by an electron in such a situation depends on the rates of both phonon absorption and phonon emission.

In the present theoretical analysis, we have studied the interaction between electrons and phonons by means of scattering via deformation potential coupling mechanism [12, 13] for two dimensional nanostructures which will be applied to dilute nitrides in the further chapters. For the sake of completeness, we briefly describe scattering by other means (for example electron–alloy, electron-electron etc.) and factors affecting these scattering mechanisms.

2.2 Confinement Effects on Phonon Interactions

Phonon interactions are altered unavoidably by the effects of dimensional confinement on the phonon modes in nanostructures [14, 15]. These effects exhibit some similarities to those for an electron confined in a quantum well. The dimensional confinement of phonons restricts the phase space of the phonons; therefore, it is certain that the carrier- phonon interactions in nanostructures will be modified by phonon confinement [16]. These confined phonons and its interaction with carriers play a critical role in determining the properties of electronic, optical and superconducting devices containing nanostructures as essential elements. Examples of such phonon effects in nanoscale devices include: phonon effects in intersubband LASERs; the contribution of confined phonons on the gain of intersubband LASERs; the contribution of confined phonons to valley current in double barrier quantum well structures; phonon-enhanced population inversion in asymmetric double barrier quantum well LASERs; and confined phonon effects in thin film superconductors [17, 18]. Because electrons confined within a small potential well experience a rapidly varying potential, the carrier's wave nature becomes important. For the quantum well illustrated in above Fig.2.1 (a), electrons are confined in the z- direction but are free to move in x-y plane. For the quantum wire illustrated in Fig. 2.1(b), electrons are confined in x-y plane but are free to move in z-direction. Such structures are produced with the semiconductor heterojunctions.

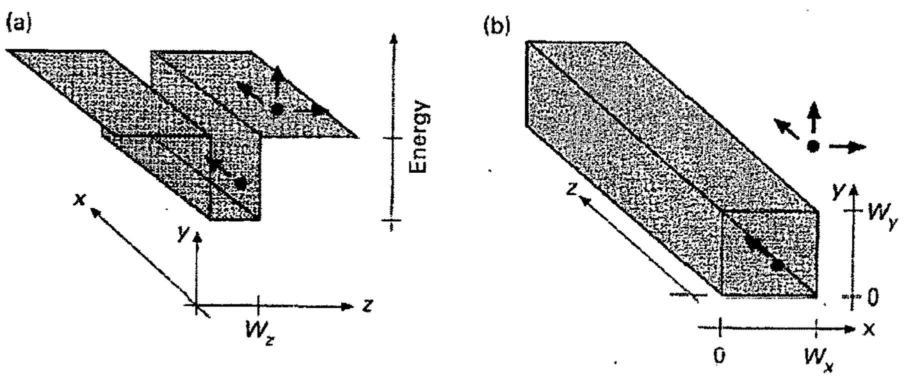

Fig.2.1 Electrons confined in a (a) quantum well and (b) quantum wire.

2.3 Density of States (DOS)

The density of states (DOS) is defined as the number of electronic states present in a unit energy range. The DOS is an important for the calculation of various optical properties such as the rate of absorption or emission [19-21]. The DOS also reflects their distribution of electrons and holes within a solid. Semiconductor heterostructures allows one to change the band energies in a controlled way and confine charge carriers to Two (well), One (wire) or Zero (dot) spatial dimensions. Due to the confinement of carriers, the dispersion relation along the confinement direction is changed. The change in dispersion relation results in a change in the density of states. For bulk, the density of states in terms of energy is expressed as

$$g(E)_{3D}\,dE=\frac{k^2dk}{\pi^2}=\frac{2mE}{\pi^2\hbar^2}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\frac{m}{\hbar^2}dE=\frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E^{1/2}dE \qquad (2.1)$$

This gives the density of states per unit volume per unit energy at a wave vector-**k** [For details see Appendix I]. Below, we describe briefly the density of states for the confined systems.

2.3.1 Density of States in 2D Structures (Quantum well)

In two dimensional structures such as the quantum well, the procedure is much the same but one of the **k**-space components is fixed instead of finding the number of **k**-states enclosed within a sphere. The problem is to calculate the number of **k**-states lying in an annulus of radius **k** to **k+dk, k**-space would be completely filled if each state occupied an area of

$$V_{2D} = \left(2\pi/L\right)^2 \tag{2.2}$$

The 'volume' of the annulus is given by

$$V_{s2D}\, d\mathbf{k} = 2\pi |k|\, d\mathbf{k} \tag{2.3}$$

In terms of energy per unit volume at an energy E, the density of states for 2D structures can be expressed as

$$g(E)_{2D}\, dE = \frac{\mathbf{k}d\mathbf{k}}{\pi} = \sqrt{\frac{2mE}{\hbar^2}} \left(\frac{2mE}{\hbar^2}\right)^{-1/2} \frac{m}{\hbar^2} dE = \frac{m}{\pi\hbar^2} dE \tag{2.4}$$

Dividing the 'volume' of the **k**-state by the area of the annulus and multiply by 2 to account for the electron spin states, the density of states can be expressed as

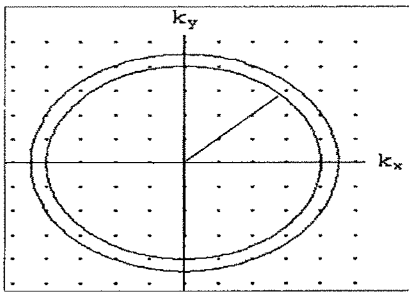

Fig. 2.2 **k**-space in 2D. The density of states at an Energy E

$$g(k)_{2D}\,dk = 2\times\frac{V_{s2D}}{V_{2D}} = \frac{|k|dkL^2}{\pi}$$

It is significant that the 2D density of states does not depend on energy. Immediately, as the top of the energy-gap is reached, there is a significant number of available states. Taking into account the other energy levels in the quantum well, the density of states takes on a staircase like function given by:

$$g(E)_{2D}\,dE = \frac{m}{\pi\hbar^2}\sum_i H\left(E-E_i\right)dE \tag{2.5}$$

where, $H\,(E\text{-}E_i)$ is the Heaviside function. It takes the value of zero when E is less than E_i and 1, when E is equal to or greater than E_i. E_i is the i-th energy level within the quantum well.

2.3.2 Density of States in 1D Structures (Quantum Wire)

In one dimension two of the k-components are fixed, therefore the area of k-space becomes a length and the area of the annulus becomes a line.

$$V_{s1D} = 2d\mathrm{k} \quad V_{1D} = {}^{2\pi}\!/_{L}$$

Therefore the density of states per unit length in 1-d and multiplying by 2 for spin degeneracy,

$$g(\mathrm{k})_{1D}\,d\mathrm{k} = \frac{V_{s1D}}{V_{1D}} = 2\times\frac{L}{\pi}d\mathrm{k} \tag{2.6}$$

For one dimension, the density of states per unit volume at energy E is given by

$$g(E)_{1D}\,dE = \frac{2d\mathrm{k}}{\pi} = \frac{2}{\pi}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\frac{m}{\hbar^2}dE = \frac{1}{\pi}\left(\frac{m}{\hbar^2}\right)\frac{1}{\sqrt{E}}dE \tag{2.7}$$

Using more than the first energy level, the density of states function becomes

$$g(E)_{1D}\, dE = \frac{1}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \sum_i \left(\frac{n_i \mathbf{H}\left(E-E_i\right)}{\left(E-E_i\right)^{1/2}} \right) dE \tag{2.8}$$

Where once again, ***H*** (*E*-*E*$_i$) is the Heaviside function and n_i is the degeneracy factor. For quantum structures with dimensions lower than 2, it is possible for the same energy level to occur for more than one arrangement of confined states. To account for this, a second factor $n_i(E)$ is introduced.

Type	Delocalized Dimension	Confined Dimension
Quantum dot	0	3
Quantum wire	1	2
Quantum well	2	1
Bulk	3	0

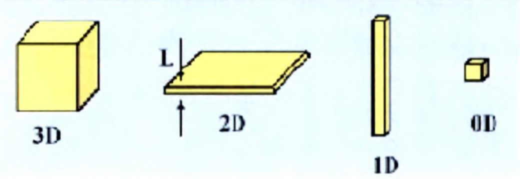

Table 2.1 Delocalized and confined dimensions of nanostructures.

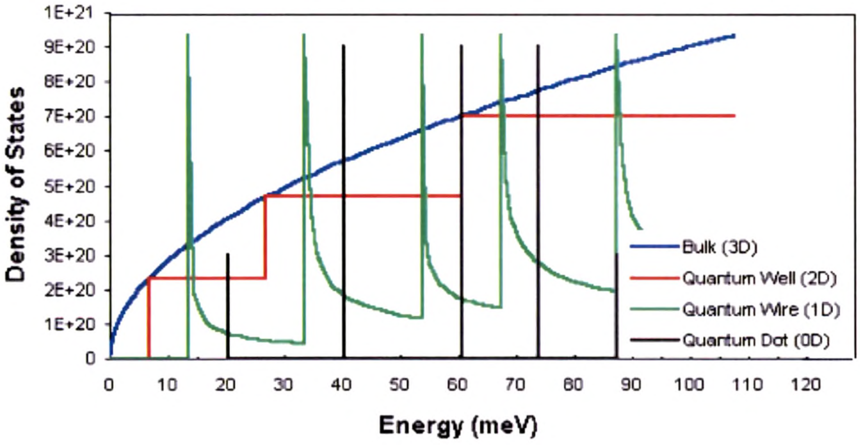

Fig. 2.3 Density of states for bulk (3D blue), quantum well (2D red), quantum wire (1D green) and quantum Dot (0D black).

2.3.3 Density of States in 0D (Quantum Dot)

In a 0-D structure, the values of k are quantized in all directions. All the available states exist only at discrete energies described and can be represented by a delta function. In real quantum dots, however, the size distribution leads to a broadening of this line function. Table 1 shows Delocalized and confined dimensions of nanostructures. In the quantum well, the ratio of the i^{th} energy level to the ground state level E_0 is proportional to k^2 and in each case since there is only one way in which k can be arranged to obtain this energy, the degeneracy is of each energy level is just 1. The quantum wire has two values of k to control its energy and therefore the energy is proportional to the sum of the squares of each **k** value. (For example: $5=1^2+2^{2)}$ and where the values of **k** are different there are two ways of obtaining the same ratio of energy. Therefore in this case the degeneracy is two. The quantum box is confined in all three **k**-directions and its energy is proportional to the sum of the squares of each ***k*** values. As one might expect, the scope for degenerate energy levels is also greater. Where the ***k*** values are all different, there are 6 ways in which the ***k*** values can be arranged to produce the same value of energy.

2.4 Relaxation Times

In the present section, we describe the most fundamental quantity of carrier scattering mechanism and evaluate the transition rate, $S(\mathbf{p_0},\mathbf{p})$, which is the probability per unit time that a carrier with crystal momentum *p* scatters to a state with crystal momentum **p'**. The approach is based on well known Fermi's golden rule. The entire description is under the parabolic band approximation. The interaction between electron and lattice vibrations is mainly considered via scattering which is considered by introducing the concept of relaxation time τ; the time which is closely related to the mean time of flight between collisions of electron with lattice vibrations. The scattering rate for the carriers is, the rate at which carriers with specific momentum $\mathbf{p}_0$ scatter to any other state and is expressed by,

$$\frac{1}{\tau(\mathbf{p}_0)} = \sum_{\mathbf{p}',\uparrow} S(\mathbf{p}_0,\mathbf{p}')[1-f(\mathbf{p}')] \qquad (2.9).$$

Here, S($\mathbf{p}_0$,$\mathbf{p}'$) is the transition rate and defined as

$$S(\mathbf{p},\mathbf{p}') = \frac{2\pi}{\hbar}\left|H_{\mathbf{p},\mathbf{p}'}\right|^2 \delta\left(E(\mathbf{p}')-E(\mathbf{p})-\Delta E\right) \qquad (2.10)$$

Here, ΔE is the change in energy caused by the scattering event. $\mathbf{H}_{\mathbf{p}_0,\mathbf{p}}$ is the matrix element defined as

$$\mathbf{H}_{\mathbf{p},\mathbf{p}'} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\mathbf{p}'\cdot\mathbf{r}/\hbar}\, U_s(\mathbf{r})\, e^{i\mathbf{p}'\cdot\mathbf{r}/\hbar}\, d^3\mathbf{r} \qquad (2.11)$$

Here, $U_s(\mathbf{r})$ is the perturbing potential. Alternatively $\tau(\mathbf{p}_0)$ is the average time between collisions (also known as life time of the state, $\mathbf{p}_0$). The vertical arrow below the sum is to indicate that the sum over final state includes only those whose spin is parallel to that of the incident carrier (the scattering mechanisms we consider do not flip the carrier's spin). The factor of [1-f($\mathbf{p}'$)],where f($\mathbf{p}'$) is the probability that the state at $\mathbf{p}'$ is occupied, gives the probability of finding an empty final state. For a non-degenerate semiconductor, there is a high probability that the state at $\mathbf{p}'$ is empty, and in this case Eqn. (2.9) is written as

$$\frac{1}{\tau(\mathbf{p}_0)} = \sum_{\mathbf{p}',\uparrow} S(\mathbf{p}_0,\mathbf{p}') \qquad (2.12)$$

To evaluate the rate at which the z-directed momentum is relaxed, we need to weight each collision by the fractional change in the z-directed momentum. Therefore, the momentum relaxation time is expressed as [21]

$$\frac{1}{\tau_m(\mathbf{p_0})} = \sum_{\mathbf{p}',\uparrow} S(\mathbf{p_0},\mathbf{p}')\left[1 - {p'_z}/{p_{z0}}\right] = \sum_{\mathbf{p}',\uparrow} S(\mathbf{p_0},\mathbf{p}')\left[1 - \left({\mathbf{p}'}/{\mathbf{p_0}}\right)\cos\alpha\right] \tag{2.13}$$

where α is the polar angle between the incident and scattered momenta. The energy relaxation rate $\tau_E(\mathbf{p_0})$, is expressed as

$$\frac{1}{\tau_E(\mathbf{p_0})} = \sum_{\mathbf{p}',\uparrow} S(\mathbf{p_0},\mathbf{p}')\left[1 - {E(\mathbf{p}')}/{E(\mathbf{p_0})}\right] \tag{2.14}$$

2.5 Carrier Scattering Mechanisms

To understand the carrier scattering mechanism, it is first essential to know the perturbing potential responsible for scattering and the distribution of carriers so that the matrix element can be evaluated. In this section, we briefly discuss the perturbing potential for the common scattering mechanisms, and obtain the scattering rates.

2.5.1 The Distribution Function

In equilibrium, the distribution function $f(\mathbf{k})$, which represents the distribution of carriers in momentum states is expressed as

$$f_0(\mathbf{k}) = \frac{1}{1+\exp\left[(E_k - E_F)/k_B T\right]} \tag{2.15}$$

where $E_\mathbf{k}$ is the energy of the state, E_F is the Fermi energy (or, strictly speaking, the chemical potential), k_B is Boltzmann's constant and T is the absolute temperature. In a spherical energy band, Eq.(2.15) is symmetric in wavevactor and we would ordinarily write this as a function of energy. Suppose now that the distribution function is displaced by **dk** in **k** space due to the application of an electric field. For a vanishingly small field, we can expand $f(\mathbf{k})$ to first order as

$$f(\mathbf{k}) = f_0(\mathbf{k} - \delta\mathbf{k}) \approx f_0(\mathbf{k}) - \nabla_k f_0(\mathbf{k}) \cdot \delta\mathbf{k} \tag{2.16}$$

(Here, the notation ∇_k just means the gradient with respect to wavevactor coordinates). Since the electron group velocity **v** (**k**) is

$$V(\mathbf{k}) = \nabla_k \omega = (1/\hbar)\nabla_k E_k \tag{2.17}$$

By using the chain rule and writing $f_0(\mathbf{k})$ as a function of E (dropping the **k** subscript), we have

$$f(\mathbf{k}) = f_0(E) - \hbar\left({df_0(E)}\big/{dE}\right)V(\mathbf{k}) \cdot \delta\mathbf{k} \tag{2.18}$$

For the time being, we shall assume that there is no magnetic field. Using the definition of average **k** displacement, $\delta\mathbf{k} = -eF\tau/\hbar$, the distribution function can be written

$$f(k) = f_0(E) + e\tau(E)\left({df_0(E)}\big/{dE}\right)V(k) \cdot F \tag{2.19}$$

This is now in the form $f(\mathbf{k}) = f(E) = f_0(E) + f_1(E)$ where, by assumption, $f_1(E) << f_0(E)$. Note that although $f_1(E)$ looks positive in Eqn.(2.19), since $df_0/dE < 1$, in fact it has the opposite sign to **F**. This is as we would expect, since we are considering the displacement of negatively charged electrons.

2.5.2 Deformation Potential Scattering

Within the spirit of Born-Oppenheimer approximation let us assume that the electrons can respond instantaneously to the ionic motion so that the electron – phonon interaction Hamiltonian can be expressed as a Taylor series expansion of the electronic Hamiltonian $\mathbf{H}_e$ (r_i, Rj):

$$\mathbf{H}_{e-ion}(\mathbf{r}_i,\delta R_j)=\sum_j\left(\frac{\partial H_e}{\partial R_j}\right)\Bigg|_{R_{j0}}\cdot\delta R_j+\ldots\ldots \qquad (2.20)$$

Usually the electronic Hamiltonian, $\mathbf{H}_e(\mathbf{r}_i,R\mathrm{j})$ is not known, and therefore approximations are required to calculate the electron-phonon interaction.

Let us assume that the electronic energies E_{nk} (where n is the band index and **k** is the wave vector) are known so that $\partial H_e/\partial R_j$ can be approximated. If we assume that band n is non-degenerate, the constant $\left(\partial En_k/\partial R_j\right)$ represents simply the shift of the electronic band energy caused by a static displacement of the atoms. In the case of long-wavelength acoustic phonons, the atomic displacements can correspond to a deformation of the crystal (Deformation Potential Theorem). Such deformation will change the electronic energies at different points in the Brillouin zone; the parameters which describe these changes in the electronic energies induced by static distortions of the lattice are known as **deformation potentials.** Thus the coefficient ($\partial E_{nk}/\partial R_j$) is related to deformation potentials of the crystal [22- 25].

Within the limit of zero wave vector or infinite wavelength, an acoustic phonon becomes a uniform translation of the crystal. Obviously such translations will not alter the electronic band structure; hence, if all ∂R_j's are identical, the change in E_{nk} is zero. Thus we have to assume that an acoustic phonon has a non zero but small wave vector in order to couple to electrons and consider the gradient of the atomic displacements as

$$\mathbf{d}_{ij}=\partial(\delta R_j)/\,\partial R_j \qquad (2.21)$$

where d_{ij} is a second rank tensor which can be decomposed in to the sum of symmetric tensor e_{ij} and an antisymmetric one f_{ij}'.

We envisage an energy band structure with valance band maxima and conduction band minima as shown in the Fig.[2.3] wherein the left figure is for normal atomic volume and the right figure is for dilation Δ resulting from the transitional change in volume. The shifts in the energy of the band extrema will in general in general be linear in the dilatation Δ. The proportionality constant is called the deformation potential constant [23].

$$\delta V_C = D_c \Delta; \quad \delta V_v = D_v \Delta \qquad (2.22)$$

Thus an electron at the conduction-band minimum will have its energy shifted by an amount proportional to the dilatation Δ. It is then assumed that if the dilatation varies with position there is an effective potential seen by the electron which also varies with position.

$$\delta V_c(r) = D_c \Delta(r) \;\; ; \;\; \delta V_v(r) = D_v \Delta(r) \qquad (2.23)$$

The effective potentials would of course be slightly different for electrons away from the band extrema. However, these constants are evaluated at the band minimum and used for all electrons near the minimum.

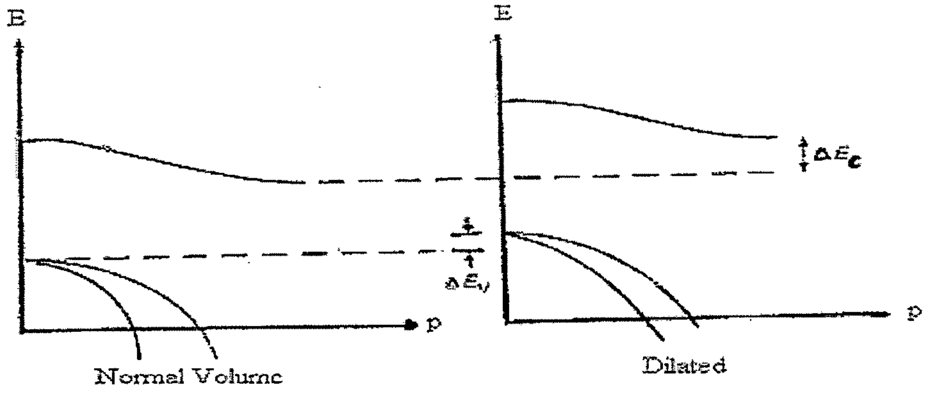

***Fig. 2.3** Effect of change in lattice spacing due to change in the atomic volume on the band structure of a semiconductor.*

2.5.2 (a) Acoustical Deformation Potential (ADP) Scattering

Because a semiconductor band structure is determined by the crystal potential, it is influenced by changes in lattice spacing. For a small change in lattice constant, there is a change in conduction and valance band energy levels which is expressed as,

$$\delta E_c = D_c \left(\delta a / a\right) \tag{2.24}$$

and

$$\delta E_v = D_v \left(\delta a / a\right) \tag{2.25}$$

where D_c and D_v are deformation potentials. Let us consider an elastic wave,

$$u(x,t) = A_\beta e^{i(\beta z - \omega t)} + A_\beta^* e^{-i(\beta z - \omega t)} \tag{2.26}$$

This wave is propagating in one dimension only. For acoustic phonons the changes in lattice spacing are produced by the strain $U_{AP}(x,t) = D_A \, \partial u / \partial x$. Therefore the interaction potential for acoustic phonons is expressed as

$$U_{AP}(x,t) = D_A \, \partial u / \partial x \tag{2.27}$$

This interaction potential applies only to longitudinal phonons as transverse elastic waves produce no first order change in lattice spacing and hence no dilation. Similarly, the interaction potential for optical phonons is expressed as

$$U_{op}(x,t) = D_o u(x,t) \tag{2.28}$$

The perturbing potential for phonon scattering is expressed as,

$$U_S = \mathbf{K}_\beta u_\beta \tag{2.29}$$

where $\mathbf{K}_\beta$ = wave vector and u_β = Fourier component of lattice vibration

$$u_{\beta} = A_{\beta} e^{i(\beta z-\omega t)} + A_{\beta}^{*} e^{-i(\beta z-\omega t)} \tag{2.30}$$

By using Eq.(2.30) and Eq.(2.29), we get the scattering potential as follows

$$U_S = \mathbf{K}_{\boldsymbol{\beta}} A_{\beta} e^{i(\boldsymbol{\beta} z-\omega t)} + \mathbf{K}_{\boldsymbol{\beta}} A_{\beta}^{*} e^{-i(\boldsymbol{\beta} z-\omega t)} \tag{2.31}$$

Now, for the plane wave confined to a normalization length, –L/2 ≤ z ≤ L/2, wave function is given as

$$\psi(z) = \frac{1}{\sqrt{L}} e^{i\mathbf{k}z} \tag{2.32}$$

The matrix element for above case will take the form

$$\mathbf{H}_{\mathbf{k'k}} = \int_{-L/2}^{L/2} \Psi(z) U_s(z) \Psi^{*}(z) dz \tag{2.33}$$

$$= \int_{-L/2}^{L/2} (1/\sqrt{L}) e^{i\mathbf{k}z} [\mathbf{K}_{\boldsymbol{\beta}} A_{\beta} e^{i(\boldsymbol{\beta} z-\omega t)} + \mathbf{K}_{\boldsymbol{\beta}} A_{\beta}^{*} e^{-i(\boldsymbol{\beta} z-\omega t)}] \, 1/\sqrt{L} \, e^{-i\mathbf{k}'z} dz$$

$$= \mathbf{K}_{\boldsymbol{\beta}} A_{\beta} \int_{-L/2}^{L/2} (1/L)[e^{i(\mathbf{k}-\mathbf{k}'\pm\boldsymbol{\beta})z}] dz \tag{2.34}$$

Now according to Fermi's golden rule, the scattering rate is given by

$$S(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} \left| \mathbf{H}_{\mathbf{k'k}} \right|^{2} \delta(E(\mathbf{k}') - E(\mathbf{k}) \pm \hbar\omega) \tag{2.35}$$

$$= \frac{2\pi}{\hbar} \left| K_{\beta} \right|^{2} \left| A_{\beta} \right|^{2} \delta_{\mathbf{k}',\mathbf{k}\pm\boldsymbol{\beta}} \, \delta(E(\mathbf{k}') - E(\mathbf{k}) \pm \hbar\omega)$$

$$= \frac{2\pi}{\hbar^{2} v\beta} \left| \mathbf{K}_{\boldsymbol{\beta}} \right|^{2} \left| A_{\beta} \right|^{2} \delta\left(\pm\cos\theta + \frac{\hbar\boldsymbol{\beta}}{2p} \pm \frac{\omega}{v\boldsymbol{\beta}}\right) \tag{2.36}$$

Here $|A_\beta|^2$ is the amplitude square of lattice vibration [See Appendix II]

$$\left|A_\beta\right|^2 = \frac{\hbar}{2\rho\omega_\beta\Omega}\left(N_\beta + \tfrac{1}{2} \pm \tfrac{1}{2}\right) \tag{2.37}$$

and

$$\left|\mathbf{K}_{\boldsymbol{\beta}}\right|^2 = \boldsymbol{\beta}^2 D_A^2 \tag{2.38}$$

By using Eq.(2.37), Eq.(2.38) and Eq.(2.36), the transition rate can be expressed as

$$S(\mathbf{k}'\mathbf{k}) = \frac{2\pi}{\hbar^2 v\beta}\beta^2 D_A^2\left(\frac{\hbar}{2\rho\omega_\beta\Omega}\left(N_\beta + \tfrac{1}{2} \pm \tfrac{1}{2}\right)\right)\delta\left(\pm\cos\theta + \frac{\hbar\boldsymbol{\beta}}{2p} \pm \frac{\omega}{v\boldsymbol{\beta}}\right)$$

$$= \frac{\pi\beta D_A^2}{\hbar v\rho\omega\Omega}\left(N + \tfrac{1}{2} \pm \tfrac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta}\right) \tag{2.39}$$

Here, $\frac{\omega_\beta}{\beta} = v_s$ = sound velocity and v = p/m* = velocity. The transition rate in terms of **k**-wave vector can be expressed in terms of momentum wave vectors as

$$S(\mathbf{p},\mathbf{p}') = \frac{\pi m^* D_A^2}{\hbar\rho\omega_\beta\Omega p}\left(N_\beta + \tfrac{1}{2} \pm \tfrac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar\boldsymbol{\beta}}{2\mathbf{p}} \pm \frac{\omega}{v\boldsymbol{\beta}}\right),$$

$$= C_\beta\left(N_\beta + \tfrac{1}{2} \pm \tfrac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar\boldsymbol{\beta}}{2\mathbf{p}} \pm \frac{\omega}{v\boldsymbol{\beta}}\right) \tag{2.40}$$

The total transition rate can be obtain by substituting S(p,p') in the expression of $1/\tau(p)$ and summing all over the momentum space

$$\frac{1}{\tau(p)} = \sum S(p,p')$$

$$= \frac{\Omega}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^{\infty} C_\beta (N_\beta + \tfrac{1}{2} \pm \tfrac{1}{2}) d\beta \int_{-1}^{1} \delta(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta}) d\cos\theta$$

By using the identity of δ-function, the scattering rate for electron – phonon interaction is expressed as

$$\therefore \frac{1}{\tau(p)} = \frac{\Omega}{(2\pi)^2} d\beta \int_{\beta_{\min}}^{\beta_{\max}} C_\beta (N_\beta + \frac{1}{2} \pm \frac{1}{2}) \beta^2 d\beta \tag{2.41}$$

The **Momentum Relaxation Time**, the scattering rate can be written as

$$\frac{1}{\tau_m(p)} = \sum_\beta S(p)(1 - \frac{p'}{p}\cos\alpha) \tag{2.42}$$

where α is the polar angle between p and p'.[See Annexure III]

$$1 - \left(\frac{p'}{p}\right)\cos\alpha = 1 - \frac{(\bar{p})\cdot((\bar{p}) \pm \hbar\beta)}{p^2} = \pm\frac{\hbar\beta}{p}\cos\theta \tag{2.43}$$

$$\therefore \frac{1}{\tau_m(p)} = \sum_\beta s(p', p)(\pm\frac{\hbar\beta\cos\theta}{p}) \tag{2.44}$$

$$= \frac{\Omega}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^{\infty} (N_\beta + \frac{1}{2} \pm \frac{1}{2}) C_\beta \beta^2 d\beta \int_{-1}^{1} \delta(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta})(\pm\frac{\hbar\beta}{p}\cos\theta) d\cos\theta$$

$$= \frac{\Omega}{(2\pi)^2} \int_{\beta_{\min}}^{\beta_{\max}} (N_\beta + \frac{1}{2} \pm \frac{1}{2}) C_\beta \beta^2 (\frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta}) \frac{\hbar\beta}{p} d\beta$$

where, $\int_{-1}^{1} \delta(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta})(\pm\frac{\hbar\beta}{p}\cos\theta) d\cos\theta = (\frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta})$. By using the δ-function identity, the momentum scattering rate is expressed as

$$\frac{1}{\tau_m(p)} = \frac{\Omega}{(2\pi)^2} \int_{\beta_{\min}}^{\beta_{\max}} (N_\beta + \frac{1}{2} \pm \frac{1}{2}) C_\beta (\frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta}) \frac{\hbar\beta^3}{p} d\beta \tag{2.45}$$

Here, for acoustic phonons, $\omega = \omega_s = \beta \upsilon_s$ and for optical phonons, $\omega = \omega_0 \approx$ constant. In this case the scattering rate for electron phonon interaction is given as

$$\therefore \frac{1}{\tau(p)} = \frac{\Omega}{(2\pi)^2} \int_{\beta_{\min}}^{\beta_{\max}} C_\beta \left(N_\beta + \frac{1}{2} \pm \frac{1}{2}\right) \beta^2 d\beta \tag{2.46}$$

where, $C_\beta = \dfrac{\pi m^* D_A^2}{\hbar \rho \omega_\beta \Omega p} = \dfrac{\pi m^* D_A^2}{\hbar \omega_\beta m p}$ $\quad \{\because \rho\Omega = m\}$

$$\therefore \frac{1}{\tau(p)} = \frac{\Omega}{(2\pi)^2} \int_{\beta_{\min}}^{\beta_{\max}} \left(N_\beta + \frac{1}{2} \pm \frac{1}{2}\right) \frac{\pi m^* D_A^2}{\hbar m \nu_s p} \beta^2 d\beta$$

$$= \frac{m^* D_A^2}{4\pi \hbar \rho \nu_s p} \int_{\beta_{\min}}^{\beta_{\max}} \left(N_\beta + \frac{1}{2} \pm \frac{1}{2}\right) \beta^2 d\beta \tag{2.47}$$

At room temperature $N_\beta = (N + \frac{1}{2} \pm \frac{1}{2}) \approx N\omega_s$

$$\therefore \frac{1}{\tau(p)} = \frac{m^* D_A^2}{4\pi \hbar \rho \nu_s p} \int_{\beta_{\min}}^{\beta_{\max}} (N\omega_s) \beta^2 d\beta$$

By using equipartition theorem, $N_{\omega s} \approx k_B T_L$, the scattering rate is expressed as

$$\frac{1}{\tau(p)} = \frac{m^* D_A^2 k_B T_L}{4\pi \hbar^2 C_l p} \left(\beta_{\max}^2 - \beta_{\min}^2\right) \tag{2.48}$$

Now for acoustic phonons, with wave vectors near the zone center $\omega \alpha \beta$, the phonon momentum is given by,

$$p = \hbar\beta = 2p\left[\pm\cos\theta \pm \frac{v_s}{v(p)}\right] \tag{2.49}$$

For phonon absorption, $\beta=\beta_{max}$ when $\theta=0$,the phonon momentum will takes the form,

$$\hbar\beta_{max}\,(aco.phonon) \cong 2p[1 \pm {}^{v_s}\!/_{v(p)}] \tag{2.50}$$

In the case of ADP scattering being approximately elastic near room temperature, $\hbar\beta_{max} = 2p = 2m^* v(p)$ and $\hbar\beta_{min} = 0$, the momentum relaxation time is expressed as

$$\frac{1}{\tau(p)} = \frac{1}{\tau_m(p)} = \frac{\pi D_A^2 k_B T_L}{\hbar C_l} g_c(E) \tag{2.51}$$

where, $g_c(E)$ is the density of states. Eq. (2.51) indicates that the scattering and momentum relaxation rates are equal but only when acoustic phonon scattering can be regarded as elastic; acoustic phonon scattering is isotropic. However, at low temperatures, ADP scattering is anisotropic.

2.5.2 (b) Optical Deformation Potential (ODP) Scattering

Since at higher temperature, the optical phonon becomes important, in the present section we briefly present the optical deformation potential scattering. For optical phonons, which displace neighboring atoms in opposite directions, the displacement produces a change in lattice spacing directly, and in this case the scattering due to optical phonons is given on the basis of Eqn. (2.28) for optical phonon deformation potential. In three dimensional semiconductors, optical phonon vibration consists of one sub lattice moving against the other. In contrast to the simple change in volume of unit cell produced by longitudinal acoustical phonons, optical phonon scattering is sensitive to the symmetry of the crystal. The scattering rate for the carrier – optical phonon scattering is written as

$$\therefore S(p',p) = \frac{\pi D_o^2}{\hbar v\beta\rho\omega\Omega}\left(N_0 + \frac{1}{2} \pm \frac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta}\right)$$

$$= \frac{\pi D_o^2}{\hbar p\beta\rho\omega_0 \Omega}(N_0 + \frac{1}{2} \pm \frac{1}{2})\delta(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta}) \tag{2.52}$$

where, $v = (p/m)$ and $D_o^2 = |K_\beta|^2$

The total scattering rate can be obtained as

$$\frac{1}{\tau_m(p)} = \sum_\beta S(p',p)$$

$$= \frac{\Omega}{(2\pi)^3}\int_0^{2\pi} d\phi \int_0^{\infty} (N_o + \frac{1}{2} \pm \frac{1}{2})C_\beta \beta^2 d\beta \int_{-1}^{1} \delta(\pm\cos\theta + \frac{\hbar\beta}{2p} \pm \frac{\omega}{v\beta})d\cos\theta$$

$$= \frac{\Omega}{(2\pi)^2}\int_{\beta_{\min}}^{\beta_{\max}} (N_o + \frac{1}{2} \pm \frac{1}{2})C_\beta \beta^2 d\beta$$

$$= \frac{\Omega}{(2\pi)^2}\frac{\pi D_o^2 m^*}{\hbar p\rho\omega_0 \Omega}\int_{\beta_{\min}}^{\beta_{\max}} [(N_o + \frac{1}{2} \pm \frac{1}{2})]\frac{\beta^2}{\beta} d\beta$$

$$= \frac{D_o^2 m^*}{4\pi\hbar p\rho\omega_0}(N_o + \frac{1}{2} \pm \frac{1}{2})\int_{\beta_{\min}}^{\beta_{\max}} [\beta d\beta] \tag{2.53}$$

Where, ħω$_o$ is optical phonon energy and $\hbar\beta = p[\pm\cos\theta \pm \sqrt{\cos^2\theta \pm {}^{\hbar\omega_o}/_{E(p)}}]$

When, θ=0, β=β$_{\max}$ and can be expressed as

$$\hbar\beta_{\max} = p[1 \pm \sqrt{1 \pm {}^{\hbar\beta}/_{E(p)}}]$$

$$\beta_{\max} = \frac{p}{\hbar}[1 \pm \sqrt{1 \pm {}^{\hbar\beta}/_{E(p)}}] \approx 2\frac{p^2}{\hbar^2}$$

and $\beta_{\min} = 0$

Now from Eq. (2.52), we have

$$\frac{1}{\tau(p)} = \frac{1}{\tau_m(p)}$$

$$= \frac{D_o^2 m^*}{4\pi\hbar p\rho\omega_0}\left(N_o + \frac{1}{2} \pm \frac{1}{2}\right)\left[\beta_{\max}^2 - \beta_{\min}^2\right]$$

By using the value of $\beta_{\max}$ we have, [See Appendix IV]

$$\frac{1}{\tau(p)} = \frac{\pi D_o^2}{2\pi\hbar p\rho\omega_0}\left(N_o + \frac{1}{2} \pm \frac{1}{2}\right)\frac{p^2}{\hbar^2} \tag{2.54a}$$

$$= \frac{\pi D_o^2}{2\rho\omega_0}\left(N_o + \frac{1}{2} \pm \frac{1}{2}\right)\frac{m^*}{\pi\hbar^2}\frac{\sqrt{2m^* E(p)}}{\pi\hbar}$$

$$= \frac{\pi D_o^2}{2\rho\omega_0}\left(N_o + \frac{1}{2} \pm \frac{1}{2}\right)g_c\left(E \pm \hbar\omega_o\right) \tag{2.54b}$$

It is seen from Eq. (2.54b) that a carrier with any energy can scatter by absorbing optical phonons but only those whose energy exceeds $\hbar\omega_o$ can emit optical phonons. The high energy carriers shed their energy by emitting optical phonons and the energy loss can be characterized by the energy relaxation time

$$\frac{1}{\tau_E(p)} = \sum_{p',\uparrow} S(p,p')\left(1 - E'/E(p')\right) \tag{2.55}$$

For very high energy carriers, phonon emission greatly exceeds absorption so,

$$\frac{1}{\tau_E(p)} = \sum_{p',\uparrow} S(p,p')\frac{\hbar\omega_o}{E(p)}$$

$$= \frac{1}{\tau(p)} \frac{\hbar\omega_o}{E(p)}$$

$$= \frac{\hbar\omega_0}{E(p)} \frac{\pi D_o^2}{2\rho\omega_0} \left(N_o + \frac{1}{2} \pm \frac{1}{2}\right) g_c\left(E \pm \hbar\omega_o\right)$$

$$= \frac{\hbar\pi D_o^2}{E(p)2\rho} \left(N_o + \frac{1}{2} \pm \frac{1}{2}\right) g_c\left(E \pm \hbar\omega_o\right) \qquad (2.56)$$

From above equation the relation between momentum and energy relaxation time can be obtained. The quantity ($E(p)/\hbar\omega_o$) signifies the number of optical phonons that must be emitted to remove the carrier's kinetic energy E(p) so that the $\tau_E(p)$ may greatly exceed $\tau_m(p)$.

2.5.2 (c) Polar Acoustical Phonon (PAP) Scattering

In polar semiconductor, acoustic phonons also produce an electrostatic perturbation known as piezoelectric scattering. This effect is much weaker than the polar optical phonon scattering, but is important at very low temperatures when the number of optical phonons is small and carriers do not have sufficient thermal energy to emit them. If a semiconductor crystal consists of dissimilar atoms (e.g. SiC), where the bonds are partly ionic and the unit cell does not contain a centre of symmetry, carriers may be scattered by longitudinal acoustic waves due to piezoelectric scattering [26]. The dielectric displacement $D = \chi_0 E + P$ vanishes, where **P** is polarization. Since there is no space charge due to the spatial displacement of carriers is negligible in comparison to that of ions. For a propagating wave of finite wavelength, a strain $\left(\nabla_r \cdot \delta r\right)$ will exist where δr is the displacement of lattice atom from its equilibrium position and is expressed as

$$\delta r = A\exp\left\{\pm i\left[(k\cdot r)-\omega t\right]\right\} \tag{2.57}$$

The dielectric displacement with this displacement is given by

$$D=\chi\chi_0 E+e_{pz}(\nabla_r\cdot\delta r) \tag{2.58}$$

The factor of proportionality e_{pz}, obtained from piezoelectric measurements, is called piezoelectric constant and is of an order of magnitude 10^{-5} Coul/cm^2.For **D**=0, the electric field will be written as

$$E=-\left({e_{pz}}/{\chi\chi_0}\right)\left(\nabla_r\cdot\delta r\right) \tag{2.59}$$

Now, since **E** is proportional to $\left(\nabla_r\cdot\delta r\right)$, the potential energy, $\delta\varepsilon=|e|\int E dr=|e|{E}/{q}$. By using the electric field, the potential energy is expressed as

$$\delta\varepsilon=\left[-|e|\left({e_{pz}}/{\chi\chi_0 q}\right)\right]\left(\nabla_r\cdot\delta r\right) \tag{2.60}$$

When this relation compared with equation for acoustical scattering reveals that instead of ADP constant D_A, we now have $\left[|e|\left({e_{pz}}/{\chi\chi_0 q}\right)\right]$, which in contrast to D_A is not a constant but depends on $\mathbf{q}=|\mathbf{k'}-\mathbf{k}|\approx 2\mathbf{k}\sin(\theta/2)=2\left({mv}/{\hbar}\right)\sin(\theta/2)$, hence the absolute magnitude of matrix element $\mathbf{H_{k'k}}$ is given by

$$\begin{aligned}\left|\mathbf{H_{k'k}}\right| &= \frac{|e|e_{pz}}{\chi\chi_0 q}\left(\frac{k_B T}{2VC_l}\right)^{1/2}\\ &= \left(\frac{e^2K^2k_BT}{2VC_l\chi\chi_0 q}\right)^{1/2}\end{aligned} \tag{2.61}$$

Here K^2 is dimensionless electromechanical coupling coefficient given by $K^2 = \left(e_{pz}^2 \big/ \chi\chi_0 c_l \right)$. The scattering probability can be obtained from Fermi's golden rule and the momentum relaxation rate is expressed as

$$\frac{1}{\tau_m} = \frac{V}{(2\pi)^3} \int \frac{2\pi}{\hbar} \left(\frac{e k_B{}^2 T}{2V\chi\chi_0 q^2} \right) \delta\left(E(k')-E(k)\right) dk (1-\cos\theta) \sin\theta d\theta (2\pi)$$

where q^2 $4k^2 \sin(\theta/2)$ and $dk = \hbar^{-1} \left(\frac{m}{2\varepsilon} \right)^{1/2} d\varepsilon$

$$\therefore \tau_m = \frac{2^{3/2} \pi \hbar^2 \chi\chi_0}{m^{1/2} e^2 K^2 k_B T} \varepsilon^{1/2} \tag{2.62}$$

The Eqn. (2.62) shows that the energy dependence of τ_m as $\varepsilon^{1/2}$.

2.5.2 (d) Polar Optical Phonon (POP) Scattering

In polar semiconductors, like GaAs, the interaction of carriers with the optical mode of lattice vibrations is known as Polar Optical Phonon (POP) scattering. In polar semiconductors, the bond between adjacent atoms is partially ionic; the As atom acquires a slight positive charge and Ga atom a small negative charge. The magnitude of this charge, termed the effective charge, q*, is a fraction of the electronic charge, q, and is determined by the degree of the ionicity of the bond. Deformation of the lattice by phonons perturbs dipole moment between atoms which results in an electric field that scatters carriers. Phonon scattering in polar semiconductor may occur from either acoustic or optical phonons. Polar optical phonon scattering which is neither elastic nor isotropic [27, 28], is a very strong scattering mechanism for compound semiconductors like GaAs. The scattering rate for POP scattering is expressed as

$$\frac{1}{\tau}=\frac{m^{*}q^{2}\omega_{0}}{4\pi\hbar k_{0}\varepsilon_{0}p}\left(\frac{k_{0}}{k_{\infty}}-1\right)\left(N_{0}+\frac{1}{2}\pm\frac{1}{2}\right)\delta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega}{v\beta}\right)$$

$$=\frac{m^{*}q^{2}\omega_{0}}{4\pi\hbar k_{0}\varepsilon_{0}p}\left(\frac{k_{0}}{k_{\infty}}-1\right)\left(N_{0}+\frac{1}{2}\pm\frac{1}{2}\right)\int_{\beta_{\min}}^{\beta_{\max}}\frac{d\beta}{\beta}$$

$$\frac{1}{\tau(p)}=\frac{q^{2}\omega_{0}\left(\frac{k_{0}}{k_{\infty}}-1\right)}{2\pi k_{0}\varepsilon_{0}\hbar\sqrt{2E(p)/m^{*}}}\left[N_{0}\sinh^{-1}\left(\frac{E(p)}{\hbar\omega_{0}}\right)^{1/2}+(N_{0}+1)\sinh^{-1}\left(\frac{E(p)}{\hbar\omega_{0}}\right)^{1/2}\right] \qquad (2.63)$$

where the first term represents POP absorption and the second term represents POP emission. The second term applies only when E(p)>ħω₀ so that emission can occur.

$$\therefore\frac{1}{\tau(p)}=\frac{q^{2}\omega_{0}}{4\pi\hbar k_{0}\varepsilon_{0}\sqrt{2E(p)/m^{*}}}\left(\frac{k_{0}}{k_{\infty}}-1\right)\left(N_{0}+\frac{1}{2}\pm\frac{1}{2}\right)\ln\left(\frac{\beta_{\max}}{\beta_{\min}}\right) \qquad (2.64)$$

To find the $\beta_{\max}$ and $\beta_{\min}$ wave vectors, δ-function in equation of S(p',p) is set to zero, which results in to a quadratic equation

$$\beta^{2}\pm\frac{(2p\cos\theta)}{\hbar}\beta\mp\frac{2p\omega_{0}}{\hbar v}=0 \qquad (2.65)$$

whose solutions give those values of β which satisfy energy and momentum conservation fro a given scattering angle θ. Solving for $\beta_{\max}$ and $\beta_{\min}$ ($-1\le\cos\theta\le+1$), we find

$$\beta_{\max}=\frac{p}{\hbar}\left(1\pm\sqrt{1\pm\frac{\hbar\omega_{0}}{E(p)}}\right) \qquad (2.66)$$

and

$$\beta_{\min} = \frac{p}{\hbar}\left(\mp 1 \pm \sqrt{1 \pm \frac{\hbar\omega_0}{E(p)}} \right) \tag{2.67}$$

The energy relaxation rate due to POP scattering is found by weighing each transition by the fractional change in energy. For high energy electrons, the POP emission dominates and the energy relaxation time is expressed as

$$\tau_E(P) = \left(\frac{E(p)}{\hbar\omega_0} \right) \tau(p) = \left(\frac{2\pi k \varepsilon_0}{q^2 \omega_0^2 \left({}^{k_0}\!/_{k_\infty} \right)} \right) \frac{E(p)\sqrt{{}^{2E(p)}\!/_{m^*}}}{(N_0+1)\sinh^{-1}\left(\frac{E(p)}{\hbar\omega_0} - 1 \right)^{1/2}} \tag{2.68}$$

The momentum relaxation time can be expressed as

$$\frac{1}{\tau_m(p)} = \frac{q^2 \omega_0 \left({}^{k_0}\!/_{k_\infty} - 1 \right)}{4\pi k_0 \varepsilon_0 \hbar \sqrt{2}[{}^{E(p)}\!/_{m^*}]} \left\{ N_0 \sqrt{1 + {}^{\hbar\omega_0}\!/_{E(p)}} + (N_0 + 1)\sqrt{1 - {}^{\hbar\omega_0}\!/_{E(p)}} \right.$$

$$\left. - \frac{\hbar\omega_0 N_0}{E(p)} \sinh^{-1}\left(\frac{E(p)}{\hbar\omega_0} \right)^{1/2} + \frac{\hbar\omega_0 (N_0+1)}{E(p)} \sinh^{-1}\left(\frac{E(p)}{\hbar\omega_0} - 1 \right)^{1/2} \right\} \tag{2.69}$$

2.5.3 Electron – Electron Scattering

When the carrier density is high, collisions between carriers are an important scattering mechanism. There are two types of processes: a binary process in which one carrier collides with another and a collective process in which a carrier interacts with the plasma comprised by carriers and carriers interact with oscillations in the carrier density. Such fluctuations are accompanied by electric fields which oppose the fluctuation and produce oscillations at the plasma frequency

$$\omega_p = \left(\frac{q^2 n}{k_s \varepsilon_0 m^*}\right)^{1/2} \tag{2.70}$$

This sustains only for $\omega_p \tau \geq 1$. The charge density oscillations of the plasma and electric field produced by this are given by $\rho_\beta = A_\beta (e^{\pm i(\beta x - \omega_p t)}$ and $\varepsilon_\beta = \int \frac{\rho_\beta dx}{k_s \varepsilon_0} = \frac{\rho_\beta}{i\beta k_s \varepsilon_0}$ respectively. The interaction potential for electron – electron scattering is expressed as

$$U_{PL} = -q \int \varepsilon_B dx = \frac{q\rho_B}{k_s \varepsilon_0 \beta^2} \tag{2.71}$$

The matrix element for carrier-plasma scattering by using the interaction potential given above can be expressed as

$$H_{p'p} = \frac{qA_\beta}{k_s \varepsilon_0 \beta^2} \delta_{p',p\pm\hbar\beta} \tag{2.72}$$

By equating the classical, electrostatic energy to its quantum mechanical counterpart, we find

$$A_\beta^2 = \frac{\hbar\omega_p k_s \varepsilon_0}{2\Omega}\left(N_p + \frac{1}{2} \mp \frac{1}{2}\right) \tag{2.73}$$

where, Nβ is the number of plasmons (quantized plasma oscillations) as given by the Bose-Einstein factor. The transition rate will be given as

$$S(p,p') = \frac{\pi q^2 \omega_p}{k_s \varepsilon_0 \beta^2 \Omega}\left(N_p + \frac{1}{2} \mp \frac{1}{2}\right)\delta_{p',p\pm\hbar\beta}\,\delta\left(E - E \mp \hbar\omega_p\right) \tag{2.74}$$

$$= C_{\beta}\left(N_{p} + \frac{1}{2} \mp \frac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega_{\beta}}{v\beta}\right)$$

where, $\delta_{p,p\pm\hbar\omega_{\beta}} \quad \delta(E - E \mp \hbar\beta) \rightarrow \frac{1}{\hbar v\beta}\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega_{\beta}}{v\beta}\right)$ and $C_{\beta} = \frac{\pi m^{*} q^{2}\omega_{p}}{\hbar k_{s}\varepsilon_{0}\beta^{2}p\Omega}$

The scattering rate for plasmon scattering can be expressed as

$$\frac{1}{\tau(p)} = \frac{q^{2}\omega_{p}}{4\pi\hbar k_{s}\varepsilon_{0}\sqrt{2E(p)/m^{*}}}\left(N_{p} + \frac{1}{2} \mp \frac{1}{2}\right)\ln\left(\frac{\beta_{\max}}{\beta_{\min}}\right) \qquad (2.75)$$

A large $\beta_{\max}$ refers to short wavelength oscillations, but about one Debye length is required to screen out the charge of a carrier. When $\beta_{\max}$ exceeds about $1/L_D$, the scattering should be treated as a binary collision. Since Eqn. (2.75) does not apply to binary collisions, $\beta_{\max}$ is replaced by β_{co}, which is equal to $\beta_{\max}$ or $1/L_D$, whichever is smaller. The scattering rate for plasmon scattering is written as

$$\frac{1}{\tau(p)} = \frac{q^{2}\omega_{p}\left(N_{p} + \frac{1}{2} \pm \frac{1}{2}\right)}{4\pi\hbar k_{s}\varepsilon_{0}\sqrt{2E(p)/m^{*}}}\ln\left[\frac{\hbar\beta_{co}}{p\left(\mp 1 \pm \sqrt{1 \pm \hbar\omega_{p}/E(p)}\right)}\right] \qquad (2.76)$$

For high carrier densities, plasmon scattering is an important component of the total scattering rate, but when the electron density exceeds about 10^{18}cm^{-3}, and the scattering rate may be high, the effects of electron plasmon scattering can be subtle.

If the plasma oscillations are not heavily damped by phonon scattering, then the total momentum to the plasma, but another will gain momentum. In this case, the electron-plasmon scattering is similar to binary electron-electron scattering, and the biggest effect may be simply to change the shape of the electron distribution function.

2.5.4 Ionized Impurity Scattering

Carriers are scattered when they encounter the electric field of an ionized impurity. The ionized impurity attracts mobile carriers which screen the potential. Therefore, the appropriate perturbing potential is the screened Coulomb potential,

$$U_s(r) = \frac{q^2}{4\pi k_s \varepsilon_0 r} e^{-r/L_D} \tag{2.77}$$

where, $L_D = \sqrt{\frac{k_s \varepsilon_0 k_B T}{q^2 n_0}}$ is the Debye length.

By using the perturbing potential for ionized impurity scattering, the matrix element can be evaluated as

$$H_{p'p} = \frac{1}{\Omega}\left(\frac{q^2}{4\pi k_s \varepsilon_0 r}\right) \int e^{-ip' \cdot r/\hbar} \left(\frac{e^{-r/L_D}}{r}\right) e^{ip \cdot r/\hbar} d^3 r$$

$$= \frac{1}{\Omega}\left(\frac{q^2}{4\pi k_s \varepsilon_0 r}\right) \int_0^{2\pi} d\phi \int_0^{\pi} \int_0^{\infty} e^{-r/L_D} e^{i(p-p')\cdot r/\hbar} r dr \sin\theta d\theta \tag{2.78}$$

According to geometry of the scattering event $\hbar\beta = p' - p$ and $\hbar\beta = 2p\sin(\alpha/2)$, the matrix element takes the form

$$H_{p'p} = \frac{1}{\Omega}\left(\frac{q^2}{4\pi k_s \varepsilon_0}\right) \int_0^{\infty} d\phi \int_0^{2\pi} \int_{-1}^{+1} r e^{r/L_D} e^{-i\beta\cos\theta} d(\cos\theta) d\phi dr \tag{2.79}$$

$$= \frac{1}{\Omega}\left(\frac{q^2}{4\pi k_s \varepsilon_0}\right) \frac{1}{\beta^2 + 1/L_D^2} \tag{2.80}$$

The scattering rate can be expressed as

$$S(p,p') = \frac{2\pi}{\hbar}\left(\frac{q^2}{\Omega k_s \varepsilon_0}\right)^2 \frac{\delta(E'-E)}{\left[\beta^2 + \frac{1}{L_D^2}\right]^2}$$

$$= \frac{2\pi N_I q^4}{\hbar \Omega k_s^2 \varepsilon_0^2} \frac{\delta(E'-E)}{\left[4\left(\frac{p}{\hbar}\right)^2 \sin^2\left(\frac{\alpha}{2}\right) + \frac{1}{L_D^2}\right]^2} \tag{2.81}$$

where, N_I/Ω is the number of impurities in the normalization volume. When mobile carriers are absent, $L_D \rightarrow \infty$, the transition rate takes the form

$$S(p,p') = \frac{2\pi N_I q^4}{\hbar \Omega k_s^2 \varepsilon_0^2} \frac{\delta(E'-E)}{\left[16\left(\frac{p}{\hbar}\right)^2 \sin^2\left(\frac{\alpha}{2}\right) + \frac{1}{L_D^2}\right]^2} \tag{2.82}$$

When the carrier density is very high L_D is small so that $\left(\frac{1}{L_D^2}\right)$ in the denominator of Eq. (2.82) dominates. For this strongly screened case,

$$S(p,p') = \frac{2\pi N_I q^4 L_D^4}{\hbar \Omega k_s^2 \varepsilon_0^2} \delta(E'-E) \tag{2.83}$$

The momentum relaxation time can be evaluated as follows:

$$\frac{1}{\tau_m(p)} = \sum_{p',\uparrow} S(p,p')\left(1 - \frac{p'}{p}\cos\alpha\right)$$

$$= \frac{\Omega}{(2\pi\hbar)^3} \int_0^{2\pi} \int_{-1}^{1} \int_0^{\infty} S(p,p')\left(1 - \frac{p'}{p}\cos\alpha\right) p'^2\, dp'\, d(\cos\theta)\, d\phi \tag{2.84}$$

If we orient the z-axis pointing to the initial momentum, then $\alpha = \theta$, $p = p_z$, $p_z' = p'\cos\theta$ and eq. (2.84) becomes

$$\frac{1}{\tau_m(p)}=\frac{N_I q^4}{2\pi\hbar^4 k_s^2\varepsilon_0^2}\int_0^{\infty}\int_{-1}^{1}\frac{(1-\cos\theta)d(\cos\theta)}{\left[4\left(p/\hbar\right)^2\sin^2(\theta/2)+1/L_D^2\right]}\delta(E'-E)p'^2\,dp' \qquad (2.85)$$

(Here we use p=p' as ionized impurity scattering is elastic). With the substitution, x=(1-cosθ), the integral in eq. (2.85) becomes

$$I_\theta=\int_0^2\frac{x\,dx}{\left[2\left(p/\hbar\right)^2x+1/L_D^2\right]^2}=\frac{1}{4(p/\hbar)^4}\left[\ln(1+4L_D^2(p/\hbar)^2-\frac{4L_D^2(p/\hbar)^2}{1+4L_D^2(p/\hbar)^2}\right]$$

$$=\frac{1}{4(p/\hbar)^4}\left[\ln(1+8m^*E(p)\left(L_D^2/\hbar^2\right)-\frac{8m^*E(p)\left(L_D^2/\hbar^2\right)}{1+8m^*E(p)\left(L_D^2/\hbar^2\right)}\right] \qquad (2.86)$$

After inserting this result for I_θ in eq. (2.85) and integrating over p', we find

$$\tau_m(p)=\frac{16\pi\sqrt{2m^*}k_s^2\varepsilon_0^2}{N_I q^4}\left[\ln(1+(8m^*E(p)L_D^2/\hbar^2)-\frac{(8m^*E(p)L_D^2/\hbar^2)}{1+(8m^*E(p)L_D^2/\hbar^2)}\right]^{-1}E^{3/2}(p) \qquad (2.87)$$

The influence of ionized impurity scattering decreases at high temperatures or at high electric field because both increase the kinetic energy of carriers.

2.6 Summary

In the present chapter, we have described the different mechanisms for the scattering of carriers for the very general case of bulk system. The chapter describes the scattering mechanisms due to carriers, phonons, ionized impurities, etc in the frame work of Fermi's golden rule. The scattering rate and the momentum and energy relaxation rates are evaluated from the transition rate, S (p, p'). Since our motive were to introduce the basic procedure to obtain scattering rates for a general case so that they can be extended for the specific problem of carrier scattering rate for two dimensional structures in the next chapter.

References

1. J E Zucker, A Pinczuk, D S Chemla, A Gossard, W Wiegmann, *Phys. Rev. Lett.* **53**(13) (1984).
2. M A Brummell, R J Nicholas, M A Hopkins, J J Harris, C T Foxon, *Phys. Rev. Lett.* **58,** 77 (1987).
3. K Putteneers, F Brosens, S N Klimin, and J T Devreese, *Solid State Comm.* **135,** 108 (2005).
4. S N Klimin and J T Devreese, *Phys. Rev. B***68,** 245303 (2003).
5. C G Rodrigues, V N Freire, A R Vasconcellos, R Luzzi, *Brazilian J. Phys.* **32,** (2002).
6. B K Ridley, W J Schaff, L F Eastman, *J. Appl.Phys.* **96,** 1499 (2004).
7. D Zanato, N Balkan, B K Ridley, G Hill and W J Schaff, *Semicond. Sci. Technol.* **19,** 1024 (2004).
8. U Goshal et al., *Appl. Phys. Lett.***80,** 3006 (2002).
9. C W Coldren, M C Larson, S G Spruytte, J S Harris, *Electron. Lett.* **36,** 951 (2000)
10. A Wagner et al, *Appl. Phys. Lett.* **76,** 271 (2000).
11. R Gupta, *Tr. J. Phys.* **23,** 551(1999).
12. A J Kent, *Phys. Stat. Sol. (b)* **241 (15),** 3439 (2004).
13. P A Knipp, T L Reinecke, *Phys. Rev. B***52,** 5923 (1995).
14. H Rucker, E Molinari, P Lugli, *Phys. Rev. B44,* 3463 (1991).
15. B A Glavin, V I Pipa, V V Mitin, M A Stroscio, *Phys. Rev. B***65,** 205315 (2002).
16. P J Price, *J. Appl. Phys.* **53,** 6863 (1982).
17. M A Stroscio, *J .Appl. Phys.* **80,** 6864 (1996).

18. J Wang, J P Leburton, Z Moussa, F H Julien, A Sa'ar, *J. Appl. Phys.* **80,** 1970 (1996).

19. S J Lee, N H Shin, J J Ko, M J Park, R Kummel, *Semicond. Sci. Technol.* **7,** 1072 (1992).

20. N W Ashcroft and N D Mermin, *Solid State Physics*, Harcourt Asia Pvt. Ltd.(First Edition (2001).

21. N A Bannov, V A Aristov, V V Mitin, *J. Appl. Phys.* **78(9),** 5503 (1995).

22. M Lundstrom, *Fundamentals of Carrier Transport,* Cambridge University Press, Cambridge (2000).

23. V.Mitin,V Kochelap, M.Stroscio, *Quantum Heterostructures*, Cambridge University Press, Cambridge (1999).

24. V I Pipa, N Z Vagidov, V V Mitin, M A Stroscio, *Phys. Rev.* ***B*64,** 235322 (2001).

25. D A Drabold, *J. Non.-Cryst. Sol.* **266,** 211 (2000).

26. P K Basu and K Bhattacharya, J. *Phys. C: Solid State Phys.* **15,** 5711(1982).

27. D Chattopadhyay, *Phys. Rev.* ***B*33,** 7288 (1986).

28. B L Gelmont, M Shur, M Stroscio, *J. Appl. Phys.* **77 (2),** 657(1995).

Appendix – I

We first calculate the available states in k-space and then use the energy-momentum relation in parabolic bands to give the density of states in terms of energy. In the consideration of the density of states, the situation is complicated due to energy degeneracy. That is, that for some of the allowed energy levels, there are more than one possible combination of components in k-space that will give the same energy. In a quantum well, there is only one restricted energy level, therefore, the degeneracy is always 1 (not taking into account the electron intrinsic angular momentum spin). In a quantum wire, the degeneracy depends on the values of two sets of energy levels. While for a quantum dot there are three sets of discrete energy levels. This is only valid if and only if,

$$k_x = \frac{2\pi n_x}{L} \quad k_y = \frac{2\pi n_y}{L} \quad k_z = \frac{2\pi n_z}{L}$$

where n_x, n_y, n_z are integers.

With this restriction in k-space, only certain values of k-space lead to acceptable electron wave-functions solutions. K-space would be filled if each position was filled with a cubic unit cell of volume. Explicitly, the volume of k-space would be:

$$V_{3D} = \left(\frac{2\pi}{L}\right)^3 \tag{a}$$

The problem of finding the number of allowed states amounts to finding the number of these allowed states between spheres of radius **k** and **k** +d **k.** In bulk, the volume between the two shells is given by:

$$V_{3D}\, dk = 4\pi |k|^2\, dk \tag{b}$$

Therefore, the number of states is given simply by dividing this volume by the volume of a single energy state. At this point it is convenient to introduce an additional factor of two to account for the intrinsic angular momentum of the electrons or spin states. The number of states is given as

$$g(k)_{3D} = 2\times\frac{V_{s3D}dk}{V_{3D}} = \frac{|k|^2\,dkL^3}{\pi^2} \qquad \text{(c)}$$

To obtain the density of states per unit volume in terms of the energy we must find a relation between E and **k**,

$$|k| = \sqrt{2mE/\hbar^2} \qquad \text{(d)}$$

The differentiation of $|\mathbf{k}|$ with respect to energy is given as .

$$dk = \left(\frac{2mE}{\hbar^2}\right)^{-1/2}\frac{m}{\hbar^2}dE \qquad \text{(e)}$$

Taking the bulk case as an example, the density of states in terms of energy is then

$$g(E)_{3D}\,dE = \frac{k^2dk}{\pi^2} = \frac{2mE}{\pi^2\hbar^2}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\frac{m}{\hbar^2}dE = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E^{1/2}dE \qquad \text{(f)}$$

This gives the density of states per unit volume per unit energy at a wave vector-k.

Appendix – II

Proof: $$\left|A_\beta\right|^2 = \frac{\hbar}{2\rho\omega_\beta\Omega}(N_\beta + \tfrac{1}{2} \pm \tfrac{1}{2})$$

Quantum mechanically, the vibrational energy is

$$E = (N + \tfrac{1}{2})\hbar\omega \qquad \text{(i)}$$

Where , N=0,1,2………

Now the kinetic energy of the vibration is

$$KE = \tfrac{1}{2}M\left|\frac{du}{dt}\right|^2 \qquad \text{(ii)}$$

where M is the mass of the oscillator. Further the lattice displacement with wave vector β is:

$$u_\beta = A_\beta e^{i(\beta z-\omega t)} + A_\beta^* e^{-i(\beta z-\omega t)}$$

$$= 2\left|A_\beta\right|\cos(\beta z - \omega_\beta t + \phi_\beta) \qquad \text{(iii)}$$

$$\frac{du_\beta}{dt} = 2\left|A_\beta\right|\omega_\beta$$

$$\left|\frac{du_\beta}{dt}\right|^2 = 4\left|A_\beta\right|^2\omega_\beta^2$$

$$KE = \tfrac{1}{2}M(4\left|A_\beta\right|^2\omega_\beta^2)$$

$$E = 2\rho\Omega\left|A_\beta\right|^2\omega_\beta^2 \qquad \text{(iv)}$$

Comparing (i) and (iv) we have,

$$(N+\tfrac{1}{2})\hbar\omega_{\beta} = 2\rho\Omega\left|A_{\beta}\right|^{2}\omega_{\beta}^{2}$$

Therefore

$$(N+\tfrac{1}{2})\hbar\omega_{\beta} = 2\rho\Omega\left|A_{\beta}\right|^{2}\omega_{\beta}^{2}$$

$$\left|A_{\beta}\right|^{2} = \frac{(N+\tfrac{1}{2})\hbar\omega_{\beta}}{2\rho\Omega\omega_{\beta}^{2}} \qquad \text{(v)}$$

which can be written as

$$\left|A_{\beta}\right|^{2} = \frac{N_{\beta}\hbar}{2\rho\Omega\omega_{\beta}} \qquad \text{; Absorption}$$

$$= \frac{(N_{\beta}+1)\hbar}{2\rho\Omega\omega_{\beta}} \qquad \text{; Emission}$$

Or more generally,

$$\left|A_{\beta}\right|^{2} = \frac{\hbar}{2\rho\omega_{\beta}\Omega}(N_{\beta}+\tfrac{1}{2}\pm\tfrac{1}{2})$$

Appendix - III

$$1-\left(\frac{p'}{p}\right)\cos\alpha = 1-\frac{\left(\vec{p}\right)\cdot(\left(\vec{p}\right)\pm\hbar\beta)}{p^2} = \pm\frac{\hbar\beta}{p}\cos\theta$$

Suppose $$\left|\vec{p}-\vec{p'}\right| = (p^2 - p'^2 - 2\vec{p}\cdot\vec{p}')^{1/2}$$

$$= p[1+(p'/p)^2 - 2\frac{(\vec{p})\cdot(\vec{p}')}{p^2}]^{1/2}$$

$$\approx p[1-2\frac{(\vec{p})\cdot(\vec{p}')}{p^2}]^{1/2}$$

$$=[1-2\frac{(\vec{p})\cdot(\vec{p}')}{p^2}]^{1/2}$$

$$=[1-\frac{(\vec{p})\cdot(\vec{p}')}{p^2}]$$

$$\therefore \frac{p}{p}-\frac{p'}{p}\cos\alpha = [1-\frac{(\vec{p})\cdot(\vec{p}')}{p^2}]$$

$$\therefore 1-\frac{p'}{p}\cos\alpha = [1-\frac{(\vec{p})\cdot(p\pm\hbar\beta)}{p^2}]$$

$$\therefore 1-\frac{p'}{p}\cos\alpha = 1-\frac{\vec{p}\cdot\vec{p}}{p^2}\pm\frac{\vec{p}\cdot\hbar\beta}{p^2}$$

$$=1-1\pm\frac{p\hbar\beta}{p^2}\cos\theta$$

$$\therefore 1-\frac{p'}{p}\cos\alpha = \pm\frac{\hbar\beta}{p}\cos\theta$$

where θ is the angle between **p** and **β.**

Appendix -IV

$$\beta^2_{\max} = \frac{p^2}{\hbar^2}\left[1 \pm \sqrt{1 \pm \hbar\beta \big/ E(p)}\right]^2$$

$$= \frac{p^2}{\hbar^2}\left[1 \pm \sqrt{\frac{E(p) \pm \hbar\omega_o}{E(p)}}\right]^2$$

$$= \frac{p^2}{\hbar^2}\left[1 \pm 2\sqrt{\frac{E(p) \pm \hbar\omega_o}{E(p)}} + \frac{E(p) \pm \hbar\omega_o}{E(p)}\right]$$

$$= \frac{p^2}{\hbar^2}\left[\{1 + 2\sqrt{\frac{E(p) \pm \hbar\omega_o}{E(p)}}(2 + \frac{E(p) \pm \hbar\omega_o}{E(p)})\}\right]$$

$$\cong \frac{2p^2}{\hbar^2}$$