APPENDIX B

B.1 Effect of Friction

The frictional forces in the zonal (F_λ) and meridional (F_φ) direction and the forces governing diffusion of heat (F_T) and water vapour (F_r) in the case of zonally averaged motion may be expressed as

$$p_* F_\lambda = -\frac{\partial}{a\cos^2\varphi\,\partial\varphi}\left(\tau^{\lambda\varphi}\cos^2\varphi\frac{\partial\Phi}{\partial\sigma}\right)+\frac{\partial}{\partial\sigma}\left(\tau^{\lambda\varphi}\frac{\partial\Phi}{a\partial\varphi}\right)-g\frac{\partial\tau^{\lambda z}}{\partial\sigma} \quad (B.1.1)$$

$$p_* F_\varphi = -\frac{\partial}{a\cos\varphi\,\partial\varphi}\left(\tau^{\varphi\varphi}\cos\varphi\frac{\partial\Phi}{\partial\sigma}\right)+\frac{\partial}{\partial\sigma}\left(\tau^{\varphi\varphi}\frac{\partial\Phi}{a\partial\varphi}\right) - \frac{\tan\varphi}{a}\tau^{\lambda\lambda}\frac{\partial\Phi}{\partial\sigma}-g\frac{\partial\tau^{\varphi z}}{\partial\sigma} \quad (B.1.2)$$

$$p_* F_T = -\frac{\partial}{a\cos\varphi\,\partial\varphi}\left(\gamma^{\varphi}\frac{\partial\Phi}{\partial\sigma}\right)+\frac{\partial}{\partial\sigma}\left(\gamma^{\varphi}\frac{\partial\Phi}{a\partial\varphi}\right)-g\frac{\partial\gamma^{z}}{\partial\sigma} \quad (B.1.3)$$

$$p_* F_r = -\frac{\partial}{a\cos\varphi\,\partial\varphi}\left(\mu^{\varphi}\frac{\partial\Phi}{\partial\sigma}\right)+\frac{\partial}{\partial\sigma}\left(\mu^{\varphi}\frac{\partial\Phi}{a\partial\varphi}\right)-g\frac{\partial\mu^{\varphi}}{\partial\sigma} \quad (B.1.4)$$

where

$$\tau^{\lambda\lambda} = -p_* k_H\left(\frac{\sigma}{RT}\right)\frac{\tan\varphi}{a}\,v \quad (B.1.5)$$

$$\tau^{\lambda\varphi} = -p_* k_H\left(\frac{\sigma}{RT}\right)^2\left\{\frac{\partial\Phi}{\partial\sigma}\frac{\partial}{a\partial\varphi}\left(\frac{u}{\cos\varphi}\right)-\frac{\partial\Phi}{a\partial\varphi}\frac{\partial}{\partial\sigma}\left(\frac{u}{\cos\varphi}\right)\right\}\cos\varphi \quad (B.1.6)$$

$$\tau^{\varphi\varphi} = -p_* k_H\left(\frac{\sigma}{RT}\right)^2\left\{\frac{\partial\Phi}{\partial\sigma}\frac{\partial v}{a\partial\varphi}-\frac{\partial\Phi}{a\partial\varphi}\frac{\partial v}{\partial\sigma}\right\} \quad (B.1.7)$$

$$\gamma^{\varphi} = -p_* k_H \left(\frac{\sigma}{RT}\right)^2 \left\{ \frac{\partial \Phi}{\partial \sigma} \frac{\partial T}{a \partial \varphi} - \frac{\partial \Phi}{a \partial \varphi} \frac{\partial T}{\partial \sigma} \right\} \quad (B.1.8)$$

$$\mu^{\varphi} = -p_* k_H \left(\frac{\sigma}{RT}\right)^2 \left\{ \frac{\partial \Phi}{\partial \sigma} \frac{\partial r}{a \partial \varphi} - \frac{\partial \Phi}{a \partial \varphi} \frac{\partial r}{\partial \sigma} \right\} \quad (B.1.9)$$

$$\tau^{\lambda z} = -g p_* k_V \left(\frac{\sigma}{RT}\right)^2 \frac{\partial u}{\partial \sigma} \quad (B.1.10)$$

$$\tau^{\varphi z} = -g p_* k_V \left(\frac{\sigma}{RT}\right)^2 \frac{\partial v}{\partial \sigma} \quad (B.1.11)$$

$$\gamma^{z} = -g p_* k_V \left(\frac{\sigma}{RT}\right)^2 \frac{\partial T}{\partial \sigma} \quad (B.1.12)$$

$$\mu^{z} = -g p_* k_V \left(\frac{\sigma}{RT}\right)^2 \frac{\partial r}{\partial \sigma} \quad (B.1.13)$$

The symbols k_H and k_V denote respectively the lateral and vertical eddy diffusion coefficients. The value of k_H is determined from the expression (Smagorinsky et al, 1965)

$$k_H = 6 \times 10^4 \, (d/300)^{4/3} \quad m^2 sec^{-1} \quad (B.1.14)$$

where d denotes the grid size expressed in km. The expression for determining k_v is given in the next section.

B.2 The surface Boundary Layer

It is assumed that within the turbulent layer, the horizontal eddy stresses $\tau^{\lambda\lambda}$, $\tau^{\lambda\varphi}$, $\tau^{\varphi\varphi}$, γ^{φ}, and μ^{φ} are zero and that the vertical fluxes of momentum, heat and water vapour are constant, so that

$$\left.\begin{aligned} \frac{p_*}{\rho}\frac{\partial \tau^z}{\partial z} &= 0 \qquad (B.2.1)\\ \frac{p_*}{\rho}\frac{\partial \gamma^z}{\partial z} &= 0 \qquad (B.2.2)\\ \frac{p_*}{\rho}\frac{\partial \mu^z}{\partial z} &= 0 \qquad (B.2.3)\end{aligned}\right\} \quad z_* \leq z \leq z_* + h$$

where $\tau^{\lambda z} = \tau^z \cos\xi$, $\tau^{\varphi z} = \tau^z \sin\xi$, and $\xi = \tan^{-1}(v/u)$

ξ is assumed to be constant throughout the turbulent layer. z_* whose numerical value is zero, denotes ground surface and h denotes the thickness of the turbulent layer which is assumed constant at 85 meters irrespective of latitude.

Based upon the mixing length hypothesis as applied to the boundary layer, the fluxes of momentum, heat, and water vapour are computed by

$$\tau^z = \rho k_v \frac{\partial U}{\partial z} \qquad (B.2.4)$$

$$\gamma^z = \rho k_v \frac{\partial T}{\partial z} \qquad (B.2.5)$$

$$\mu^{z} = \rho k_v \frac{\partial r}{\partial z} \qquad (B.2.6)$$

where $U = \sqrt{u^2+v^2}$, and

$$k_v = l^2 \frac{\partial U}{\partial z} \qquad (B.2.7)$$

where l is the mixing length. The vertical variation of l is expressed as (smagorinsky et al, 1965)

$$l = k_0 (z + z_0), \qquad z < h$$

$$l = k_0 (h + z_0) \frac{H - z}{H - h}, \qquad h \leq z \leq H$$

$$l = 0, \qquad H \leq z$$

k_0 is the karman constant and z_0 is the roughness parameter.

Assuming that ρ is constant within the turbulent layer and is equal to the surface density the solutions of (B.2.4) through (B.2.6) can be obtained as

$$U = \frac{U_f}{k_0} \ln \frac{z + z_0}{z_0} \qquad (B.2.8)$$

$$T = T_* + \frac{T_f}{k_0} \ln \frac{z + z_0}{z_0} \qquad (B.2.9)$$

$$r = r_* + \frac{r_f}{k_0} \ln \frac{z + z_0}{z_0} \qquad (B.2.10)$$

where T_* is the surface temperature and r_* is the mixing ratio of water vapour at the earth's surface.

U_f, T_f, and r_f are respectively the velocity, temperature, and mixing ratio obtained by scaling due to friction and are defined as

$$U_f^2 = \tau^z/\rho_* \quad , \quad U_f T_f = \gamma^z/\rho_* \quad , \quad U_f r_f = \mu^z/\rho_* \qquad (B.2.11)$$

U_f, T_f, and r_f are determined from the condition of continuity of vertical derivaties of U, T, and r at the top of the turbulent layer. As shown in Figure 4.2.1 (B) and also in Table A.1 of Appendix A, the top of the turbulent layer coincides with the level "$8\frac{1}{2}$". Equations (B.2.8) through (B.2.10) may be written in difference form to express the relation of continuity between the turbulent layer and the layer above it.

$$\frac{U(8) - U(8\frac{1}{2})}{\Delta z} = \frac{U_f}{k_c}\frac{1}{h+z_0} \qquad (B.2.12)$$

$$\frac{T(8) - T(8\frac{1}{2})}{\Delta z} = \frac{T_f}{k_c}\frac{1}{h+z_0} \qquad (B.2.13)$$

$$\frac{r(8) - r(8\frac{1}{2})}{\Delta z} = \frac{r_f}{k_0}\frac{1}{h+z_c} \qquad (B.2.14)$$

where Δz is the height difference between level "8" and level "$8\frac{1}{2}$". The right-hand-side refers to the turbulent layer while the left-hand-side refers to the level above it. From equations (B.2.8) through (B.2.14) and substituting h for z, it follows that

$$U_f = m k_0 U(8) \tag{B.2.15}$$

$$T_f = m k_0 (T(8) - T_*) \tag{B.2.16}$$

$$r_f = m k_0 (r(8) - r_*) \tag{B.2.17}$$

where

$$m = \left[\ln \frac{h+z_0}{z_0} + \frac{\Delta z}{h+z_0} \right]^{-1} \tag{B.2.18}$$

and

$$\tau^z = \rho_* k_0^2 m^2 U(8)^2 \tag{B.2.19}$$

$$\gamma^z = \rho_* k_0^2 m^2 U(8)(T(8) - T_*) \tag{B.2.20}$$

$$\mu^z = \rho_* k_0^2 m^2 U(8)(r(8) - r_*) \tag{B.2.21}$$

$$U(8\tfrac{1}{2}) = m U(8) \ln \frac{h+z_0}{z_0} \tag{B.2.22}$$

$$T(8\tfrac{1}{2}) = m \left(T(8) \ln \frac{h+z_0}{z_0} + T_* \frac{\Delta z}{h+z_0} \right) \tag{B.2.23}$$

$$r(8\tfrac{1}{2}) = m \left(r(8) \ln \frac{h+z_0}{z_0} + r_* \frac{\Delta z}{h+z_0} \right) \tag{B.2.24}$$

For calculating $\tau^{\lambda z}$ and $\tau^{\varphi z}$ from τ^z, ξ is first determined from the condition of continuity of the ratio $(\partial v/\partial z)/(\partial u/\partial z)$ for the turbulent layer and the layer above it, so that

$$\frac{v(8) - v(8\frac{1}{2})}{u(8) - u(8\frac{1}{2})} = \tan \xi \qquad (B.2.25)$$

Due to the assumption that ξ is constant throughout the turbulent layer, it follows that

$$\frac{v(8\frac{1}{2})}{u(8\frac{1}{2})} = \tan \xi \qquad (B.2.26)$$

and, therefore,

$$\frac{v(8)}{u(8)} = \tan \xi \qquad (B.2.27)$$

Finally,

$$\tau^{\lambda z}(8\tfrac{1}{2}) = \rho_* k_o^2 m^2 U(8) u(8) \qquad (B.2.28)$$

$$\tau^{\varphi z}(8\tfrac{1}{2}) = \rho_* k_o^2 m^2 U(8) v(8) \qquad (B.2.29)$$

$$u(8\tfrac{1}{2}) = m \ln \frac{h+z_o}{z_o} u(8) \qquad (B.2.30)$$

$$v(8\tfrac{1}{2}) = m \ln \frac{h+z_o}{z_o} v(8) \qquad (B.2.31)$$

$$K_v(8\tfrac{1}{2}) = m k_o^2 (h+z_o) U(8) \qquad (B.2.32)$$