APPENDIX C

C.1 Non-adiabatic Heating

In the thermodynamic energy equation (4.3.3), $\mathring{q}$ which represents the source of heat due to radiation and condensation, may be expressed as

$$\mathring{q} = \mathring{q}_r + \mathring{q}_c \qquad (C.1.1)$$

where $\mathring{q}_r$ is the heating due to radiation and $\mathring{q}_c$ is the heating due to condensation.

C.1.1 Heating due to radiation

The heating due to radiation $\mathring{q}_r$ may be separated into two components as

$$\mathring{q}_r = \mathring{q}_{rS} + \mathring{q}_{rL} \qquad (C.1.2)$$

where $\mathring{q}_{rS}$ is the heating due to the absorption of solar radiation and $\mathring{q}_{rL}$ that due to long wave radiation.

Heating due to solar radiation : Heating due to solar radiation consists of two components; one due to the absorption of direct solar radiation ($\mathring{q}_{rSd}$), and the other due to the absorption of the reflected (diffused) radiation ($\mathring{q}_{rSr}$).

$$\mathring{q}_{rS} = \mathring{q}_{rSd} + \mathring{q}_{rSr} \qquad (C.1.3)$$

The heating due to direct solar radiation is obtained by

$$\mathring{q}_{rSd} = \frac{g}{p_*} \frac{\text{day time}}{\text{whole day}} \cos\bar{\xi} \frac{\partial}{\partial \sigma} a^G (y_0 \sec\bar{\xi}) \qquad (C.1.4)$$

where a^G is the amount of absorption of solar radiation as a function of optical depth (y) and $\bar{\xi}$ is the mean zenith angle of the sun, which is defined as

$$\cos\bar{\xi} = \frac{\int_0^{\text{day time}} \cos\xi \, dt}{\int_0^{\text{day time}} dt} \qquad (C.1.5)$$

The heating due to reflected solar radiation is given by

$$\mathring{q}_{rSr} = -\frac{g}{p_*} \frac{R_e}{S} \frac{\partial}{\partial \sigma} a^G (1.66 y_*) \qquad (C.1.6)$$

where y_* is the total optical depth from the surface to a given σ-level. S is the solar constant ($= 2$ Ly min^{-1}) and R_e is the amount of reflected solar radiation.

Let $\alpha_1 (=0.2)$, $\alpha_2 (=0.5)$, and $\alpha_3 (=0.7)$ be the reflectives and n_h, n_m and n_l be the amounts of high, medium, and low clouds respectively. The amount of solar energy reflected by these clouds is

$$R_c = \frac{\text{day time}}{\text{whole day}} \cos\bar{\xi}\, S\left[\alpha_1 n_h + \alpha_2 (1-\alpha_1 n_h) n_m + \alpha_3 (1-\alpha_1 n_h)(1-\alpha_2 n_m) n_l\right] \qquad \text{--- } (C.1.7)$$

Since R_c does not contribute to heat up the earth's surface, it is assumed that R_c is reflected at the earth's surface and not at the top of each cloud. Hence, the total reflection at the earth's surface may be written as

$$R_c = \frac{\text{daytime}}{\text{whole day}} \cos\bar{\xi}\, \alpha_{*e}\left[S - a^T(y_{o,*}\sec\bar{\xi})\right] \qquad (C.1.8)$$

where α_{*e} is the effective albedo defined by

$$\alpha_{*e} = \alpha_* + \frac{\alpha_1 n_h + \alpha_2(1-\alpha_1 n_h)n_m + \alpha_3(1-\alpha_1 n_h)(1-\alpha_2 n_m)n_l}{1 - a^T(y_{o,*}\sec\bar{\xi})/S} \qquad (C.1.9)$$

In the above, α_* is the reflectivity of the earth's surface, $y_{o,*}$ is the optical depth from the top to the bottom of the atmosphere and a^T is the sum absorption due to water vapour, carbon dioxide, and ozone. The net solar radiation absorbed at the surface is

$$S_* = \frac{\text{daytime}}{\text{whole day}} \cos\bar{\xi}\,(1-\alpha_{*e})\left[S - a^T(y_{o,*}\sec\bar{\xi})\right] \qquad (C.1.10)$$

Heating due to long wave radiation: $\mathring{q}_{rL}$ in (C.1.2) may be computed from

$$\mathring{q}_{rL} = \frac{g}{p_*}\frac{\partial F}{\partial \sigma}$$

where the superscript G stands for water vapour (w) of carbon dioxide (c), or ozone (o). The net flux F^G

is the difference between the upward and downward fluxes.

$$F^G = U^G - D^G \qquad (C.1.12)$$

For water vapour, the upward and downward fluxes of radiation at any level σ are computed by (Godbole et al, 1970)

$$D_\sigma^W = \pi B_0 \tilde{e}_f(y_0, T_c) - \int_{B_0}^{B_\sigma} \tilde{a}_f(y_0, T)\pi dB - \int_{B_\sigma}^{B_0} \tilde{a}_f(y, T)\pi dB \qquad (C.1.13)$$

and

$$U_\sigma^W = \pi B_* - \int_{B_\sigma}^{B_*} \tilde{a}_f(y, T)\pi dB \qquad (C.1.14)$$

where

$$B(T) = \int_0^\infty B_\nu(T) d\nu = \frac{1}{\pi} \sigma T^4 \qquad (C.1.15)$$

$B_\nu(T)$ is the black body radiation of frequency ν and temperature T; $B_c = B(T_c)$, $B_0 = B(T_0)$, $B_\sigma = B(T_\sigma)$, and $B_* = B(T_*)$, where $T_c = 220°$ K, T_0 is the temperature at $\sigma = 0$, T_σ is the temperature at any given level σ, and T_* is the temperature of the earth's surface.

$y_0 = |u_\sigma - u_0|$, $y = |u_\sigma - u|$

where u_σ is the optical depth measured from the top of the atmosphere to any level σ.

The mean slab emissivity $\widetilde{e}_f(u,T)$ and the mean slab absorptivity $\widetilde{a}_f(u,T)$ are defined as (Yamamoto, 1952)

$$\widetilde{e}_f(u,T) = \frac{1}{B(T)} \int_0^\infty B_\nu(T)\left[1 - \tau_f(k_\nu, u)\right] d\nu \qquad (C.1.17)$$

$$\widetilde{a}_f(u,T) = \frac{1}{dB/dT} \int_0^\infty \frac{dB_\nu}{dT}\left[1 - \tau_f(k_\nu, u)\right] d\nu \qquad (C.1.18)$$

where $\frac{dB}{dT} = \int_0^\infty \frac{dB_\nu}{dT} d\nu$. τ_f is the transmission function of the slab, and k_ν is the absorption coefficient corresponding to frequency ν .

Carbon dioxide and ozone have relatively narrow absorption bands at 15μ and 9.6μ respectively, and hence the dependence of dB_ν/dT upon ν may be neglected. Under these conditions, the mean slab absorptivity in respect of these two gases may be defined as (Manabe et al, 1961)

$$\widetilde{E}_f(u,T) = \frac{1}{\nu_2 - \nu_1} \int_{\nu_1}^{\nu_2} \left[1 - \tau_f(k_\nu, u)\right] d\nu \qquad (C.1.19)$$

where T is equal to 218° K for carbon dioxide and 293° K for ozone. The downward and upward fluxes in respect of carbon dioxide and ozone take the form

$$D_\sigma^G = \pi b_0 \widetilde{E}_f(y_0, T) - \int_{b_\sigma}^{b_0} \widetilde{E}_f(y,T)\, \pi\, db \qquad (C.1.20)$$

$$U_\sigma^G = \pi b_* - \int_{b_\sigma}^{b_*} \widetilde{E}_f(y,T)\,\pi\,db \qquad (C.1.21)$$

where

$$b(T) = \int_{\nu_1}^{\nu_2} B_\nu(T)\,d\nu \qquad (C.1.22)$$

Optical depth : The optical depth for any absorbing gas in a layer of thickness $\Delta\sigma$ is obtained by

$$\Delta u = \frac{p_*}{g}\left(\frac{\sigma p_*}{p_0}\right)^k r\,\Delta\sigma \qquad (C.1.23)$$

where p_0 = 1000 mb. The values of k used for water vapour, carbon dioxide, and ozone are 0.6, 0.86, and 0.3 respectively (Manabe et al, 1961).

C.1.2 Condensation and convection

Dry convective adjustment : The required temperature change δT is obtained by the following two conditions.

$$\frac{\partial}{\partial\sigma}\theta(T+\delta T,\sigma) = 0 \qquad (C.1.24)$$

$$\frac{c_p}{g}p_*\int_{\sigma_T}^{\sigma_B} \delta T\cdot d\sigma = 0 \qquad (C.1.25)$$

where θ is the potential temperature, and σ_T and σ_B represent the values of σ at the top and the bottom of the unstable layer respectively.

Wet. convective adjustment : The required change in temperature δT and mixing ratio δr is obtained by satisfying the conditions

$$\frac{\partial}{\partial \theta}\,\theta_e\left(T+\delta T,\, r+\delta r,\, \sigma\right) = 0 \qquad (C.1.26)$$

$$r+\delta r = r_s\left(T+\delta T,\, \sigma\right) \qquad (C.1.27)$$

$$\frac{c_p}{g}\, p_* \int_{\sigma_T}^{\sigma_B} \left(\delta T + \frac{L}{c_p}\,\delta r\right) d\sigma = 0 \qquad (C.1.28)$$

where θ_e is the equivalent potential temperature.

Non-convective adjustment : The required change in mixing ratio δr may be obtained as

$$\delta r = r_s\,(1 - R.H.) + \left(\frac{\partial r_s}{\partial T}\right)_p \delta T \qquad (C.1.29)$$

where R.H. is the relative humidity and $\left(\frac{\partial r_s}{\partial T}\right)_p$ represents the rate of change of r_s with respect to T at constant pressure. The temperature change due to release of condensation heat is expressed as

$$\delta T = -\,\frac{L}{c_p}\,\delta r \qquad (C.1.30)$$

The amount of precipitation due to both convective and non-convective condensation is computed from the relationship

$$C = -\,\frac{p_*}{g} \int_0^1 \delta r \, d\sigma \qquad (C.1.31)$$